ONE SOLAR CYCLE LATER: REFLECTIONS AND SPECULATIONS ON DIRECTIONS IN
HELIO- AND ASTEROSEISMOLOGY IN A NEW MILLENNIUM

J. R. Kuhn

Institute for Astronomy, University of Hawaii, 2680 Woodlawn Dr., Honolulu, HI, USA

ABSTRACT

This talk reflects on our progress since the last helioseismology symposium held here in Tenerife over one solar cycle ago. While even a superficial inspection shows that the last decade of seismic investigation of the Sun and other stars has been enormously revealing – it appears that new observations are generating new questions at a faster rate than our ability to solve old problems. Here we briefly review some of this progress, and highlight questions and research directions that might possibly be described at the next Tenerife helioseismology meeting.

Key words: Helioseismology; Solar Cycle.

1. INTRODUCTION

My job here is to attempt a review and preview of helio- and asteroseismology since the last, and for the next, Tenerife workshop. You may note that we intermittently attempt this at these SOHO/GONG meetings – this one is timely because it was about one solar cycle ago that one of the first international helioseismology workshops was held in Tenerife. This talk will try to adopt the tone and format of a review from 5 years ago during SOHO 4 (Harvey 1995). In the same workshop, forward looking views of helioseismology were described by Gough (1995), and asteroseismology was reviewed by Brown (1995). More recent reviews of asteroseismology (Kurtz 1998) and helioseismology (Thompson 1998) along with recent GONG and asteroseismology reviews by Christensen-Dalsgaard (1998) and Günther (1998) bring us almost up-to-date. I will try not to duplicate the earlier discussions. Perhaps, if there is only one message to be taken from this, it is that we are justified in sharing the optimism within each of these retro- and pro-spec-tives.

Several issues we take for granted now (like “if” solar oscillation frequencies change with the solar cycle) were avidly debated when about half of us met here in Tenerife 12 years ago. The newest data from the ground- and space-based experiments have precision that far exceeds what was available at that meeting and, while spanning a longer duration, have “almost” settled some of the questions we asked then about the acoustic structure of the Sun. On the other hand, it is clear that important problems related to the physical origins of these results are still with us, in part because our hopes for “non-acoustic” solar interior diagnostics have not materialized.

Solar/Stellar seismology is still a young field, and clearly is not in an equilibrium state. Figure 1 shows how the yearly publication rate has fluctuated since 1982. While these data are almost certainly incomplete, and it is easy to overinterpret them, I can’t resist making a few observations. The most obvious is that solar/stellar seismology is a field which continues to grow (at times explosively). It is also notable that an increasing fraction of the published stellar seismology literature concerns stars other than the Sun. This trend might have reached its logical conclusion a few years ago if it weren’t for the stimulus of GONG and SOHO in the mid-90’s. The decline in research output near 1990 and very recently seems to suggest that we are distracted by other questions near solar maximum. The IAC organizers should be congratulated on their prescience in hosting these meetings in the timely periods near the peaks of rapid growth.

Others will reflect on our progress with asteroseismology in section 5, later in this meeting. The only point I would make is that astro- and helioseismology exhibit some interesting correlations and anticorrelations which might suggest that these joint meetings are a “good thing.” Figure 1 hints that research progress in these two subdisciplines is coupled. For example, in the mid-90’s the drop in helioseismic publication effort was contemporaneous with stellar research gains. Interestingly, near solar maximum, dips in research productivity appear simultaneously in both subdisciplines.

2. THEMES

Harvey (1995) describes a “helioseismic food chain” in terms of the production of seismic data and its
Figure 1. Helio- and Asteroseismology publication rates. Any paper including the term “helioseismology” or “asteroseismology” in the text was counted using the NASA ADS. The yearly rate was computed from two year averages.

Figure 2. Frequency differences for \( l=1 \) modes are compared from 1988 and 1998 experiments.

Figure 3. Frequency errors between GOLF and MDI experiments are plotted along with BBSO internal mode frequency uncertainties.

What we have gained with the newer data is not simply an increase in signal-to-noise. Bertello et al. (2000) found that individual low-\( l \) modes between 1.5 mHz to 3 mHz are measured consistently (with the same frequency) to within 100 nHz from GOLF and MDI. These are of course distinct experiments, GOLF being a full-disk velocity instrument, and MDI a relatively high spatial resolution doppler/imaging system. In contrast, about the time of the last Tenerife meeting, Duvall et al. (1988) published a comparison of ground-based frequency measurements. While this early comparison was complicated by the fact that there was uncertainty over whether or not mode frequencies changed with time, the mode frequency differences for a subset of measurements obtained from nearly simultaneous experiments were substantially larger. Figure 2 shows that the consistency in measuring low-\( l \) mode frequencies has improved by an order of magnitude. This improvement is not only due to the longer timeseries, but because of enhancements in analysis techniques – like accounting for the mode asymmetry noticed by Duvall et al. (1993) only “recently.”

While Fig. 2 suggests that mode frequency observations have improved dramatically, this is not uniformly true over the full mode spectrum. It is the mitigation of the systematic differences between datasets (and improved understanding of the real differences) which accounts for the new-found consistency. As an example, Libbrecht’s (1992) derived internal frequency uncertainty within the BBSO measurements is plotted in Figure 3 against comparable recent satellite data. It shows that the most important gains in frequency resolution have been at high frequencies, while between 1.5 mHz and 3 mHz, the frequency resolution has only improved approximately by the square-root of the ratio of the timeseries’ duration.

In another important way the new data far surpass the usefulness of earlier experiments. Because of the long timeseries, now more than a solar cycle if we consider all of the helioseismology experiments, we have the potential to critically study the acoustic so-

2.1. Data Quality

Progress in each of these areas is evident, but it is the first category that drives the “food chain”, i.e., I think none of us were surprised to see the rapid growth in the volume of research soon after SOHO and GONG began distributing their high quality data.
lar cycle. Figure 4 shows how the space experiments compare with some long-running low-l ground helioseismology studies. Intermittent high spatial resolution ground instruments (which effectively started in the early 1980’s with the NSO effort) have also sampled the solar cycle. Only recently has it been possible to compare high resolution seismic data from the same instrument over a significant fraction of a solar cycle.

2.2. Diagnostic Tools

The quality of the seismic data has improved with the higher signal-to-noise in the longer SOHO and GONG timeseries, but the largest improvements in measured mode characteristics may be due to changes in analysis techniques. For example, the use of multi-parametric peak fitting (e.g. to account for line asymmetry and background power) and leakage matrix corrections (to correct for non-orthogonality of the spatial basis functions) were fundamental milestones (Bertello et al. 2000; Schou 1998; Rabello-Soares and Appourchaux, 1998; Hill and Howe, 1998). This new technology has systematically improved splitting and mode frequency measurements.

Another measure of progress is the new vocabulary which did not exist at the last Tenerife workshop. Terms like “local helioseismology” and “tachocline” were absent. A review of this subject by Duvall (2000) should be definitive.

2.3. Physical Theories and Conclusions

Presumably there are two possible outcomes of an improving analysis of helioseismic data: 1) a more complete understanding of the Sun’s structure, and/or 2) more complete physical theories. Recall that over time, studies involving the Sun (even more

Figure 4. Past “helioseismic coverage” of the solar cycle. This plot is incomplete but it samples a few of the intermittent spatially resolved experiments (dashed line) and the long-running ground (low resolution) and space experiments.

Figure 5. Past and current sound speed discrepancies in the solar core.

so than studies of distant stars) have been important for nuclear, atomic, plasma, and gravitational physics. Inferences that lead to a better solar model (item 1 above) are essential to forming more general physical conclusions (item 2 above). Thus, while the immediate benefits of improved one- or two-dimensional mean thermodynamic models of the Sun may not have touched the general astrophysical community, they will eventually!

Christensen-Dalsgaard (1998) recently reviewed the current state of solar models. While he justifiably complains about lacking observations of low frequency modes which penetrate the core, great progress has been made. For example, the agreement between the theoretical sound speed and the seismic data has improved dramatically. Figure 5 shows how the unmodeled sound speed has decreased by an order of magnitude. Unfortunately, the 0.2% c^2 dip near 0.2 solar radii is still larger than all known data and modeling errors.

In the outer envelope of the Sun, where there are many seismic model constraints, improvement has been more gradual. For example, Shibahashi and Sekii (1988) found sound speed discrepancies no larger than 1.5% between 0.4 and 0.9 solar radii – with even a hint of the tachocline in their early inversions. Christensen-Dalsgaard (1998) has shown how the “classical” model improvements (heavy-element diffusion, more accurate solar radius/age, and opacity improvements) have not been entirely successful at reducing such sound speed discrepancies near the base of the convection zone.

Boundary layers should be fertile ground for constraining the physics of the convection zone. The layer between the differentially rotating convection zone and the nearly uniformly rotating radiative interior, which we now call the tachocline, is particularly important, and was unknown to us in 1988. This is a region where the radial velocity shear is largest in the Sun and is most likely the accumulation point during the solar cycle for a differentially wound toroidal magnetic field. It is also a layer where convective overshoot is likely to mix the lighter material from above with the heavier and stably stratified radiative
zone. Clearly this layer is physically “rich” (Spiegel and Zahn 1992; Elliot and Gough 1999; Brun et al. 1999) and enormously interesting.

There can be no doubt that solar models are confronting more general physical problems. For example, despite lingering uncertainty over the solar age, radius, and opacities, the helioseismic measurements of the photospheric helium mass fraction are converging. Figure 6 shows the evolution toward $Y = 0.246$. This is an interesting measurement because, although it falls within the uncertainty in the cosmic He abundance, if corrected for settling and diffusion (Brun et al. 1998), it is likely to be higher than other astrophysical He abundance determinations.

The possibility of solving problems in gravitational physics or cosmology with improving solar models is very real. Some years ago, when there seemed to be evidence that the gravitational constant varied with distance, even the helioseismic data available in 1987 were sufficiently precise to place interesting physical limits on spatial variations in the gravitational constant, or deviations from a Newtonian force law (Kuhn 1988). More recently Guenther et al. (1998) derived significant limits on possible cosmic temporal variations in the gravitational constant. We should not lose sight of the fact that the Sun is not just an interesting astrophysical laboratory, but a laboratory of fundamental physical interest.

3. PERSISTENT PROBLEMS, PROGRESS IMMINENT?

This section highlights two problems which were recognized as puzzles back in 1988, which have been well studied observationally since then, but which still lack accepted solutions. These questions have been with us for years with only incremental advances in their theoretical understanding, yet we have reason to hope that under the weight of the helioseismic data, they really ought to give way to clear solutions. After all, in the convection zone, we now know the internal density, temperature, and rotation profile far more accurately than we did at the last Tenerife meeting.

3.1. Solar Rotation

Christensen-Dalsgaard (1998) described the state of our understanding of the solar rotation. It is fair to summarize it as “well known down to about half a solar radius.” Recall that at the last Tenerife meeting (where we spent one complete session discussing rotation) we were starting to convince ourselves that there was almost no radial differential rotation in the convection zone. This of course is now well established, but what can be said about the cause of this rotation profile?

This problem is a good example of how helioseismology and numerical convection experiments are both needed to generate new physical insight. We digress a bit here to make the connection between small-scale convection, as resolved in numerical simulations, and the observed large scale solar differential rotation. The derivation requires an estimate of the turbulent Reynolds stress in the convection zone. We make use of the fact that the Reynolds stress tensor, $T_{ij}$, may be computed from local velocity correlations of independent fluid elements of density $\rho$ and velocity $\mathbf{v}$, $T_{ij} = \rho v_i v_j$. The two critical parts of the calculation we describe below are: 1) the well known fact that small scale (local) turbulence in the presence of a mean shear flow can generate a Reynolds stress which is non-diagonal (cf. Durman and Spruit 1979), 2) the fact (deduced from numerical experiments) that solar turbulence is strongly anisotropic (dominated by the vertical $t_z$).

The first issue is illustrated by considering a point $P$ within a turbulent fluid with no mean velocity shear. Figure 7 illustrates the spherical surface where idealized distinct fluid elements originated one mean lifetime ($\tau$) earlier, before they collided at $P$. It is evident that in this case $\nabla \cdot \mathbf{v}$ is zero if $\mathbf{v}$ is not equal to $\mathbf{0}$. The second panel in this figure shows where fluid elements must have originated from, and what their velocities were, when the fluid above $P$ was traveling to the left and the fluid below was traveling to the right (shear flow). Elements which collide at $P$ now have a non-isotropic velocity distribution. In this case it is obvious that $\nabla \cdot \mathbf{v} > 0$. Elsasser (1966)
Figure 7. Turbulent fluid elements arrive at \( P \) on the left within a medium with no mean velocity shear. In the right half of this figure the upper region is moving to the left while the lower region moves in the opposite direction.

showed how the Reynolds stress, \( T^{ij} \), in a sheared medium could be related to the mean flow \( \bar{U} \) and the stress in the absence of rotation, \( t^{ij} \),

\[
T^{ij} = -\tau(t^{ik} \partial_k U^j + t^{jk} \partial_k U^i + U^k \partial_k t^{ij}).
\]

The second point has been established from numerical solar convection simulations (cf Kuhn 1991, Kuhn and Georgiobiani 2000) and is simply that the vertical velocity dispersion dominates the horizontal. Contrary to simple mixing length conclusions, it is now well established that turbulence in the convection zone is characterized by strong vertical downflows and more gentle, broader upflows. Applying the above equation locally (where \( \hat{z} \) is vertical, \( -\hat{x} \) points toward the pole, and \( \hat{y} \) points in the local rotational velocity direction), we find that the stress is dominated by the term, \( T^{xy} = -\tau t^{xy} \partial_x U^y \) (cf. Kuhn 1996). Since the local force density generated by the Reynolds stress is given by \( -\nabla \cdot T \) we can compute the transverse acceleration that balances \( z \)-gradients in the rotational velocity

\[
\rho \partial U^y / \partial t = \tau t^{xy} \partial_x U^y / \partial z^2.
\]

This is, of course, just the diffusion equation, and it tells us that any wiggles or bumps in the rotation profile defined by \( U^y(z) \) will be smoothed out over a “diffusion time” of \( T \approx L^2 / \tau \), where \( v_0 \) is a characteristic eddy velocity and \( L \) is the length scale of the rotation profile deviation. Choosing granulation velocity and lifetime scales for \( v_0 \) and \( \tau \) (using 1 km/s and 300s respectively) implies that \( T \) is only a few years or less. Thus, in this simple model, the lack of rotational shear in the vertical direction is a consequence of the strong anisotropy of the local convection. Global helioseismic model results and numerical simulations are evidently both critical parts of the puzzle. Full mean-field calculations by Kitchatinov and Rudiger (1999) and numerical experiments in rotating shells (Elliott et al. 2000) seem to be converging on a complete solution of the rotation problem in the convection zone. Perhaps at the next Tenerife meeting this problem will have only historical interest.

3.2. The Solar Cycle

One-dimensional solar models and helioseismic inferences agree to a few parts in \( 10^{-3} \) down to about 40% of a solar radius. One might ask where the ultimate limits to these models lie – is it in the data or the physical assumptions they’re based on? An obvious limitation comes from the simple fact that the Sun is neither static nor is it one-dimensional. Even though there is evidence for acoustic solar changes on many timescales, it is the 11/22 year variations which are most likely to elucidate the Sun’s solar cycle (henceforth abbreviated SC), and it is this regime which we restrict this discussion to.

Twelve years ago we were in the midst of appreciating the evidence for the SC in the helioseismic data. Recent measurements of this changing solar asphericity are summarized in Howe et al. (1999). Evidently this solar cycle is acoustically similar to the previous and is well described by “even” frequency splitting coefficients which determine the solar asphericity. These observables are of order a few hundred nanohertz but vary by the same magnitude during the SC. This implies that the solar structural asphericity is of the same order as its temporal variation. From a strictly dimensional point-of-view it suggests that the dominant solar asphericity may be a consequence of the SC. It is also interesting because the asphericity corrections are only a factor of 10 smaller than current uncertainties in 1D models – leaving little room for improvement here without incorporating a better understanding of the temporal and aspherically variable Sun.

A great advantage of our new experimental helioseismic perspective is the existence of uniform timeseries from long running experiments. While we did have measurements from a sequence of experiments spanning most of a cycle in 1988, we had nothing like the MDI, GONG, LOWL and IRIS data we now have to consider. For the most part these data confirm and extend our qualitative view of the previous acoustic solar cycle. Thus, as for the solar rotation problem, it is not the seismic data which limits our understanding – we have much to learn about the acoustic solar cycle.

At this point in a solar cycle discussion it is customary to reflect on how the solar “dynamo” is relevant. Of course the term “dynamo” is tautological when used synonymously with the generic concepts of flux transport and field generation – key issues we obviously must grapple with to describe the SC. As the helioseismic data, and the last 20 years of precision solar photometry have so aptly shown, we should recognize that the SC is more than a “dynamo” problem. In concert with surface magnetic field changes any meaningful explanation must also account for large scale solar irradiance, luminosity, and acoustic changes. This 11 or 22 year solar oscillation mode is highly non-linear and physically “rich” – it is an oversimplification to refer to it as the “solar dynamo.”
Unfortunately, mean field solar dynamo theories have been remarkably unsuccessful at predicting observables, even the photospheric magnetic field configuration. Petrovay (2000) charitably noted in his review during the meeting here last week that "...no breakthrough has been made in the solar dynamo problem for decades...And the prospects do not seem to promise a spectacular change in this situation in the near future." This state seems to indicate that current models are fundamentally incomplete, in that the kinematic and magnetic configuration of the Sun is insufficient to determine the SC. Recognizing that there are global acoustic and irradiance signatures to the SC, this statement seems entirely uncontroversial.

3.2.1. Solar Cycle Observables

A useful solution to the SC problem requires a causal description of irradiance (or surface brightness), acoustic, and magnetic changes. As we noted above (except for the centrifugal contributions) the solar asphericity appears to be intimately related to the solar cycle. While magnetic and brightness asphericity measurements only sample the surface, the acoustic measurements reach the interior and so are a unique and particularly powerful tool for exploring the SC. To make quantitative sense of the various asphericity observables we need a formalism for relating the frequency splitting observations to the surface measurements (and interior models). A reasonable ansatz is to consolidate all of the perturbative local physics at work throughout the solar interior into an "effective local sound speed." To the extent that the solar shape and radius do not vary significantly during the course of the solar cycle this ansatz is practical and we are safe to ignore surface boundary effects. With SOHO/MDI came the ability to measure very small astrometric changes in the radius (Emilio et al. 2000) and shape (Kuhn et al. 1999) of the Sun. In fact, these data have established that SC mode frequency changes are not caused by changes in the "position" of the solar limb.

We can connect models of sound speed asphericity with splittings by taking

$$\delta c^2(\tau, \theta) = c^2(\tau)(1 + \sum_s c_s f_s(\tau) P_s(\cos\theta)),$$

$$\delta f_{nlm} = \sum_s b_{nlm} P_s(-m/l).$$

Here the coefficients $c_s$ and $b_{nlm}$ parametrize the Legendre polynomial angular dependence of the sound speed, and the Legendre polynomial angular harmonic index dependence of the observed frequency splittings. Only even values of $s$ contribute to each of these sums. The functions $f_s(\tau)$ describe the depth dependence of $c^2$ perturbations. It is a straightforward first order perturbation calculation to show that $\delta c^2$ produces frequency shifts of $\delta f_{nlm} = \sum_s R_{nlm} S_{nlm} f_{nl}$ where $f_{nl}$ is an unperturbed mode frequency and $S_{nlm} \approx (-1)^{l/2} (l-1)!! R_m(l).$ Here $R_{nlm}$ is a radial integral of the mode energy density and the radial perturbation $f_s(\tau)$ and these expressions are valid for even indices $s$ (cf. Kuhn 1996).

Since the 5-min modes have their energy density concentrated near the surface and the functions $f_s$ are, by construction, normalized to the surface we can derive an approximate relation between $b$ and $c$ coefficients by setting $f_s(\tau) = 1.$ In this case we obtain,

$$b_{2nl} = -0.2c_2 f_{2nl}, \quad b_{4nl} = 0.2c_4 f_{4nl} \ldots$$

In general the $c_s$ coefficients alternate in sign and decrease slowly in amplitude with larger even $s$ index. Two important aspects of this calculation are that: 1) frequency splitting coefficients in the Legendre expansion for a given multiplet are directly related to the corresponding surface asphericity (Kuhn 1988), and 2) there are no arbitrary scaling coefficients needed to relate splittings and surface asphericity data. For example, the scale of the surface brightness variations is one degree Kelvin. To the extent that this measures $\delta c^2$ at the surface this implies $c_s$ of about $2 \times 10^{-4}.$ At a frequency $f_{nl} = 3\text{MHz}$ this implies $b_{2nl}$ of order 100 nHz – which is what was observed (Kuhn 1988) during the last cycle and the current (Howe et al. 1999, Dziembowski et al. 2000).

Splitting data from this solar cycle are in reasonable agreement with the brightness asphericity measured during the last cycle. In Fig. 8 we've compared the splittings in the current cycle (determined by GONG, Howe et al. 1999), against the temperature asphericity from the previous cycle. Exact agreement isn't possible given the simplicity of this model, but it is evident that this SC is qualitatively very similar to the previous.

The most recent measurements of splitting coefficients (Howe et al. 1999 and Dziembowski et al. 2000)
2000) have been used to show that temporal variations in the magnetic flux and the Ca II K limb brightness, when they are resolved into angular Legendre coefficients, are also well correlated in time with splitting coefficients. Goode et al. (2000) also noted that recent facular brightness observations correlate well with splittings. While figure 8 shows that the brightness asphericity from the previous cycle matches the splittings from this SC, it also shows that the facular-only asphericity has the same temporal dependence as the overall surface brightness asphericity. Furthermore, during much of the cycle the error resulting from using a facular-only brightness asphericity to compute splittings would be relatively small. Given the photospheric spatial correlation between CaII K, magnetic, and facular brightness determinations, perhaps it should not be surprising that each of these asphericity observables qualitatively (to within an arbitrary scale factor) describe the helioseismic splittings. Of course the question still remains as to what the causal relationship between the observables is. I again emphasize that the brightness asphericity calculations do not incorporate any scaling parameters to match splittings and asphericity coefficients, unlike comparisons with magnetic flux, and CaII K asphericity which lack quantitative models for these relationships.

3.2.2. Key Questions

A realistic SC model must account for all the observable clues — including irradiance, helioseismic, and magnetic changes. Essentially all models account for these changes as a consequence of the SC evolution of the magnetic field, although the physics of how this takes place is not well developed. Note, for example, that the magnetic field alone cannot describe helioseismic frequency shifts (Jiménez-Reyes et al. 1998).

One of the most interesting questions is if (and how) the asphericity has any direct coupling to changes occurring near the base of the convection zone. The alternative is that the irradiance and seismic changes are an incidental consequence of the SC magnetic field evolution in the very outer regions of the convection zone. There is some evidence, from both Howe et al. (1999) and Dziembowski et al. (2000), for asphericity perturbations beneath the outer superadiabatic zone but the empirical results are not obviously consistent.

It has been argued that the SC toroidal field which accumulates in the radiative zone must perturb the locally stable stratification. A consequence of this is that emerging photospheric flux should be accompanied by enhanced luminosity (Kuhn 1996, 1998). In this model the perturbation at the base of the convection zone drives the SC changes in luminosity, so we can estimate the magnitude of the variable interior SC asphericity near the tachocline. The emergent $10^{-3}$ luminosity perturbation integrated over a SC is about $10^{35}$ erg. If this energy originates in the tachocline (with a thickness of 35,000 km) we expect the sound speed there to vary by $\delta c/c$ of order $10^{-6}$ or $10^{-5}$. This might soon be observable (cf. Howe et al. 1999) — especially if the tachocline is thinner in radial extent than 0.05 solar radii.

Perhaps the greatest qualitative advance in experimental helioseismology since our last Tenerife meeting is the extension of high precision solar acoustic observations over most of a solar cycle. With these data we are approaching the sensitivity required to sample interior SC changes that are relevant to the irradiance. This promises to be an area of exciting new developments during this and future meetings.

4. FUTURE PROGRESS

Most helioseismology reviews written over the last decade lament our failure to detect solar gravity modes. Summaries from the 1998 GONG/SOHO meeting were no exception as it seems well established by now that the most important clues for understanding how the Sun works will come from measurements of low frequency g modes. As Provost et al. (2000) demonstrate through their calculations, even a 0.1% measurement of mode frequencies (we measure p mode frequencies with an accuracy two orders of magnitude better) will place significant constraints on the solar age, metallicity, and luminosity. The current state of this hunt is summarized by Appourchaux et al. (2000) which I will not duplicate in this discussion. Suffice it to say that a careful search of MDI doppler, GONG, VIRGO, and BiSON data failed to detect g modes with an amplitude limit of about 1 cm/s near 200 $\mu$Hz.

4.1. Gravity Mode Detection Prospects

Figure 9 illustrates the range of predicted g mode amplitudes. In most cases these have been estimated by assuming equipartition with the background convective velocity field, or from some other mode energy constraint. The rapid rise in surface amplitude above 200$\mu$Hz is a consequence of the radial mode structure (note the reciprocal mode inertia from PBM). The most positive conclusion we can draw from these estimates is that the largest g mode the Sun is likely to harbor will have a surface amplitude of a few tenths of a mm/s, but these amplitudes could be lower by one or two orders of magnitude. In comparison the solar velocity noise background has an amplitude of several mm/s.

This doesn't bode well, given current detection limits of about 10 mm/s. Fortunately, expected g mode damping times are 1000's to millions of years so that the duration of any observation will always be short in comparison. It follows that the relative spectral signal to noise will continue to improve linearly with observation duration. One conclusion from this is that if we're patient enough to continue current ob-
Figure 9. Estimates of g mode amplitudes are plotted here for a range of frequencies and assumptions. Calculations are from Toutain et al. (1996-TBPG), Kumar et al. (1996-KQB), Anderson (1996-A), Provost et al. (2000-PBM). The dotted line shows the expected solar velocity noise background in a one year observation. Parenthetical notations indicate angular g mode order. TBPG amplitudes are plotted assuming a mode energy 100 times smaller than the authors'. PBM estimates show an arbitrary scaling of the reciprocal mode inertia.

observations for another 10 to 20 years, we may achieve the necessary g mode sensitivity.

Perhaps we can do better by refining our search techniques. For example, do low frequency f and p modes have been convincingly measured by others (cf. Schou 1998b) down to frequencies below 600$\mu$Hz. Schou effectively demonstrates how prior knowledge of where to search can dramatically increase the detection sensitivity — the $p_1$ and $f$ modes he finds apparently have velocity amplitudes near 1 mm/s in the full-disk MDI doppler data. These amplitudes are significantly lower than the g mode sensitivity of Appourchaux et al. (2000). It is also interesting that low order f modes may have a radial structure similar to the centrally concentrated g modes (cf. Provost et al, 2000).

4.2. Better Techniques?

It seems that our best prospects for non-incremental improvements in g mode detection sensitivity must come from new techniques. One interesting possibility is to look for low frequency modes using astrometric methods. While these ideas were pioneered by Dicke, Hill, and collaborators from the ground, useful sensitivity levels which improve on the state-of-the-art must be achieved from above the Earth's atmosphere.

There is a significant advantage to detecting low frequency (coherent) oscillations from the corresponding limb displacement. Consider a solar oscillation with period $P$ and radial velocity amplitude $s$. Faced with detecting this oscillation over duration $T$, against an incoherent background characterized by velocity noise with an amplitude $v$, and mean time between zero crossings of $t$ (a "random telegraph signal" Rice, 1944) we obtain

$$P_{\text{noise}}(\omega) = \frac{2v^2t}{1 + 4\omega^2/t^2},$$

and

$$P_{\text{sig}} = \frac{s^2T\delta(\omega - 2\pi/P)}{2\pi},$$

for the noise background and signal velocity power spectral densities. The velocity power to background ratio is then $r = \frac{s^2T}{P^2}$. Similarly, the displacement power to background ratio is $r_{\text{disp}} = \frac{s^2T}{P^2} (P/2\pi)^2$. It follows that the relative improvement in signal/background ratio for displacement observations (also modeled as a random telegraph) in comparison to velocity measurements, $r$, is

$$r = \left(\frac{P}{2\pi}\right)^2.$$

This shows that there is a significant advantage to astrometric observations over velocity measurements for detecting oscillations with periods long compared to the characteristic timescale of the background noise. This reflects the fact that a coherent oscillation yields a displacement amplitude which increases linearly with its period, while the incoherent background, characterized by a velocity and shorter timescale, generates noise which increases only like the square root of the period. At low enough frequencies the velocity and displacement noise power spectra are Lorentzian and should approach a constant. Such a displacement power spectrum implies a decreasing velocity noise background with decreasing frequency. In support of this conclusion figure 10 shows the measured rms velocity amplitude determined from MDI limb astrometry (Kuhn et al. 2000).

To the extent that granulation noise (with a velocity scale of 1 km/s and timescale of a few minutes) dominates the solar background there is an advantage to astrometric searches for g modes, especially at shorter periods (notwithstanding the fact that the tangential/radial mode velocities scale like $lP^2$, cf. Kumar et al. 1996). Figure 11 shows the spatial order $k = 6$ amplitude spectrum derived from MDI limb astrometry. The plotted symbols show the expected low order f and p mode frequencies for $l > 5$ modes from Christensen-Dalsgaard (2000). We find no evidence for these modes in the astrometric data. Good evidence for longer period oscillations (r modes) has been detected from astrometric data (Kuhn et al. 2000).

4.3. New Experiments?

The SOHO/MDI experiment has opened the door on high precision solar astrometry and there are now at least two space experiments being planned or actually funded for construction. The French PICARD
The root-mean-square limb shape velocity amplitude is plotted here versus frequency. The average displacement power between angular frequencies of 20-256 has been converted to an rms velocity by scaling the amplitude by the temporal frequency. This is plotted on a logarithmic scale here to reveal the mean effective velocity background from 800 days of MDI astrometric observations.

Some of the research reported here was supported by NASA through the SOHO Guest Investigator program.

REFERENCES

Christensen-Dalsgaard, J. 2000 (personal communication).
Kuhn, J.R. 1996 The Structure of the Sun: VI Winter School at Instituto d’Astrophysica de Canarias, T. Roca Cortes, F. Sanchez, eds., Cambridge, 23.
Petrovay, K. 2000 The Solar Cycle and Terrestrial Climate, ed. A. Wilson, ESTEC