THE DETERMINATION OF MDI HIGH-DEGREE MODE FREQUENCIES

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ABSTRACT

As mode lifetimes get shorter and spatial leaks get closer in frequency, individual p-modes can only be resolved up to some degree \( \ell \) (around 150). At higher degrees, individual modes blend into ridges and the power distribution of the ridge masks the true underlying mode frequency. To recover the underlying mode frequency from fitting the ridge, an accurate model of the amplitude of the peaks that contribute to the ridge power distribution is needed.

Using full-disk data from the Michelson Doppler Imager data on the Solar and Heliospheric Observatory, we present and discuss the differences between the observations and the spatial leakage calculation (including the horizontal component) and estimate the horizontal-to-vertical displacement ratio for medium-degree modes using sectoral modes for different observational periods. We show how time variations in the instrument calibration affect the spatial leakage and discuss their importance in the spatial leakage calculation. By constructing a physically motivated model (rather than some ad hoc correction scheme) can we hope to produce an unbiased determination of the high-degree modes in the near future.

Key words: Sun: oscillations.

1. INTRODUCTION

Since we observe only half of the solar surface, helioseismic power spectra computed for a specific target mode with degree \( \ell \) and azimuthal order \( m \) also contains modes with nearby \( \ell \) and \( m \) (see top panel of Figure 1). These spatial leaks complicate the fitting of the observed spectrum and degrade the resulting mode parameter estimates, especially if the leaks have frequencies similar to that of the target mode. As mode lifetimes get shorter and spatial leaks get closer in frequency (i.e., \( dv/\ell \) becomes small), individual p-modes can only be resolved up to some degree \( \ell \) (around 150 at moderate frequency and around 250-300 for the f-mode) — see Figure 1. At higher degrees, individual modes blend into ridges and the power distribution of the ridge masks the true underlying mode frequency.

The amplitudes of the spatial leaks have been shown to be asymmetric (see, for example, Korzennik, 1999). A direct consequence of this leakage asymmetry is to offset the power distribution of unresolved power ridges resulting in a significant difference between the central frequency of the ridge and the frequency of the targeted individual mode. To recover the underlying mode frequency from fitting the ridge, an accurate model of the amplitude of the peaks that contribute to the ridge power distribution (i.e., the leakage matrix) is needed. This has so far limited the use of high-degree data for helioseismic inversions for the near-surface structure. Since experiments like the Michelson Doppler Imager (MDI) on board the Solar and Heliospheric Observatory (SOHO) (Scherrer et al., 1995), with a two-arcsec-per-pixel spa-
tial resolution, allows us to detect oscillation modes up to \( \ell \approx 1500 \), the number of high-degree modes that are not used is very large.

The high-degree modes are trapped near the solar surface (for example, a mode of degree 500 and frequency 3000 \( \mu \text{Hz} \) has its lower turning point at 0.99 of the solar radius \( R_\odot \)) which makes them exceptional tools to probe the near-surface region of the Sun. This region is of very great interest since it is there that the effects of the equation of state are felt most strongly, and that dynamical effects of convection and processes that excite and damp the solar oscillations are predominantly concentrated. Rabello-Soares et al. (2000) showed that the inclusion of high-degree modes (\( \ell \) up to 1000) has the potential to improve dramatically the sound-speed inversion in the outermost 2 - 3\% of the solar radius. Furthermore, inversion of frequency differences between models computed with the MHD and OPAL equations of state recovered the intrinsic difference in \( \Delta \gamma \) throughout the second helium ionization zone and well into the first helium and hydrogen ionization zones, with errors far smaller than the actual differences between those two equations of state. Thus data of this type have the potential to probe very subtle effects in the thermodynamic properties of this region.

Libbrecht & Kaufman (1988) were the first to publish estimates of high-degree mode frequencies, using \( m \)-averaged Big Bear Solar Observatory data. To recover the underlying mode frequency from fitting the ridge of a given \((n, \ell)\) mode, they used, as an approximation for the \( m \)-averaged leakage matrix, a Gaussian profile:

\[
C_\ell^2 (\ell, \ell') = \exp \left( -\frac{(\Delta \ell - \ell')^2}{2s^2} \right)
\]

where \( \Delta \ell = \ell' - \ell \) and the \( \ell e \) term represents the asymmetry introduced by an image scale error of a fraction \( \epsilon \) of the image size. The ridge centroid frequency was estimated using a simple weighted average:

\[
\hat{\nu}_{n, \ell} = \frac{\sum_{\ell'} C_\ell^2 (\ell, \ell') \nu_{n, \ell'} A_{n, \ell'} A_{n, \ell}}{\sum_{\ell'} C_\ell^2 (\ell, \ell') A_{n, \ell'}}
\]

where \( A_{n, \ell'} \) is the individual mode amplitude and \( \nu_{n, \ell'} \) is the mode frequency. Using \( A_{n, \ell'} \approx A_{n, \ell} + dA/d\ell \Delta \ell \) and \( \nu_{n, \ell'} \approx \nu_{n, \ell} + d\nu/d\ell \Delta \ell \) in the equation above, the frequency difference between the ridge and the mode frequency, \( \delta \nu_{n, \ell} = \hat{\nu}_{n, \ell}(\text{ridge}) - \nu_{n, \ell}(\text{mode}) \), is estimated. The observed frequency difference \( \delta \nu \) can be obtained for medium-\( \ell \) modes from reducing the frequency resolution of the observed power spectra forcing individual modes to blend into ridges. Using these observed \( \delta \nu, s \) and \( \epsilon \) can be calibrated for medium-\( \ell \) modes and the correction extrapolated to high-\( \ell \) modes.

As Libbrecht & Kaufman (1988) stated in their paper, this is only a first step and there is room for substantial improvement particularly at high \( \ell \). Improving this method but following the same idea, Korzennik (1990) and Rhodes et al. (1999) also estimated high-degree mode frequencies using Mount Wilson and MDI data respectively. However, Bachmann et al. (1995) instead of using a Gaussian profile approximation to model the amplitude of the peaks that contribute to the ridge power distribution, calculated the radial component of the \( m \)-averaged leakage matrix in order to estimate high-degree mode frequencies for the High-L Helioseismometer at Kitt Peak.

In this paper, we present improved estimates of the effective leakage matrix. We have concentrated our efforts in constructing a physically motivated model — rather than an ad hoc correction scheme — in order to produce an unbiased determination of the high-degree modes. Since Korzennik (1999) has shown that the inclusion of the horizontal component of the leakage matrix calculation partially explains its observed asymmetry, we have included the horizontal component in the leakage matrix calculation to model the amplitude of the leaks.

In section 2, we describe the used data. In sections 3 and 4, we present and discuss the differences between the spatial leakage calculation (including the horizontal component) and the observations, with an emphasis on its asymmetry and estimate the ratio between the radial and horizontal components of the displacement on the solar surface for medium-degree modes, using different MDI data sets. Finally, in section 5, we show how time variations in the instrument calibration affect the spatial leakage and discuss their importance in its calculation.

## 2. ESTIMATION OF LIMIT SPECTRA

### 2.1. Data

The data that were used in this work consist of four time series of Doppler velocity images obtained by the MDI instrument operating in full-disk mode, with a 4″ resolution. The instrument operates in this mode — known as the Dynamics Program — during approximately 3 months every year, when the provided telemetry bandwidth is big enough to bring down full disk images. The starting time and duration of each Dynamics data set is listed in Table 1.

<table>
<thead>
<tr>
<th>Data Set</th>
<th>Starting Date</th>
<th>Duration [days]</th>
</tr>
</thead>
<tbody>
<tr>
<td>DYN96</td>
<td>1996.05.23</td>
<td>63</td>
</tr>
<tr>
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<td>1997.04.14</td>
<td>61</td>
</tr>
<tr>
<td>DYN98</td>
<td>1998.01.09</td>
<td>92</td>
</tr>
<tr>
<td>DYN99</td>
<td>1999.03.13</td>
<td>77</td>
</tr>
</tbody>
</table>

Estimates of limit spectra were computed from averaged power spectra using the following procedure. Time series of spherical harmonic coefficients were first detrended using a 21-minute-long running mean. The 9th order sinc multi-tapered power spectrum was then computed. Finally, a small fraction of each power spectrum (about 300 \( \mu \text{Hz} \)), centered around each mode frequency, was averaged for a subset of modes defined by limiting the range

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in $\ell$ ($15\ell$'s) and keeping $n$ constant (i.e., averaging along a ridge in the $\ell - v$ diagram).

The mode frequencies used in this work were obtained by the MDI Science Team for the Structure Program for the period corresponding to each data set in Table 1 (Schou, 1999).

2.2. Sectoral Modes

The leakage matrix for sectoral modes is diagonal, resulting in a simple leakage pattern. On the right side of the target peak with degree $\ell$ and azimuthal order $m = \ell$ (see top panel of Figure 1), there are several leaks with progressively smaller relative amplitudes as frequency increases: $(\ell + 1, m + 1), (\ell + 2, m + 2), \ldots$. Similarly, on the left side, there are several leaks with progressively smaller relative amplitudes as frequency decreases: $(\ell - 1, m - 1), (\ell - 2, m - 2), \ldots$. For zonal and tesseral modes the leakage pattern is more complex resulting in leaks very close to each other, separated only by even multiples of $dv/dm$. For these modes, only for very low degrees (where the lifetime is long enough), and if the time series is long enough, can the main peak be resolved from its spatial leaks. Henceforth, for their simplicity, only sectoral spectra have been considered in this work.

3. LEAKAGE MATRIX

The complete leakage matrix for the observed line-of-sight component of the velocity has a radial component $C^r$ plus a horizontal component $C^\theta + C^\phi$. Thus a mode with given $(\ell', m')$ will appear in the $(\ell, m)$ power spectrum with a relative amplitude in relation to $(\ell, m)$ given by the square of $C_{\ell,m;\ell',m'}$ where:

$$C_{\ell,m;\ell',m'} = C_{\ell,m;\ell',m'}^r + \beta_{n,t} \left(C_{\ell,m;\ell',m'}^\theta + C_{\ell,m;\ell',m'}^\phi\right),$$

(3)

$\beta$ is the horizontal-to-vertical displacement ratio. Using a simple outer boundary condition, i.e., that the Lagrangian pressure perturbation vanish ($\delta p = 0$, Christensen-Dalsgaard, 1997) leads to an estimate of the ratio $\beta$ given by:

$$\beta_{n,t} = \frac{GM_\odot}{R_\odot^3} \frac{\omega_{n,t}^2}{v_{n,t}^2} L = \frac{v_{n,t}^2}{v_{n,t}^2}$$

(4)

where $G$ is the gravitational constant, $M_\odot$ is the solar mass, $R_\odot$ is the solar radius, $\omega$ is the angular mode frequency ($2\pi v$) and $L = \sqrt{\ell(\ell + 1)}$. Since for a given $\ell$, the $f$-mode frequency is smaller than any other $n$ mode, Equation 4 implies that $0 \leq \beta < 1$.

As solar oscillations can be described in terms of spherical harmonics $Y^m_\ell(\theta, \phi)$ of co-latitude $\theta$ and longitude $\phi$, the velocity oscillations can be written as:

$$\bar{v}_{n,t,m} = (V_r Y^m_\ell, V_i \partial_\theta Y^m_\ell, V_i \partial_\phi Y^m_\ell, V_h 1/\sin \theta \partial_\phi Y^m_\ell).$$

(5)

Figure 2. Synthetic (dashed line) and observed (continuous line) power spectra for D\small{Y}N\small{96} data set for the $n = 2$, $\ell = 77$ and $n = 2, \ell = 122$ in the top and bottom panels respectively. The synthetic spectra shown in the right and left panels were computed using leakage matrix calculations with and without including the horizontal component respectively.

Ideally, the leakage matrix components for the observed line-of-sight velocity are given by:

$$C^r_{\ell,m;\ell',m'} = \int W Y^m_\ell Y^{m'}_{\ell'} \sin \theta \cos \phi d\Omega$$

(6)

$$C^\theta_{\ell,m;\ell',m'} = \frac{1}{L} \int W Y^m_\ell \partial_\theta Y^{m'}_{\ell'} \cos \theta \cos \phi d\Omega$$

(7)

$$C^\phi_{\ell,m;\ell',m'} = \frac{1}{L} \int W Y^m_\ell \partial_\phi Y^{m'}_{\ell'} \sin \phi \sin \theta d\Omega$$

(8)

where $Y^m_\ell$ is the spherical harmonics complex conjugate representing the filter applied to separate the individual $(\ell, m)$ modes (i.e., the spherical harmonic decomposition) and $W(\theta, \phi)$ is the spatial window function.

Figure 2 shows some typical examples of observed spectra and synthetic spectra where the amplitude of the leaks are given by the leakage matrix calculated with and without the horizontal component (right and left panels respectively). It is clearly seen that the observed amplitudes of the spatial leaks are asymmetric, as well as the importance of the contribution of the horizontal component in the leakage matrix calculation, since the radial component of the leakage matrix is nearly symmetric. It should also be noticed that, although the complete leakage matrix agrees in broad terms with the observed spectra, it does not agree in detail, as already pointed out by Korzeni (1999).

4. ESTIMATE OF $\beta$ FROM MEDIUM-$\ell$ MODES

The horizontal-to-vertical displacement ratio $\beta$ is both a function of the outer boundary condition used, as well
as a function of the depth of the solar spectral line observed by the instrument. The estimation of \( \beta \) from the eigenfunctions of a standard solar model (Christensen-Dalsgaard et al., 1996) at the correct solar atmospheric height for the MDI instrument and using a better upper boundary condition than Equation 4 gives a very similar result. At frequencies higher than 2000 \( \mu \)Hz, it tends slightly faster to zero than Equation 4 as frequency increases (at \( \nu = 3000 \mu \)Hz, it is 15% smaller than Equation 4). However, the non-adiabaticity of the oscillations at the solar atmosphere are not taken into account in the standard solar models or in Equation 4.

Korzennik (1999) estimated \( \beta \) from the observations using the leakage asymmetry for the DYN97 data set. First, a seven-component profile was fitted in a least-squares sense to the observed power spectra (section 2) using the following equation:

\[
P_{n,t}(\nu) = \sum_{\ell' = \ell - 3}^{\ell + 3} A_{n,\ell'} \left( \frac{1 + \alpha_{n,\ell}(x - \alpha_{n,\ell}/2)}{1 + x^2} \right)
\]

(9)

\[
z = \frac{\nu - \nu_{n,\ell'}}{\gamma_{n,\ell}}
\]

(10)

where \( A_{n,\ell}, \gamma_{n,\ell}, \nu_{n,\ell} \) and \( \alpha_{n,\ell} \) are the mode amplitude, width, frequency and asymmetry parameter. To quantify the leakage asymmetry, Korzennik (1999) defined the following factor:

\[
S_z(n, \ell) = \frac{\sum_{\ell' = \ell - 3}^{\ell + 3} (\ell' - \ell) A_{n,\ell'}}{\sum_{\ell' = \ell - 3}^{\ell + 3} A_{n,\ell'}}
\]

(11)

Figure 3 shows the resulting observed leakage asymmetry factor \( S_z \) as a function of frequency (top) and the theoretical one as a function of \( \beta \) (bottom) calculated using the

\[
S_z(\beta, \ell) = \frac{\sum_{\ell' = \ell - 3}^{\ell + 3} (\ell' - \ell) C_{n,\ell',\ell}^2(\beta)}{\sum_{\ell' = \ell - 3}^{\ell + 3} C_{n,\ell',\ell}^2(\beta)}
\]

(12)

Finally, the observed \( \beta \) was inferred from the measured \( S_z \) (Figure 4, top panel).

The ratio \( \beta \) can also be estimated by comparing the observed ridge frequency with its theoretical prediction. As mentioned in the Introduction, the observed individual frequency can be determined for medium-\( \ell \) modes, but by reducing the frequency resolution of the observed spectra, the individual modes in the range 50 < \( \ell \) < 150 are blended into ridges. The ridge frequency was determined for the DYN97 data set by averaging 128 sectoral-mode power spectra, each computed using 1024-minute-long time-series. An asymmetric profile (as in Equation 9) plus a background term (\( B_\ell(\nu) \)) was fitted using least squares:

\[
P_{\ell}(\nu) = \sum_{n} A_{n,\ell} \left( \frac{1 + \alpha_{n,\ell}(x_{n,\ell} - \alpha_{n,\ell}/2)}{1 + x_{n,\ell}^2} \right) + B_\ell(\nu)
\]

(13)

where:

\[
x_{n,\ell} = \frac{\nu - \nu_{n,\ell}}{\gamma_{n,\ell}}
\]

(14)

and

\[
\log B_\ell(\nu) = \sum_{j=0}^{2} b_{j,\ell} \nu^j
\]

(15)
$A_{n,t}$, $\gamma_{n,t}$, $\nu_{n,t}$ and $\alpha_{n,t}$ are the ridge amplitude, width, frequency and asymmetry parameter.

A theoretical ridge frequency can also be calculated constructing a ridge power distribution model from the overlap of the target mode with its spatial leaks using the leakage matrix:

$$\varphi_{n,t}(\nu) = \sum_{t'=t-3}^{t+3} C^{2}_{t,t',t'}(\beta_{n,t}) \frac{1 + \alpha_{n,t}(x - \alpha_{n,t}/2)}{1 + x^2},$$  

(16)

where $x$ is defined in Equation 10, and fitting this model as described above (Equation 13). The observed $\beta$ is estimated modifying $\beta$ in Equation 16 until the theoretical ridge frequency matches the observed one (Figure 4, top panel). Both estimates of $\beta$ agree in broad terms with each other and with the values from Equation 4. Schou & Bogart (1998) using ring-diagram analysis for two months of MDI data (starting on 1996 May 24) also found $\beta$ in good agreement with the theory. At frequencies higher than 2000 $\mu$Hz, both estimations of $\beta$ in Figure 4 (top panel) tend faster to zero than the theoretical value; as does $\beta$ calculated using the eigenfunctions of a standard solar model, although here the effect is much larger.

The value of $\beta$ was also estimated for the others data sets listed in Table 1 using the asymmetry factor $S_{a}$ and are plotted in Figure 4 (bottom panel). There is a clear agreement between the values estimated for the DYN96 and DYN97 data sets. However, DYN98 and especially DYN99 clearly have different values than the other two. There is no physical explanation, that we know of, for the value of $\beta$ to change by that much from one year to the other. Thus instead of trying to improve the estimation of $\beta$, which will end up to be an ad hoc correction, we attempted to improve the leakage matrix calculation to produce an unbiased determination of the high-degree mode frequencies. In fact, we will show that the apparent different values of $\beta$ for DYN98 and DYN99 are associated with a time variation of the plate scale error in the MDI images.

5. IMPLICATIONS OF THE MDI CALIBRATION IN THE LEAKAGE MATRIX

Bush et al. (2001) reported time variations in the MDI instrument calibration, specifically in the image plate scale and in the instrumental modulation transfer function (MTF). Continuous exposure to solar radiation has increased the instrument's front window absorption resulting in an increase in its temperature (see Figure 2 in Bush et al., 2001). Moreover, there is indirect evidence for a temperature gradient from center to the edge of the front window, associated with the temperature increase. This gradient is causing a small curvature that converts the window into a weak lens. Evidence of this is a drift in the instrument focus (Figure 5), where the small variations in the focus are correlated with the annual temperature change of the front window due to the satellite orbit around the Sun (see Figure 5 in Bush et al., 2001).

The focus of the MDI instrument can be adjusted by inserting glass blocks of varying thickness into the light path. There are 9 possible focus positions (one focus step corresponds to approximately 0.3 waves). Also plotted in Figure 5 is the configuration that the instrument was operating at a given time and the corresponding amount of defocus which will affect the instrumental MTF.

In addition to affecting the MTF, another consequence of a change in focus is a variation in the image scale. Figure 6 shows the relative variation of the mean image radius. The discontinuities are due to change in instrument focus position (the vertical dashed lines indicate when there was a change in the instrumental focus position). The observed annual variation plus a small systematic increase with time are correlated with the aforementioned changes in the front window temperature. The plotted values are relative to the initial plate scale that was used in the spherical harmonic coefficients time-series calcula-
Figure 7. Observed power spectra (top panel) and simulated power spectra (bottom panel) where the leakage matrix was calculated without and with a plate scale error of $-0.1\%$, i.e., the power spectra were calculated assuming that the solar image radius was $0.1\%$ larger than observed.

Figure 8. Ratio $A_{l+1}/A_{l-1}$ for observed (top panels) and simulated (middle and bottom panels) power spectra. The theoretical leak amplitudes were calculated with and without a plate scale error (middle panels) and with and without distortion (bottom panels). Note that the values for theoretical spatial leaks for low $n$ are outside the plot.

The leakage matrix equations in section 3 are oversimplified, for example, they do not take into account the pixel size of the detector or the instrumental MTF, which are particularly important for high-degree modes. Instead of numerically integrating those equations, a simulated solar image is created using spherical harmonics and it is processed in the exactly same way as the observed images to determine the leakage matrix. The finite size of the pixels in CCD detector are de facto taken into account. This method is not only more realistic, but it also allows us to include effects like plate scale error and distortion easily. In the bottom panel of Figure 7, we plotted simulated power spectra using the leakage matrix with and without a plate scale error. The same effect as in DYN98 and DYN99 power spectra is seen here: $A_{l+1}/A_{l-1}$ decreases when a negative plate scale error is introduced. Figure 8 (top and middle panels) shows that this is true for a large number of modes. The observed amplitudes of the leaks were estimated fitting Equation 9 to the observed spectra and the theoretical amplitudes of the leaks are given by the square of the leakage matrix. Note that for $n \geq 13$, the observed ratio is always smaller than 1 and, for $n = 0$, it is not as large as the theoretical one, indicating that there are further issues that have to be taken into account other than the plate scale error.

Figure 9 shows the expected distortion for the MDC images, for the Dynamics Program, given the optical properties of all elements in the instrument (Scherrer, private communication). The effect of the distortion on the sectoral spatial leaks is similar to the effect of a plate scale error of about half the amount of distortion at the solar limb (Figure 8, bottom panels). As the approximate shape of the observed solar image is an ellipse, the distortion of the instrument is also a function of the angle from the solar equator and should be present in all data sets.

In Figure 10, we plotted an azimuthally averaged estimate of the modulation transfer function (MTF) obtained by Bush et al. (2001) for different years and the corresponding point spread function (PSF) calculated from the MTF. Although they are very similar, the width of the PSF is
increasing with the amount of defocus, as expected. The DYN97 data set which is more out of focus has a slight wider PSF. In fact, this can be seen in the DYN97 power spectrum in comparison with the DYN96 power spectrum (Figure 11). The difference between them is qualitatively similar to the difference between two simulated power spectra: one using a Gaussian PSF with a width of 0.8 pixels and another five times wider. A change in the PSF does not affect the leakage asymmetry much (Figure 12), except at very low \(n\). However, as expected, an increase in the PSF width increases the size of the leaks (Figure 13).

Although DYN97's azimuthally averaged estimate of the PSF is only slightly wider than DYN96's, the difference between their leaks is qualitatively similar to the difference between the amplitude of the leakage matrices calculated using two Gaussian PSF one being 5 times wider than the other. Even though the estimated PSF has not a Gaussian profile (Figure 10), the sectoral spatial leaks calculated using the actual profile or a Gaussian fit are very similar and can not explain the discrepancy. A possible explanation is that an azimuthally averaged estimate PSF is not a good approximation to the true PSF of the instrument. Tarbell et al. (1997) estimated the PSF of the MDI's higher resolution field (which magnifies the image by a factor of 3.2 having a pixel size of 0.63 arcsec), using phase-diversity analysis (Figure 14). A secondary peak besides the main one is clearly seen (with an amplitude equal to 0.15) caused by optical aberrations of the instrument components. We have evidence that the PSF for our data sets are similar to Figure 14, as one should expect since the only difference between them is the presence of a magnifying lens in the optical light path. One possible explanation at this point is that, when the instrument is more out of focus, the effect of the optical aberrations will increase, making the secondary peak at the PSF more important and it will affect the amplitude of the leaks in the power spectra.

### 6. CONCLUSION

The estimation of the horizontal-to-vertical displacement ratio from the observations seems to agree with the simple equation: \( \beta = \nu_{\ell} / \nu_{n,\ell} \), except maybe at frequencies higher than 2000 \( \mu \)Hz where it seems that the observations tend slightly faster to zero than the theory. However, a better calculation of the leakage matrix is needed to draw further conclusions.

The correct calibration of the MDI instrument and its time variations are essential to estimate the spatial leaks in the power spectra. Any ad hoc correction scheme, such as, making a simple approximation of the spatial leaks for medium-degree modes or estimating the horizontal-to-vertical displacement ratio for medium-degree modes and, then, extrapolating it to high-degree modes, will always result in an unreliable extrapolation, which will not give us confidence to determine unbiased high-degree mode frequencies. The only way is to calculate the full leakage matrix taking into account the instrument calibra-
Figure 12. Ratio $A_{i+1}/A_{i-1}$ for observed power spectra (top panels) and for theoretical spatial leaks (bottom panels). Note that the values for theoretical leaks for low $n$ are outside the plot.

Figure 13. Relative leak average amplitude defined as: $[\sum_{\nu} A_{\nu}(\text{DYN97}) - \sum_{\nu} A_{\nu}(\text{DYN96})] / \sum_{\nu} A_{\nu}(\text{DYN96})$ (on the left) and for the spatial leaks that were calculated using a Gaussian PSF with a width of 0.8 pixels and using a PSF five times wider (on the right).

We think that we are on the correct track to improve our knowledge of the instrument calibration and to improve the spatial leakage calculation and estimate unbiased high-degree mode frequencies in the near future.

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