INFLUENCE OF EQUILIBRIUM FLOWS AND THE ATMOSPHERIC MAGNETIC FIELD ON SOLAR OSCILLATION MODES

B. Pintér¹, R. Erdélyi², R. New¹, and M. Goossens³

¹School of Science and Mathematics, Hallam University, 1 Howard Street, Sheffield, S1 1WB, England (UK)
²Space & Atmosphere Research Center, Department of Applied Mathematics, University of Sheffield, Hicks Building, Hounsfield Road, Sheffield, S3 7RH, England (UK); Email: Robertus@sheffield.ac.uk
³CPA, Dept. of Mathematics, K.U.L., Celestijnenlaan 200-B, B-3001, Leuven-Heverlee, (Belgium)

ABSTRACT

The competing effects of an atmospheric magnetic field and an equilibrium flow in the internal regions of the Sun are studied on the helioseismic f- and p-modes. The Sun is modeled as a multi-layered plasma, where the upper parts, representing the chromosphere and corona, are embedded in a unidirectional though inhomogeneous magnetic field, meanwhile the lower part, representing the sub-photospheric polytropic region, is in a steady equilibrium state. The steady state sub-surface region can be considered as a first approximation of dynamic motions (e.g., convective motion, differential rotation, sub-surface flows, meridional flows, etc.). The obtained frequency shifts of the different eigenmodes are associated with flow and magnetic effects. We also found damping of the eigenfrequencies which apparently can be associated with the universal mechanism of resonant absorption. Resonant absorption (already known as a viable heating mechanism in the solar corona) is present due to inhomogeneities in the atmosphere which give rise to Alfvén and slow continua. Damping of helioseismic modes occurs when the modes are coupled into these continua. When both atmospheric magnetic field and sub-surface flows are present, a complex picture of competition of these two effects is found. The theoretically predicted frequency shifts in a steady state are in excellent agreement with the observed values. For related works see also the papers by Erdélyi & Taroyan and Varga & Erdélyi in the present Volume.

Key words: helioseismology; magnetic canopy; flow.

1. INTRODUCTION

The helioseismic f- and p-modes are global oscillations of the Sun providing important information about its structure. These modes are influenced by, e.g., the atmospheric magnetic field, the sub-surface and atmospheric large-scale motions. The importance of the dynamic nature of the solar plasma is becoming increasingly un-veiled because of recent high-resolution satellite measurements.

A simple model of the complex dynamic nature could be the consideration of uniform homogeneous equilibrium flows, i.e., steady states. This approximation is valid when motions are not changing rapidly on time scales of, e.g., measurements of the solar f- and p-mode oscillations (e.g., meridional flows, sub-surface motions, chromospheric downflow, convective motion). Observations (e.g., Braun & Fan, 1998) show a significant shift in p-mode frequencies between poleward- and equatorward-traveling waves measured over the solar latitudes 20° - 60°. These shifts are associated with large-scale motions (meridional flow).

Observations (e.g., Woodard & Noyes, 1985) also show that changes in solar magnetism influence the frequency-wavenumber relationship of solar helioseismic oscillations.


The influence of convective flows such as granulation and supergranulation on the f-mode frequencies has been studied by, e.g., Murawski & Roberts (1993a,b), Murawski & Goossens (1994), Ghosh et al. (1995). Erdélyi et al. (1999), Varga & Erdélyi (2000) studied the effect of a uniform homogeneous equilibrium flow on MAG surface waves. Erdélyi & Taroyan (1999, 2000a,b) studied analytically the combined effects of atmospheric magnetism, atmospheric temperature changes and sub-surface uniform homogeneous equilibrium flows on the frequencies of f- and p-modes.

Here we focus on the combined effect of flow and resonant absorption (the latter due to the inhomogeneous nature of the atmosphere) on the solar f- and p-modes.
There is a coupling of global solar oscillations to local continuum eigenoscillations (Alfvén and slow continua), i.e., some of the helioseismic $f$- and $p$-mode frequencies may fall within these continua. The helioseismic $f$- and $p$-modes coupled to the continua suffer damping because of resonance. The damping rate of the global oscillations is changing in the presence of an equilibrium flow.

2. EQUILIBRIUM MODEL

A three-layer model (see Fig. 1) represents the solar interior, chromosphere and corona in a Cartesian geometry (this means our analysis is valid only for high degree modes, for which the horizontal wavelength is small compared to the solar radius, i.e., $l \geq 6$).

We consider an adiabatic temperature profile, where the temperature increases linearly with depth with a gradient of $(\gamma - 1)g$. The solar interior is in a steady state (e.g., there is a homogeneous equilibrium flow in the horizontal direction).

We assume a constant flow $v_0(z) = (v_0(z), 0, 0)$ along the horizontal $x$-axis,

$$v_0(z) = \begin{cases} 0 & z < 0, \\ v & z \geq 0, \end{cases}$$

The solar chromosphere and corona are embedded in a unidirectional horizontal magnetic field $B_0(z) = (B_0(z), 0, 0)$. The chromosphere is a transition layer, where the strength of the magnetic field increases continuously from zero, giving rise to slow and Alfvén continua in the frequency spectrum.

The magnetic field strength decreases exponentially with height in the corona resulting in a constant coronal Alfvén speed (see Fig. 1).

3. DISPERSION RELATION

Let us introduce the Eulerian perturbation of the total pressure (as the sum of the thermal plasma pressure and the magnetic pressure), $P \equiv p + B_0 \cdot B/\mu$, and the vertical component of the Lagrangian displacement, $\xi_z$.

The linearised MHD equations can be reduced to two ordinary differential equations of the first order for the vertical component of the Lagrangian displacement and for the Eulerian perturbation of total pressure in Fourier space:

$$D \frac{d\xi_z}{dz} = C_1 \xi_z - C_2 \frac{\Omega}{\omega} P, \quad D \frac{\Omega}{\omega} \frac{dP}{dz} = C_3 \xi_z - C_4 \frac{\Omega}{\omega} P.$$  \tag{1}

The coefficient functions $D, C_1, C_2$ and $C_3$ are

$$D(z) = \rho_0 (v_0^2 + v_A^2)(\Omega^2 - \omega_z^2)(\Omega^2 - \omega_A^2),$$

$$C_1(z) = -g \rho_0 \Omega^2 (\Omega^2 - \omega_A^2),$$

$$C_2(z) = \Omega^2 - k^2 (v_0^2 + v_A^2) (\Omega^2 - \omega_z^2),$$

$$C_3(z) = \left[ \rho_0 (\Omega^2 - \omega_A^2) - \frac{d\rho_0}{dz} \right] D + g^2 \rho_0 (\Omega^2 - \omega_A^2)^2.$$  

Figure 1. (a) The magnetic induction, (b) the plasma density and (c) the sound ($v_s$), slow ($v_0$) and Alfvén ($v_A$) speeds as function of height. The solar interior is field-free and polytrope; the magnetic field strength increases from zero in a characteristic transition layer (from 0 to \(-L\), where $L = 2$ Mm); the characteristic velocities are constant in the corona.
where $\Omega(z) = \omega - kv_0(z)$ is the Doppler-shifted frequency. Here $\omega_A$ and $\omega_c$ are the local Alfvén frequency and the local cusp frequency. Their squares are given by

$$\omega_A^2(z) = k A(z), \quad \omega_c^2(z) = k_c^2 v_c^2(z).$$

The squares of the characteristic velocities – the sound speed ($v_s$), the Alfvén speed ($v_A$) and the cusp speed ($v_c$) – are given by

$$v_s^2 = \frac{\gamma p}{\rho}, \quad v_A^2 = \frac{B^2}{\mu \rho}, \quad v_c^2 = \frac{v_s^2 v_A^2}{v_s^2 + v_A^2}.$$  

Eqs. (1) govern the linear motions of a one-dimensional magnetic plasma in a gravitational field. The coefficient function $D$ vanishes at $z = z_c$ and at $z = z_A$ where the local slow or Alfvén frequency equals to the Doppler shifted eigenfrequency, $\Omega$. The singularities of Eqs. (1) can be removed by including dissipation (i.e., magnetic diffusivity) in the MHD equations. Dissipation damps the eigenmodes and the related eigenfrequencies become complex. The condition for slow resonance is $\text{Re} (\Omega) = \omega_c(z_c)$, and for Alfvén resonance it is $\text{Re} (\Omega) = \omega_A(z_A)$. When Eqs. (1) are supplemented with boundary conditions they define an eigenvalue problem for the global frequency $\omega$.

In the special case when the horizontal component of the wave vector, $k$, is parallel to the magnetic field lines ($k_y = 0$), Eqs. (1) show that the global oscillations interact resonantly only with local slow oscillations. The ideal and dissipative MHD equations can be found, e.g., in Pintér et al. (1998).

4. RESULTS

According to observations, the meridional flow is of the order of 10 m s$^{-1}$, nonetheless, we let the velocity of the steady flow run between $\pm 0.9 v_{sh}$ where $v_{sh}$ is the photospheric velocity, $v_{sh} = 0.676$ km s$^{-1}$ in the present calculation – in order to have a more general view of flow effects on the frequency spectrum. The profile of the magnetic field is fixed by taking $\beta_z \equiv \beta(z = -2$ Mm) = 0.5. We show results for oscillations with a harmonic degree of $l = 100$.

Analytical solutions can be found for $\xi_z$ and for $P$ in the solar interior ($z \geq 0$) and in the corona ($z \leq -L$). Eqs. (1) are integrated numerically in the chromospheric transitional layer ($-L \leq z \leq 0$). We solve the dissipative equations in a thin layer where the resonant condition applies by using Taylor-series expansion of the equilibrium quantities. The solutions in the adjacent layers are matched at the boundaries (at $z = 0$ and at $z = -L$) by taking into account the continuity condition for the Lagrangian displacement, $\xi_z$, and for the Lagrangian perturbation of the total pressure, $P + \rho g \xi_z$. Our analysis is restricted to trapped waves, i.e., of which the kinetic energy tends to zero for $|z| \to \infty$. These conditions yield a dispersion relation. The solution of the dispersion relation is the frequency spectrum of global oscillations. More details of the numerical methods are given by, e.g., Pintér & Goossens (1999).

Figure 2. Frequency spectrum for $l = 100$ and $L = 2$ Mm. Frequencies of $p$-modes of the order of 3 to 8 take place between the lower ($v_f$) and the upper ($v_{f1}$) cut-off frequencies. The Lamb- or $a$-mode frequency is located between $v_f$ and $v_c$. The frequencies of the damped $f$, $p_1$- and $p_2$-modes are in the slow continuum, i.e., below $v_c$. Stronger plasma flow in the polytropic layer results in larger shifts of the eigenmode frequencies.

Figure 3. The negative imaginary parts of the $f$, $p_1$- and $p_2$-modes measure the damping rate of the global oscillations. The damping is higher for stronger plasma flows.
We present the results in terms of frequency, \( \nu = \omega / (2\pi) \), which is preferred by observers, rather than the cyclic frequency, \( \omega \).

Fig. 2. shows the frequencies obtained for the \( f \)- and \( p \)-modes together with the characteristic frequencies which describe the equilibrium state. The sound (\( \nu_s \)), slow (\( \nu_c \)) and Alfvén (\( \nu_A \)) frequencies are taken at \( z = -L \). Non-leaky or trapped modes can appear between the lower (\( \nu_L \)) and upper (\( \nu_U \)) cut-off frequencies or in the slow continuum (\( \nu < \nu_c \)). The cut-off frequencies do not vary with the plasma flow, because they characterize the top of the chromospheric transition layer, while the flow occurs below the photosphere.

A general tendency is that an increasing flow, which is parallel to the magnetic field lines, decreases the \( f \)- and \( p \)-mode frequencies by about \( -k_x v \). This implies for a flow of 10 m s\(^{-1}\) a shift of 1.44 \( \mu \)Hz, which is observable. The deviation from the value of frequency shift, \( -k_x v \), is less than 0.1 \( \mu \)Hz for the \( p_2 \)-mode and for \( p \)-modes of the order of 4 to 8 for \( |v| < 0.9 v_{\text{ap}} \).

Besides the \( f \)- and eight \( p \)-modes, we also found the Lamb or \( a \)-mode, which is almost insensitive to the plasma flow. Its frequency falls off slightly, staying between the lower cut-off frequency, \( \nu_L \), and the sound frequency, \( \nu_s \), until the mode is terminated reaching \( \nu_U \). For negative flow velocities, the frequency of the \( p_3 \)-mode decreases with the same gradient as that of the other \( p \)-modes, but for positive flow velocities, it approaches the \( a \)-mode frequency, and finally the \( p_3 \)-mode couples to the \( a \)-mode.

Higher order \( p_n \)-modes, \( n \geq 9 \), do not exist for the present model as their frequency would be beyond the upper cut-off frequency, \( \nu_U \), and so they would not be trapped modes.

Modes with frequencies lower than \( \nu_U \) are also leaky modes, except if they couple resonantly to a local slow or Alfvén oscillation. We study only oscillations of which the horizontal wave vector, \( k \), is parallel to the magnetic field lines (\( k_y = 0 \)), hence they do not interact with Alfvén modes. The \( f \)-, \( p_1 \)- and \( p_2 \)-modes are coupled to local slow modes, as their frequency takes place within the slow continuum, as it can be seen in Fig. 2. Dissipation, which is significant around the resonant layer only, makes these modes damped. The negative imaginary part of their frequency, which measures their damping, is displayed in Fig. 3. The damping rate increases with an increasing flow. This effect is the strongest for the \( f \)-mode.

5. CONCLUSION

We found that a sub-photospheric plasma flow in a three-layer magnetic solar model has a significant effect on global oscillations which are trapped in the top of the convection zone. A steady flow parallel (anti-parallel) to the atmospheric horizontal magnetic field lines causes a negative (positive) frequency shift, and in case of resonant coupling, it also causes a stronger (weaker) damping of the helioseismic modes. The obtained frequency shifts due to flows which are of the order of \( \mu \)Hz, are observable in helioseismology. Formation, termination and transformation (i.e., coupling) of modes were also found as consequences of a change in a flow velocity.

ACKNOWLEDGMENTS

The authors thank B. Roberts for stimulating discussions and for his comments. BP is grateful to PPARC for financial support. RE acknowledges M. Kéray for patient encouragement. BP & RE also acknowledge financial support obtained from the NSF, Hungary (OTKA, ref. nr. TO32462).

REFERENCES

Erdélyi, R. & Taroyan Y., 1999, ESA-SP, 448, 81
Erdélyi, R. & Taroyan Y., 2000a, in Proc of IAU, 203, in press
Erdélyi, R. & Taroyan Y., 2000b, this Volume
Erdélyi, R., Varga, E. & Zétényi, M., 1999, ESA-SP, 448, 269
Varga E. & Erdélyi, R., 2000, this Volume
Woodard, M. F. & Noyes, R.W., 1985, Nat., 318, 449