3-D General Relativistic MHD Simulations of Generating Jets

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Abstract. We have performed the first full 3-D GRMHD simulation of a Schwarzschild black hole with a freely falling corona. The preliminary simulation results show that the accretion disc is falling with the corona and that the proper pressure increases near the black hole, as in the previous axisymmetric simulations. We expect that in this case an instability around the black hole will occur as a result of the steep pressure gradient and the twisted magnetic fields. We plan to investigate how the instability affects jet formation.

1. Introduction

Relativistic jets have been observed in active galactic nuclei (AGNs) and microquasars in our Galaxy, and it is believed that they originate from the regions near accreting black holes. To investigate the dynamics of accretion discs and the associated jet formation, we use our newly developed full 3-D GRMHD code.

Recently, Koide, Shibata & Kudoh (1999) have investigated the dynamics of an accretion disc initially threaded by a uniform poloidal magnetic field in a non-rotating corona (either in a state of steady fall or in hydrostatic equilibrium) around a non-rotating black hole. The numerical results show that the disc loses angular momentum by magnetic braking, then falls into the black hole. The infalling motion of the disc, which is faster than in the non-relativistic case because of general-relativistic effects below $3r_S$ ($r_S$ is the Schwarzschild radius), is strongly decelerated at the shock formed by the centrifugal force around $r = 2r_S$ by the rotation of the disc, and plasma near the shock is accelerated by the $\mathbf{J} \times \mathbf{B}$ force, forming bipolar relativistic jets. Inside this magnetically driven jet, the gradient of gas pressure also generates a jet over the shock region (gas-pressure-driven jet). This two-layered jet structure is formed both in a hydrostatic corona and in a steady-state falling corona. Koide et al. (2000)
have also developed a new GRMHD code in Kerr geometry and found that, with
a rapidly rotating \((a = 0.95)\) black-hole magnetosphere, the maximum velocity
of the jet is \(0.9c\) and its terminal velocity \(0.85c\). All of the full GRMHD sim-
ulations described here were made assuming axisymmetry with respect to the
\(z\)-axis and mirror symmetry with respect to the plane \(z = 0\); the axisymmetric
assumption suppresses the azimuthal instabilities.

2. 3-D GRMHD Simulations: Equations and Numerical Techniques

Our basic equations are those of Maxwell for the fields and a set of general-
relativistic equations representing the plasma, namely the equations of conserva-
tion of mass, momentum, and energy for a single-component conducting fluid
(Weinberg 1972; Thorne et al. 1986). In making the simulations, we use these
equations with the 3+1 formalism (for details, see Koide, Shibata & Kudoh
1999).

3. Preliminary 3-D GRMHD simulations with a Schwarzschild black
hole

In order to investigate how accretion discs near black holes evolve under the
influence of accretion instabilities such as the magnetorotational instability, the
use of a full 3-D GRMHD code is essential.

3.1. Initial and boundary conditions

In the assumed initial state, the simulation region is divided into two parts: a
background corona around a black hole, and an accretion disc (Fig. 1a). The
coronal plasma is set in a state of transonic free-fall flow, as in the case of the
transonic flows with the ratio of specific heat capacities \(\Gamma = 5/3\) and \(H = 1.3\);
here the sonic point is located at a radius of \(r = 1.6r_S\). The Keplerian disc
in the corona is set up in the following way. The disc region is located at
\(r > r_D \equiv 3r_S, |\cos \theta| < \delta = 1/8\). Here the density is 100 times that of the
background corona (Fig. 1a), while the orbital velocity is relativistic and purely
azimuthal: \(v_\phi = v_K \equiv c/[2(r/r_S - 1)]^{1/2}\). (Note that this equation reduces
to the Newtonian Keplerian velocity \(v_\phi = \sqrt{GM/r}\) in the non-relativistic limit
\(r_S/r \ll 1\). The pressure of both the corona and the disc are assumed equal
to that of the transonic solution. The initial conditions for the entire plasma
around the black hole are:

\[
\rho = \rho_{fc} + \rho_{dis}
\]

\[
\rho_{dis} = \begin{cases} 100\rho_{fc} & (r > r_D \text{ and } |\cot \theta| < \delta) \\ 0 & (r \leq r_D \text{ or } |\cot \theta| \geq \delta) \end{cases}
\]

\[
(v_r, v_\theta, v_\phi) = \begin{cases} (0, 0, v_K) & (r > r_D \text{ and } |\cot \theta| < \delta) \\ (-v_{fc}, 0, 0) & (r \leq r_D \text{ or } |\cot \theta| \geq \delta) \end{cases}
\]
where we set $\delta = 0.125$; the smoothing length is $0.3r_S$.

In addition, there is a magnetic field crossing the accretion disc perpendicularly. We set it to the Wald solution (Wald 1974), which represents the uniform magnetic field around a Kerr black hole: $B_r = B_0 \cos \theta$, $B_\theta = -\alpha B_0 \sin \theta$ (where $\alpha$ is the lapse function, $\alpha = (1 - r_S/r)^{1/2}$). At the inner edge of the accretion disc, the proper Alfvén velocity is $v_A = 0.015c$ in a typical case with $B_0 = 0.3\sqrt{\rho_0 c^2}$, where the Alfvén velocity in the fiducial observer’s frame is

$$v_A \equiv B\sqrt{\rho + [\Gamma \rho/(\Gamma - 1) + B^2]/c^2}.$$  \hspace{1cm} (5)

The plasma beta of the corona at $r = 3r_S$ is $\beta \equiv \rho/B^2 = 1.40$. The simulation is performed in the region $1.1r_S \leq r \leq 20r_S$, $0 \leq \theta \leq \pi$, $0 \leq \phi \leq 2\pi$ with $200 \times 140 \times 30$ meshes. The effective linear mesh widths at $r = 1.1r_S$ and at $r = 20r_S$ are $2.9 \times 10^{-3}r_S$ and $0.55r_S$, respectively, while the angular mesh widths are $0.2 \times 10^{-2}$ in the polar direction and $20.8 \times 10^{-2}$ in radiative boundary condition is imposed at $r = 1.1r_S$ and at $r = 20r_S$:

$$u_{n+1}^0 = u_0^n + u_1^{n+1} - u_1^n,$$ \hspace{1cm} (6)

where the superscripts $n + 1$ and $n$ denote the time steps and the subscripts 0 and 1 refer to the boundary and to its neighbor meshes, respectively. The computations were made on an ORIGIN 2000 computer with 1.04 GB internal memory, and they used about 34 hours of CPU time for 2000 time steps with $200 \times 140 \times 30$ meshes.

3.2. Simulation results

Fig. 1 shows the evolution of 3-D simulation performed in the region $1.1r_S \leq r \leq 20r_S$, $0 \leq \theta \leq \pi$, and $0 \leq \phi \leq 2\pi$ with $200 \times 140 \times 30$ cells. The parameters used in this simulation are the same as those of the axisymmetric simulations shown in Fig. 6 of Koide, Shibata & Kudoh (1999). In this figure, the shading shows the proper mass density [(a) and (c)] and the pressure [(b) and (d)] on logarithmic scales; the vector plots show the flow velocity.

The black circle represents the black hole. Figs 1a and 1b present the initial conditions. At $t = 34.1r_S$, the free-falling corona drags and bends the magnetic field lines near the black hole as shown in Fig. 1d. Due to the coarseness of the grid in the azimuthal direction, a numerical instability occurred at $t = 34.1r_S$. Recently we have been making some calculations with a finer grid ($120 \times 100 \times 120$). After checking the initial simulation results more thoroughly we will increase the resolution again, probably to $200 \times 200 \times 200$ cells.

4. Discussion

This simulation result is preliminary and we need to perform more simulations with better resolution and to find an optimum resolution to generate a reliable solution.

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Figure 1. For the full 3-D simulation, these panels present the time evolution of the proper mass density with velocity \((v_x, v_z)\) [(a) and (c)] and the proper pressure with the magnetic field \((B_x, B_z)\) [(b) and (d)] in a transonic free-fall (steady-state falling) corona with an initially uniform magnetic field, at [(a) and (b)] \(t = 0.0\tau_S\), and [(c) and (d)] \(t = 34.1\tau_S\).
References


Weinberg, S. 1972, Gravitation and Cosmology (New York: Wiley)