Extraction of CMB Polarization B and E Modes

Tzihong Chiueh, Cheng-Jiun Ma, and Huichun Lin

Department of Physics, National Taiwan University, Taipei 106, Taiwan

Abstract. The CMB polarization B and E modes can be clearly conceived in a projected space, where the polarization tensor is projected into a genuine vector, the G-vector, that contains a divergence-free and curl-free component. Each component exhibits easily recognizable B or E-mode pattern. Extraction of B and E modes can be conducted either from the G-vector through integration along an arbitrary closed contour of the Stokes Q and U map, or by a modified method involving contour integration of the measured Stokes Q and U along a circle. The advantages of the latter method include that no spatial derivative on the measured data is needed, and that it can be conveniently implemented for the dual-polarization receivers that are mounted on a turntable. AMiBA has dual-polarization receivers and is also mounted on a turntable platform; when adopting this observing strategy, the expected performance of AMiBA is presented.

1. Introduction

Linear polarization is described by a $2 \times 2$ tensor, which assumes different values at different parts of the sky, thus forming angular patterns. The CMB fluctuations contain linear polarization patterns of two different parities: a tensor pattern (E-mode) and a pseudo-tensor pattern (B-mode) (Kamionkowski, Kosowsky, & Stebbins 1997; Zaldarriaga & Seljak 1997; Hu & White 1997). The two modes have different origins and can thus be used to probe different physics at the epoch of photon-electron decoupling near $z \sim 1100$. Both polarization modes are caused from the Thomson scattering of anisotropic CMB photons. However, for the E mode, the photon anisotropy is given by the scalar-mode primordial density fluctuations, whereas for the B-mode, the photon anisotropy is caused by the primordial gravitational waves. Therefore, detection of B modes has an obvious profound implication.

In addition, the two kind of polarization patterns reveal themselves on their own favored angular scales. The E-mode is well-correlated with the CMB primary anisotropy; being a tensor, it has an angular pattern a few times finer than that of the scalar primary anisotropy, and peaks at the multipole mode number $l \sim 600 - 1200$. (It manifests the fact that the most symmetric pattern for a scalar is a monopole pattern, but can at best be a quadrupole pattern for a tensor.) On the other hand, the B-mode peaks at $l \sim 200$, since the primordial gravitational waves have a power spectrum weaker, on small scale, than the primordial scalar fluctuations. Partially due to the smaller phase space area
at low \( l \), and partially due to the weaker intrinsic fluctuations, the B mode is expected to have a smaller overall amplitude, about 10% of E mode. It makes the B-mode power about 1% of E-mode, which is in turn about 1% of the primary anisotropy. This small amplitude thus poses a great challenge for any measurement that aims to detect B modes.

2. E and B Modes through G-vector

How does one measure the tensor E-pattern or pseudo-tensor B-pattern? The convenient way to visualize the two polarization patterns is not to construct the conventional polarization-"vector" \( \mathbf{p} \), whose strength is \((Q^2 + U^2)^{1/4}\) and the alignment, not the direction, along the angle \((1/2)\tan^{-1}(Q/U)\), from the Stokes \( Q \) and \( U \). Instead, one may construct a genuine vector \( \mathbf{G} \), which has a well-defined direction, by taking a spatial derivative on the Stokes \( Q \) and \( U \). The spatial pattern of \( \mathbf{G}(x, y) \) on the sky contains a curl-free vector component and a divergence-free pseudo-vector component, with the former arising solely from the E-mode polarization and the latter only from the B-mode polarization. In Chiuhe (2000) and Chiuhe & Ma (2000), the \( G \)-vector has been derived and is found to be

\[
\mathbf{G} = \left( \frac{\partial Q}{\partial x} + \frac{\partial U}{\partial y} \right) \hat{x} + \left( \frac{\partial U}{\partial x} - \frac{\partial Q}{\partial y} \right) \hat{y}
\]

A 1deg\(\times\)1deg, \( G \)-vector pattern for the CMB E-mode polarization is shown in Fig.(1). For the purpose of illustration, on may rotate the E-mode \( G \)-vector by 90 degrees, either clockwise or anti-clockwise, to obtain the \( G \)-vector pattern for B mode.

In the polarization measurement that aims to detect B modes, one should make full use of the different geometric natures and angular scales of the two patterns so as to isolate the weaker B mode from the stronger E mode. Although the E and B modes can be naturally separated in the Fourier space and seem to be well-suited for interferometric measurements, it turns out that the wavenumber of the polarization patterns must be finely resolved in the conventional interferometric measurements in order to achieve this goal. Such a task is in fact impractical, as all measurements always collect fluctuation power within a finite unresolved Fourier band. Here we report a way to conduct a clean B-mode measurement and furthermore access how AMiBA may perform for such a B-mode measurement.

Due to their different geometrical natures, the E and B-mode polarization signals can be separately measured via a topological method. Much like how one may measure a remote electric charge, or current, by measuring the electric, or magnetic, field around the source, one can adopt a similar procedure to measure the E and B modes. Since the sky patterns are two-dimensional, the topological measurements is especially simple: one measures the component of \( \mathbf{G} \) parallel to any arbitrary closed contour to obtain the B mode and the component perpendicular to the contour to obtain the E mode. This topological measurement obtains the total strength of source into the measurement contour, and therefore is a real-space measurement. Despite its being a real-space measurement, the expected signal strength can be evaluated from the standard polarization power spectra generated by the CMBFAST, by integrating the expected power spectra.
weighed by the corresponding $l$-space filter function of the real-space contour (Chiueh 2000; Chiueh & Ma 2000).

3. Measuring E and B Modes through Circular Scans

There can, however, be a potential drawback for the measurement of B and E modes by means of the $G$-vector, due to its need for differencing the data of adjacent pixels. To circumvent this problem, one may need to adopt a slightly different method, which avoids data differencing. It turns out that this is a possible task. The idea is that to project the polarization tensor into a vector, it is not necessary to use spatial derivative, $\nabla$, to provide a local direction onto which the tensor is projected. Instead, one may choose to use a position vector $\mathbf{r}$ to provide the direction, onto which the tensor is projected. However, this position vector cannot be arbitrary. It has been shown that such a vector
must be the position of a sky circle, along which the Stokes $Q$ and $U$ are measured (Chiuheh & Ma 2001). That is, the sky contour can no longer be any arbitrary one, and it must be a circle for this method to work properly.

In fact, the E and B-mode measurements can be conducted in an even more convenient manner than the measurements using $G$-vector, when the following circular scan strategy is adopted. The detector continuously scans around a fixed sky circle and integrates the CMB signal without readout. In the mean time, the two polarized receivers in the detector should always be aligned in the local radial and azimuthal directions and measure the local Stokes $Q$ and $U$ in this rotating frame. The integrated $Q$ is contributed entirely by the E mode and $U$ by the B mode. A detector mounted firmly on a turntable and observes a sky circle centering around the rotation axis of the turntable will perform the task! The measured signal is actually the quadrupole-moment of polarization fluctuations on the sky circle. Higher multipole-moments of polarization fluctuation on this circle cannot be separated in such a clean manner (Zaldarriaga 1998); that is, B modes of higher multipole moments can be heavily contaminated by E modes. Hence, for investigations on the angular-scale dependence of B modes, it is helpful to do separate observations on circles of different radii, instead of extracting the higher-moment modes from the same circle.

4. Beam Effects

So far, there has been no mention on the requirement of observing beams. The circular scan strategy always has an axi-symmetric annulus beam when the signals are integrated over one continuously scanned circle, even though the instantaneous beam may be of any arbitrary shape. To make this statement, we have assumed that the beam shape does not drift and remains constant in time. The axi-symmetric annulus beam warrants the E and B modes to be cleanly separated.

A proper beam size can help tremendously in isolating B modes from E modes, especially when the axi-symmetry of the annulus beam is mildly broken, due perhaps to an imperfect instrument. As has been mentioned earlier, the B mode power is concentrated at the scale of a couple of degrees, whereas the E-mode power at the scale less than few tens of arcminutes. The scale separation allows one to choose a sufficiently large beam to filter out the smaller-scale E modes, while retaining most B-modes.

An idealized instantaneous beam is a symmetric Gaussian beam, and the Fourier k-space window function for the annulus beam traced by the instantaneous beam is $J_2(kR) \exp(-k^2d^2/2)$, where $R$ and $d$ are the radius of sky circle and the Gaussian beam width, respectively. Shown in Fig.2 is the surface brightness of B and E modes for various $R$ and $d$. Clearly seen here is that when $d \sim 0.5$ degree, the E-mode amplitude is suppressed by almost one order of magnitude at $R \sim 3$ degrees, whereas the B-mode power is reduced by only a factor of 1.5. Although the present circular-scan strategy ideally can separate out the B and E modes in a clean manner, the use of a large beam double guards the extraction of B modes against the contamination of E modes. The large beam makes the B and E-mode powers of comparable size, and can effectively suppress incidental small leakage of E modes into B modes.
5. AMiBA Performance for B and E-mode Measurements

The AMiBA is built to be an interferometry array. The interferometric measurements collect CMB power within a finite unresolved Fourier band. As was mentioned earlier, a finite $k$-space resolution makes the E-B separation difficult to carry out. Although it is possible to assimilate the real-space circular scan described in the last section with an interferometric observation, the efficiency turns out to be low (Chiueh, Ma, & Lin 2002). The reason seems to stem from the fact that the interferometry measurement intrinsically involves the plane-wave expansion, whereas the circular scan attempts to capture the ring-like patterns, and the conversion efficiency of a ring-like pattern to plane waves is low.

Nevertheless, AMiBA can also operate in a single-dish mode, where all 19 dishes are taken to be 19 independent single-dishes. The 19 dishes are mounted on a platform, which is rotatable about the axis that tracks the sky. Moreover, AMiBA has dual-polarization receivers, and hence the instantaneous Stokes $Q$ and $U$ can be measured simultaneously. These make AMiBA ideal for the circular scan measurement described in the last section. However, AMiBA has been designed to measure E modes at the multipole moment $l \sim 600$ and its dish size, 30 cm, is optimized for interferometry measurements at this angular scale for 90 GHz. This dish size corresponds to a primary beam with $d \sim 18'$. Such a dish size is not far from the optimization for B-mode measurements in filtering out E modes to a satisfactory level. The optimized dish size for B-mode measurements...
at 90 GHz is about 20 centimeters, corresponding to \( d \sim 27' \). Despite the dish size being not optimized, the purpose of having a large beam is to double guard against the E-mode leakage; the circular scan method can in principle cleanly filter out the E modes without a large beam. We therefore assume that AMiBA can conduct a perfect circular scan and the following sensitivity estimate ensues.

The system temperature of AMiBA is 70K, the antenna efficiency 0.5 and the receiver bandwidth 20GHz. The noise variance per antenna per polarization is hence \( 0.9/\sqrt{t_{\text{sec}}} \text{mK} \). Having 19 antennae, it reaches the polarization sensitivity \( 0.2/\sqrt{t_{\text{sec}}} \text{mK} \). On the other hand, with a primary beam of \( d \sim 18' \), a sky circle of \( R \sim 3 \) degrees yields an expected B-mode amplitude about 0.15\( \mu \)K, whereas the amplitude of the expected leakage source, E modes, is about 0.22\( \mu \)K. The \( 1 - \sigma \) B-mode (not accounting for the cosmic variance) thus takes about 500 hours of integration. Several tens of such a time span is needed to make successful statistical detection of B modes over several different sky circles of 3-degree radius. This kind of observing time scale is marginally feasible for a dedicated experiment.

On the other hand, the AMiBA 30cm dish is far from optimization for E-mode measurements using the present circular scan scheme. Nonetheless, AMiBA has also been designed to conduct galaxy cluster surveys via the thermal Sunyaev-Zeldovich effect. The needed higher resolution for cluster detection requires another set of AMiBA dish of size 1.2 meters, giving \( d \sim 4.5' \). The 1.2m dish is still not optimized for E-mode measurement but can perform much better than the 0.3m dish does. At the circle radius \( R \sim 0.3 \) degree, the 1.2m dish yields an expected E-mode amplitude about 1\( \mu \)K. The \( 1 - \sigma \) E-mode needs an expected integration time about 12 hours.

6. Conclusion

Detection of the CMB polarization E mode is on the verge of success. (See Timbie in this Proceeding.) It is foreseeable that in the next few years, a detailed E-mode power spectrum can be measured in the range of \( l = 600 \) to 1200, provided that the standard \( \Lambda CDM \) model with scale-invariant perturbations well approximates the correct cosmology. However, these cosmological measurements tell us only a partial story about our universe, which likely underwent an inflationary phase in the very early epoch. The scale-invariant scalar perturbation with a power index \( n = 1 \) is a natural outcome of inflation, and has been confirmed by recent CMB observations (de Bernardis et al. 2000; Hanany et al. 2000; Pryke et al. 2001) and likely by future detailed E-mode measurements. Another natural outcome of the early inflation is the scale-invariant primordial gravitational waves with a power index \( n = 0 \). The imprint of the latter outcome is encoded in the CMB polarization B modes. Only when the B modes are detected with the expected power spectrum can the inflation scenario be called a theory that stands against experimental scrutiny.

Unfortunately, the CMB polarization B mode has such a small amplitude. This makes the detailed power-spectrum measurement difficult, if not impossible, to conduct in a foreseeable future. Nevertheless, it is expected that positive detection of B modes, without spectral resolution, can be within reach in the next several years, if the correct cosmology is near the one we currently believe.
This lecture reports a convenient strategy for B-mode detection. It requires the instrument to be appropriately tailored mechanically and the detector to be equipped with dual polarization in order for the experiment to entertain the convenience. The AMiBA can in principle conduct such a B-mode measurement; with the designed sensitivity, AMiBA can achieve $1 - \sigma$ B-mode detection above the instrument noise in about 500 hours. AMiBA may, however, have too small a beam to double guard against incidental leakage of E modes into B modes. Nevertheless, this problem is among the many, such as the CMB photons scattered by the instrument as well as its surroundings to become polarized and be received by detectors, that need to be overcome in actual experiments, and the somewhat smaller beam may turn out not to be a serious problem in the AMiBA experiment.

References

Chiueh, T., Ma, C.J. & Lin, H. 2002 to be submitted.