What Do “Filling Factors” of Wind X-Ray Sources Tell Us?

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Abstract. We discuss the concept of volume filling factors of X-ray emitting material in the atmospheres of hot early-type stars. The spatial distribution of material is described by an irregularity factor. Under specific assumptions, the irregularity factor is proportional to the ratio of the volumes filled by hot and cool material. By determination of lower limits on volume filling factors it is possible to put constraints on parameters of the wind fine structure.

1. X-ray emission of hot early-type stars

It is widely accepted now that the acceleration of high mass-loss WR winds is due to multi-line scattering of photons (e.g. Gayley et al. 1995). According to a phenomenological model (e.g. Lucy 1982), as a consequence of the dynamical instabilities of the line driven stellar wind, shocks are generated throughout the outflow. Gayley & Owocki (1995) have shown that the instability mechanism that leads to shock formation in the relatively low mass-loss OB star winds should still operate in the dense WR winds. The formation of extensive wind structures, including strong clumping, can be expected. Model computations predict shock velocity-jumps ranging from 500 km s\(^{-1}\) to 1000 km s\(^{-1}\), implying post-shock temperatures adequate for the production of the observed X-ray emission.

The first detection of an O star as a discrete X-ray emitter was made by the EINSTEIN (0.2–4.0 keV) mission (Seward et al. 1979). One of the most intriguing features of these X-ray detections was the discovery that the observed X-ray flux from O stars is roughly proportional to the stellar bolometric luminosity, \(L_X \approx 10^{-7} L_{\text{bol}}\) (e.g. Seward & Chlebowski 1982). Such scaling does not follow from detailed simulations of the line-driven instability (Owocki & Cohen 1999). Subsequent observations, the most robust by ROSAT (0.2–2.4 keV), confirmed this scaling result (Kudritzki et al. 1996). The ROSAT data have also revealed that, unlike their O-star progenitors, the X-ray luminosities of single WR stars are not correlated with their bolometric luminosity \(L_{\text{bol}}\), wind momentum \(\dot{M} v_{\infty}\), wind kinetic luminosity \(0.5 \dot{M} v_{\infty}^2\), or WR subtype (e.g. Ignace & Oskinova 1999).
Understanding the origins and properties of X-ray emission from stellar winds demands that we quantify accurately and meaningfully the physical properties (density, temperature, etc.) of the emitting material. The diagnostic study of such properties has recently become possible for stellar plasmas, especially with the launch of new generation X-ray observatories (e.g., Chandra and XMM). In parallel with these observational advances, there has been intensive theoretical modeling of small scale structures in stellar winds. One common feature in modeling different aspects of variable spectra of early type stars is considering the mass outflow as a superposition of a steady spherically symmetric wind and density (and temperature) fluctuations. We apply this approach and consider a spherically symmetric and time independent stellar wind permeated by radiatively driven shocks. The shock heated material is assumed to be distributed within the atmosphere in the form of optically thin filaments embedded in the ambient steadily expanding cold wind which is opaque to X-ray emission and described by standard wind models. To avoid reference to the density and volume of heated material, the parameter \( f_X \) is introduced as the ratio of emission measures of the hot and cold components of the outflow:

\[
f_X = \frac{EM_X}{EM_w}.
\]

This quantity is the only spectroscopically observable property describing the “amount” of matter in hot and cool components of the stellar wind. By knowing \( f_X \) one cannot separately infer the density enhancement of X-ray emitting material or the fraction of the volume filled by hot plasma, but only the product of these two.

In absolute terms the density of a plasma may be written as

\[
\rho = \mu_e m_H n_e,
\]

where \( n_e \) is the electron number density, \( \mu_e \) is the mean mass per free electron and \( m_H \) is the mass of a proton. This depends on the abundances and ionisation stage of the elements. For the steady cold component of the outflow, mass conservation leads to the continuity equation:

\[
4\pi r^2 \rho v = \dot{M},
\]

where \( \dot{M} \) is the mass-loss rate through an entire spherical surface and \( v \) denotes the velocity in the radial direction. For electron number densities \( n_X \), \( n_w \) of hot and cold material within two components of total volumes \( V_X \), \( V_w \) respectively, the parameter \( f_X \) is

\[
f_X = \frac{\langle n_X^2 \rangle}{\langle n_w^2 \rangle} \frac{V_X}{V_w}.
\]

Under the assumption that \( \mu_e \) and the density ratio \( (\rho_X/\rho_w = D_s) \) of shocked and steady components are constant in space we may rewrite (4) in the form

\[
f_X = D_s^2 \left( \frac{\mu_w}{\mu_e} \right)^2 \frac{V_X}{V_w}.
\]
This is sometimes referred to in the literature as “the volume filling factor” though it also includes a dependence on $D_s$ and can, for large $D_s$, be greater than unity.

Proposed explanations of observed properties of X-ray emission of hot stars involve special assumptions about the distribution of heated material described by the filling factor $f_X$. A radius-dependent filling factor allows us to explain the correlation of the X-ray and bolometric luminosities for OB-stars according to Owocki & Cohen (1999). A density-dependent filling factor allows us to explain the lack of correlation between the X-ray luminosity and other parameters of WR stars (Ignace & Oskinova 1999). The assumption $f_X \sim (M/v_\infty)^{-1}$ is not inconsistent with the ROSAT observations (Ignace et al. 2000). In this case, because of mutual cancellation of emission and absorption density dependent factors, a lack of correlation between $L_X$ and wind parameters would be expected. Thus, the emergent X-ray emission of WR stars depends on factors relating to relative abundances and ionisation only.

2. The spectrum of an optically thin inhomogeneous hot plasma.

Thermal X-ray emission arises from the decay of collisionally excited levels to lower levels of atoms or to free-free scattering. Therefore at X-ray energies, the emission coefficient scales as the square of the plasma density.

For such a plasma the emission from its total unocculted volume $V$ is

$$\int_V \frac{dL_\nu}{dV} \, dV = \int_V \frac{1}{(\mu_em_H)^2} \rho^2(r) \Lambda_\nu(T) \, e^{-\tau_\nu(r)} \, dV \quad \text{[erg s}^{-1}], \quad (6)$$

where the function $\Lambda_\nu(T)$ denotes the theoretical spectral distribution function for an isothermal source element. Detailed computations of $\Lambda_\nu(T)$ have been made (e.g., Raymond & Smith 1978) on the assumption of steady collision ionisation equilibrium and of solar element abundances. The optical depth $\tau_\nu(r)$ is along the line of sight from the point $r$.

As pointed out by Craig & Brown (1976), since the source is not spatially resolved and since the temperature distribution is convolved with $\rho^2$, it is clearly impossible to deconvolve the triple integral in eqn. 6 to get $T(r)$ and $\rho(r)$ from spectral data alone. However, for a source with a continuous spatial distribution of temperature $T(r)$ it is feasible to derive a weighted distribution function for the source, namely the emission measure differential in temperature. This concept has been used widely and eqn. 6 is commonly rewritten as

$$L_\nu = \int_T \text{DEM}(T) \Lambda_\nu(T) \, e^{-\tau_\nu(T)} \, dT. \quad (7)$$

The proper definition of the emission measure differential in temperature requires the volume differential in a system which involves coordinates in and perpendicular to surfaces $S_T$ of constant temperature. If $s$ is a coordinate along the local normal to $S_T$ at $r$ then one can write

$$dV = dS_T \, ds = dS_T \frac{dT}{|\nabla T|}$$
and the precise meaning of DEM(T) in eqn. 7 becomes

\[
DEM(T) = \int \int_{S_T} n_e^2(r) \frac{dS_T}{\nabla T},
\]

(8)

where \( \int \int_{S_T} \) includes a summation over all disjoint surfaces \( S_T \) at the same \( T \), (such as over multiple independent shocks in a stellar wind).

Due to the poor X-ray spectral information for early-type stars available till now it was sufficient to use simplified models of the temperature structure of X-ray emitting plasma in stellar winds instead of DEM(T). In Cohen et al. (1996), a power law was used for the differential emission measure on a temperature interval \( T = [T_1 : T_2] \) to fit EUV/X-ray spectral data for a B star. This parameterization is a generalisation of a two-temperature model, in which case

\[
DEM(T) = EM_1 \delta(T - T_1) + EM_2 \delta(T - T_2),
\]

(9)

where \( \delta \) is the \( \delta \)-function and

\[
[\mu_{e,1} m_H]^2 EM_{1,2} = \int_{V_{1,2}} \rho_{1,2}^2(r) e^{-\tau_{v_{1,2}}(r)} dV,
\]

(10)

where \( \rho_{1,2}(r) \) and \( V_{1,2} \) are the densities and total volumes of the two components of hot plasma with different temperatures, respectively.

Although no reasonable models predict the existence of isothermal X-ray emitting sources, the quality of data sometimes allows us to neglect a temperature distribution in the X-ray emitting material (e.g. Owocki & Cohen 1999). By using only one term in eqn. 9 with temperature \( T_2 = T_X \gg T_1 \) and combining eqn. 7 and eqn. 9, the X-ray spectrum becomes

\[
L_\nu(E) = f_X EM_\nu \Lambda_\nu(E, T_X).
\]

(11)

Equation 11 allows us to obtain spectra of material heated to temperature \( T_X \) by referring to the density of the cool material via the filling factor \( f_X \).

3. Irregularity factor

Allen (1973) defined a quantity describing the irregularity in the spatial distribution of gaseous material — the “irregularity factor \( \mathcal{X} \)” for the particle number density \( n \) given by \( \mathcal{X} = \langle n^2 \rangle / \langle n \rangle^2 \) and \( \langle n \rangle = \int_V n dV / \int_V dV \). By definition, the irregularity factor is unity for a medium with uniform density and exceeds unity for an inhomogeneous medium. We can see from eqn. 11 precisely where the irregularity factor \( \mathcal{X} \) comes from. Let us consider a source of total volume \( V_{tot} \) containing \( N = \langle n \rangle V_{tot} \) particles. The material in the atmospheres of early-type stars is highly ionized; therefore, we use for estimation of density irregularities the number density of electrons only. First of all let us assume that the total volume \( V_{tot} \) is filled by material with uniform density. Then the total emission measure of the whole volume is \( EM_{tot} = \langle n_e \rangle^2 V_{tot} = N^2 / V_{tot} \).

If instead the particles are all concentrated in a subvolume \( \Delta V \) (\( \Delta V < V_{tot} \)) so that the rest of the total volume is void, then the emission is enhanced by
factor $\mathcal{X}$ compared with $EM_{\text{tot}}$. In this special case the irregularity factor is $\mathcal{X} = V_{\text{tot}}/\Delta V > 1$. The inverse quantity

$$\mathcal{F} = \frac{1}{\mathcal{X}} \approx \frac{(n_e)^2}{\langle n_e^2 \rangle} \approx \frac{\Delta V}{V_{\text{tot}}}$$

(12)

is the fraction of the volume filled by material. Note that the relation 12 between $\mathcal{F}$ and irregularity factor $\mathcal{X}$ is applicable if the density of the small fraction of the volume $\Delta V$ is much bigger than the density of the surrounding medium (i.e. most particles are concentrated in $\Delta V$).

Thus we describe three dimensionless variables which carry information about the distribution of the material in a stellar wind:

1. $f_X$ – ratio of hot to cold emission measure. The only directly observable diagnostic for a spatially unresolved source which reflects the special model assumption of a two-component atmosphere structure.

2. $\mathcal{X}$ – irregularity factor which reflects the enhancement of emission in a clumped medium as compared with the case of a uniform density distribution of the same matter.

3. $\mathcal{F}$ – fraction of the volume filled by the material if the rest of the volume is void.

It is important to clearly understand that the meaning of $f_X$ lies in the assumption of a two (shocked isothermal hot and steady cool) component medium and has physical sense as a consequence of the neglect of temperature gradients in eqn. 7. By performing exact integration of eqn. 7 over temperature for radiatively cooling shocks, the ratio $f_X$ of emission measures of isothermal hot medium and cool medium does not provide precise information about the volume filled by hot gas with temperature $T_X$, since $T_X$ is not unique any more.

On the other hand, parameters $\mathcal{X}$ and $\mathcal{F}$ describe spatial distributions of the material as measures of the clumpiness in the medium without any assumptions about the temperature structure. Under a plausible assumption of a significant density jump between shocked and steady material, parameter $\mathcal{F}$ is the fraction of the volume filled by shocked material.

Let us next express the X-ray luminosity using the irregularity factor $\mathcal{X}$ to describe the distribution of hot material in an X-ray emitting source. In equation 11 we will account for the attenuation using the exospheric approximation (Owocki & Cohen 1999; Ignace et al. 2000). In this approximation the observed X-ray emission arises from the hot gas only from radii exterior to the surface of optical depth unity with radius $r_1$, with X-rays at smaller radii taken to be completely attenuated. So, one should regard the volume in eqn. 11 as being for the hot material above the surface of radius $r_1$. Under the assumptions we have made about an isothermal X-ray emitting source and for a fixed amount of emitting material:

$$L_\nu = \frac{1}{(\mu_e^X m_H)^2} \mathcal{X} \langle \rho_X \rangle^2 \Lambda(E, T_X) V_{\text{tot}},$$

(13)
where $\langle \rho_X \rangle$ is over $V_{tot}$. One may see that larger irregularity factors provide higher X-ray luminosities. This just reflects the fact that the emission measure is proportional to the square of the density. So concentrating material in small volumes increases the emission measure. Although clumpiness of the medium can be inferred, snapshot spectra can provide only limited information about the fine structure of the source of the X-ray emission.

Nevertheless, though no realistic hydrodynamical models predict the existence of isothermal shocks, one may make use of the isothermal approximation. Maximizing the temperature of the hot material and its amount (by assuming that all the X-ray emitting material is heated to maximum possible and/or detectable temperatures), lower limits to the volume filling factor can be derived (e.g., as in Ignace et al. 2000). Such lower limits on volume filling factors put constraints on the irregularity factors for the two-component medium as well as on $\mathcal{X}$ for the atmosphere with radiatively cooling shocks. Using the definitions of $\mathcal{X}$ and $f_X$ the relation between these quantities in a two-component approximation may be written as $\mathcal{X}(\rho_X)^2 V_X = f_X E M_w$.

Thus, we may conclude that the widely used concept of “volume filling factor” $f_X$ in the literature does not bear unique information about the fractional volume filled by hot material and its spatial distribution except under special assumptions about the density and temperature. If the density and temperature of the hot gas do not depend on the location inside the source then a lower limit on the volume filling factor can be derived by maximizing the density ratio and temperature of a source. This information can be used to put constraints on the irregularity factor $\mathcal{X}$. Irregularity factors are a measure of the inhomogeneity of the medium and under special assumptions are inverse to the ratio of the volumes of shocked and embedded material. To get further insight into the structure of the wind we need additional information such as provided by the detection of time variability of X-ray flux (Oskinova et al. 2000).

References

Hillier, D. J., Kudritzki, R. P., Pauldrach, A. W., Baade, D., Cassinelli, J. P.,
Seward, F. D., Forman, W. R., Giacconi, R., Griffiths, R.E., Harnden, F. R.,
Discussion

Sergey Marchenko: What kind of chemical composition did you use for the cooling function?

Lidia Oskinova: A Raymond-Smith cooling function corrected for non-solar abundances.

Sergey Marchenko: ... different for WC and WN?

Lidia Oskinova: Yes.

Gregor Rauw: What's the typical time scale of the variability?

Lidia Oskinova: Well, nobody knows of course - it's not observed. [Laughter.] We don't model any time scales.

Gregor Rauw: It could be anything?

Lidia Oskinova: No. The point is that if we get observations we would be able to estimate a characteristic time for the variability. It will tell us a lot of information about the shock distribution.

Allan Willis: The observations you looked at were the sky-survey observations. There are a few pointed ROSAT observations of the same star looking for variability and I can think of at least one case where significant variability has been seen over time scales of a few hours up to a few days and at a level of variability of ~30%, which is much bigger incidently than the amplitude on your plot.

Lidia Oskinova: I was just trying to illustrate that the star was variable. But that star, WR6, is a very close binary ...

Allan Willis: No, it's not a binary.

Lidia Oskinova: Oh, I'm sorry.

Sergey Marchenko: That is very controversial. [Laughter.]

Lidia Oskinova: The variability I'm talking about here is rather on a much shorter time scale of ~ hours; it is stochastic, not periodic.

Allan Willis: The variability is not periodic; it is stochastic and it is 30%. And there's no change in the hardness ratio in the variability. That's why it's not a binary.

Svet Zhekov: It doesn't cover the whole so-called period of 3.7 days.

Allan Willis: Yes it does.

Tony Moffat: So then what is the general level of variability that you predict for WR stars?

Lidia Oskinova: I would say that, if we would assume that it is these shocks propagating here at constant velocity, i.e. $v_\infty$, and they will, let's say, emit in a flow time of ~ hours, the variability should be ~10%.

Tony Moffat: We should have seen that by now.
Watson, Sherlock's faithful assistant on the search for fingerprints in the dust