Recent Developments for Balmer Line Photopolarimetry

C. Stehlé
*DASGAL et UMR 8633 du CNRS, Observatoire de Paris, 5 Place J. Janssen, 92195 Meudon, France*

G. Mathys, S. Brillant
*European Southern Observatory, Casilla 19001, Santiago 19, Chile*

T. Lanz
*NASA Goddard Space Flight Center, Code 681, Greenbelt, MD 20771, USA and Department of Astronomy, University of Maryland, College Park, MD 20771, USA*

**Abstract.** Mean longitudinal magnetic fields of stars are determined from observations of circular polarisation in their spectral lines. Two methods have been extensively used: the “photographic” technique, based on consideration of the polarisation of metallic lines, and Balmer line photopolarimetry, which relies on polarisation in the wings of hydrogen Balmer lines. For a number of Ap stars, these two methods yield different values of the longitudinal field. Recent theoretical developments on hydrogen line formation indicate that determinations of the longitudinal field by Balmer line photopolarimetry have so far been based on an incorrect interpretation of the observed polarisation. Using an appropriate description of the formation of the polarised hydrogen Stark broadened profiles, we have revisited this interpretation. Our improved treatment of hydrogen line formation is required to understand the reasons for the differences between existing photographic and photopolarimetric measurements of the magnetic field.

1. **Introduction**

The observation of circular polarisation in spectral lines allows one to measure the longitudinal magnetic field, which is the component of the magnetic field along the line of sight. Due to the lack of spatial resolution, current measurements cannot give information about the longitudinal field value $H_z(x, y)$ at any point $(x, y)$ of the stellar disk. Thus they are restricted to the mean longitudinal magnetic field $\langle H_z \rangle$, which is the average of the local field $H_z(x, y)$ on the stellar disk, using an appropriate weighting factor.

In the “photographic” technique (Babcock 1947), the spectrum is recorded simultaneously in right (RCP) and left (LCP) circular polarisation. The mean longitudinal magnetic field $\langle H_z \rangle_{\text{photo}}$ is derived from measurements of wave-
length shifts between the centres of gravity of spectral lines recorded in RCP and LCP through the relation:

$$\lambda_{\text{RCP}} - \lambda_{\text{LCP}} = -\frac{2}{W_\lambda F_\ell} \int \mathcal{F}_V \lambda d\lambda = 2 \tilde{g} \Delta Z < H_z >_{\text{photo}},$$  \hspace{1cm} (1)

where $\mathcal{F}_V$ and $F_\ell$ denote respectively the integral over the stellar disk of the emergent intensity in Stokes $V$ (in the line) and $I$ (in the continuum close to the line), and $W_\lambda$ is the equivalent width of the line. $\tilde{g}$ is the effective Landé factor of the transition and $\Delta Z = 4.67 \times 10^{-13}$ Å G\(^{-1}\). The mean longitudinal field $< H_z >_{\text{photo}}$ is defined by:

$$< H_z >_{\text{photo}} = \frac{\int dx \int H_z(x, y) \epsilon(x, y) dy}{\int dx \int \epsilon(x, y) dy},$$  \hspace{1cm} (2)

where $\epsilon(x, y)$ is a weighting factor accounting for the limb darkening effect and defined as:

$$\epsilon(x, y) = \frac{I(x, y, \mu, \lambda)}{I(x, y, \mu = 1, \lambda)}.$$  \hspace{1cm} (3)

$I$ is the emergent intensity at the wavelength $\lambda$ (at point $(x, y)$ of the disk), for a direction of radiation making with the normal to the disk an angle whose cosine is equal to $\mu$.

In “Balmer line photopolarimetry” (Angel & Landstreet 1970), one measures alternatively the right and left circular polarisation signal in the wings of the Balmer lines, through a narrow interferometric filter. The mean longitudinal magnetic field is derived by application of the relation:

$$\mathcal{F}_V = -\Delta Z < H_z >_{\text{Balmer}} \frac{dF_\ell}{d\lambda}.$$  \hspace{1cm} (4)

For a number of Ap stars, differences are found between the mean longitudinal magnetic field values derived by the photographic technique and by Balmer line photopolarimetry. The nature and magnitude of the differences vary from star to star, and sometimes along the rotation cycle of a given star (see e.g. Mathys 1991).

2. The limb-darkening problem

Equation (4) rests on the assumption that, locally, the emergent intensity in Stokes parameters $V$ and $I$ obeys the following relation:

$$V(x, y) = -\Delta Z H_z(x, y) \frac{dI(\lambda, x, y)}{d\lambda}.$$  \hspace{1cm} (5)

Then $< H_z >_{\text{Balmer}}$ is obtained by integrating $H_z$ over the stellar disk:

$$< H_z >_{\text{Balmer}} = \frac{\int dx \int H_z(x, y) \epsilon'(x, y) dy}{\int dx \int \epsilon'(x, y) dy},$$  \hspace{1cm} (6)
with a definition of the limb-darkening factor different from that of Eq. (3):
\[
\epsilon'(x, y) = \frac{dI(x, y, \mu, \lambda)}{dI(x, y, \mu = 1, \lambda)}
\] (7)

The difference in the expression of the limb darkening factor accounts only partly for the difference in the values of the mean longitudinal field derived through the two methods.

3. Microscopic processes

Equation (5) rests on the assumption that the magnetic splitting of the Balmer lines is adequately described by the general formalism of linear Zeeman effect. This is not the case in the line wings. In a plasma, the hydrogen atom is also submitted to interactions with the perturbing protons and ions. These interactions can be described in terms of random electric fields that increase with the electronic density. In the line wings, broadening is attributed to strong electric fields, which to first order induce a linear Stark splitting. The split Stark sublevels are perturbed by the magnetic field.

We have computed the line shapes of Lyman lines with an accurate description of these effects. We have also included the effect of the fine structure. We have shown that the correct relation in the line wings is (Brillant, Mathys, & Stehle 1998; Stehle, Brillant, & Mathys 2000):
\[
V(x, y) = -\frac{4}{5} \Delta Z H_z(x, y) \frac{dI(x, y)}{d\lambda}.
\] (8)

Thus, the correct relation to be used for the diagnosis of the mean longitudinal field through Balmer line photopolarimetry is (Mathys et al. 2000):
\[
\mathcal{F}_V = -\frac{4}{5} \Delta Z < H_z >_{\text{Balmer}} \frac{d\mathcal{F}_I}{d\lambda}.
\] (9)

4. Numerical experiment

For a typical stellar model, we have computed synthetic H\beta line profiles in the Stokes parameters I and V. From these profiles, we have derived \(< H_z >_{\text{Balmer}}\) by application of Eq. (9). We have compared this “measured value” \(< H_z >_{\text{Balmer}}\) with the input magnetic field of the problem, using the form given by Eq. (7) for the limb-darkening function (Mathys et al. 2000).

The characteristics of the atmosphere are \(\log(L/L_\odot) = 2.78, T_{\text{eff}} = 16400\, \text{K}, M/M_\odot = 5.0, \log g = 4.3\), metal abundances: 10 times solar, angle between rotation axis and line of sight \(i = 43^\circ\), \(v \sin i = 35\, \text{km s}^{-1}\). The magnetic field topology is given by the superposition of collinear dipole, quadrupole and octupole (respective polar field strengths at the surface: 16700 G, 9500 G and −4000 G). The angle between magnetic and rotation axes is \(\beta = 87^\circ\).

A good agreement is found between the input and measured field values, as can be seen from Fig. 1.
5. Conclusions

We have identified two contributions to the discrepancy between the values of the mean longitudinal field derived through the photographic technique and Balmer line photopolarimetry. The first one is a difference in the weighting of the disk-average of $H_\alpha$. The second one comes from the microscopic description of the line polarisation in the line wings. The description of the atomic states must take into account the electric fields affecting the hydrogen atom, the order of magnitude of which is comparable to or even larger than the Zeeman effect. Future work will cover application of this more realistic physical description of the line formation to actual observations.

References