Magnetic Doppler Imaging of CP Stars

N. Piskunov

Department of Astronomy and Space Physics, Uppsala University,
Box 515, S-751 20 Uppsala, Sweden

Abstract. We present the new Magnetic Doppler Imaging (MDI) code INVERS10 capable of reconstructing the vector distribution of magnetic field over the surfaces of CP stars. We study the performance of the code with numerical experiments and present the first application to a real data set for $\alpha^2$ CVn.

1. Introduction

The knowledge of the detailed structure of magnetic fields on the surface of Ap and Bp stars is very important for understanding the origins of magnetic fields, their relation to the chemical composition spots and to the pulsations on roAp stars. It may also shed some light on the evolution of magnetic fields.

For many years the fields on magnetic CP stars were assumed to be dipolar. In the same time we have multiple evidences that they are not! We also know that dipolar fields can mimic many different observational signatures if only partial data is available (e.g. only $I$ and $V$ Stokes profiles). Observations of all four Stokes parameters may be sufficient to reconstruct the true field geometry without additional assumptions.

The main goal of this project is to build a tool which is able to reconstruct the geometry of the magnetic field without explicit assumptions about field model (e.g. multipolar geometry), to verify that we get a unique solution, to specify the requirements for the observations (S/N, resolution, phase coverage etc.) and to test the method on a real star where a certain field geometry is to be expected.

We started from a well understood Doppler Imaging (DI) technique, generalized the radiative transfer part to four Stokes parameters and optimized the code for parallel execution using Message Passing Interface (MPI). Then the new code INVERS10 was extensively tested using numerical experiments for different field geometries (e.g. multipolar, magnetic spots) with and without simultaneous reconstruction of the abundance map. In the following sections we briefly describe the code, show the results of the most important tests and the reconstruction of the magnetic field on $\alpha^2$ CVn using $I$ and $V$ Stokes data.
Figure 1. Block diagram of MDI iteration: starting maps provide the vector of magnetic field, abundance and radial velocity for each grid cell. Local Stokes vector is computed by solving the equation of magnetic radiation transfer. The Stokes vectors are integrated over the visible part of the stellar surface and compared with the observations. The surface maps are modified to minimize the discrepancy.

2. The code

The basic principle of the MDI (Fig. 1) is similar to the conventional DI (e.g. Piskunov & Rice 1993). The main difference is the number of simultaneously reconstructed maps and the radiative transfer. INVERS10 minimizes a regularized discrepancy function:

\[
\Phi(\vec{B}, Z) = \sum_{\phi, \lambda} \omega^I \left[ I^c(\vec{B}, Z) - I^o \right]^2 + \sum_{\phi, \lambda} \omega^Q \left[ Q^c(\vec{B}, Z) - Q^o \right]^2 + \\
+ \sum_{\phi, \lambda} \omega^U \left[ U^c(\vec{B}, Z) - U^o \right]^2 + \sum_{\phi, \lambda} \omega^V \left[ V^c(\vec{B}, Z) - V^o \right]^2 + \Lambda \cdot R(\vec{B}, Z) = \min
\]

where \( R \) is the regularization function. The integral Stokes components \( I, Q, U \) and \( V \) are computed via disk integration of the local Stokes profiles. The solution of the radiation transfer equation for the local Stokes profiles dominates the computations, therefore it is logical to distribute different surface elements between parallel processors. INVERS10 is using MPI to organize the parallel processing. It achieves automatic load balance and linear scaling of the performance up to a few hundred processors.

Three methods have been tested for solving the MRT equation: magnetic Feautrier (Auer, Heasley, & House 1977), DELO (Rees, Murphy, & Durrant 1989) and Hermitian method (Ruiz Cobo, Bellot Rubio, & Collados 1999). As a reference we used the Runge-Kutta algorithm. The comparison of the methods shows that:

- Magnetic Feautrier is accurate even on a sparse grid but requires the solution of \( 3N \) systems of linear equations each with \( 4 \times 4 \) matrix and \( 4+5+1 \) different RHS. The potential numerical stability problem can be handled with pivoting (Piskunov 1999).
Figure 2. The comparison of the accuracy achieved with the three MRT solvers for the continuum using 10 points per decade of optical depth. The Hermitian method seems to perform below expectations at the high optical depths which affects the accuracy at the surface.

- DELO is a fast converging short characteristics method, with only 2N systems of linear equations with 4+1 RHS. The parabolic approximation for the source function (Socas-Navarro, Trujillo Bueno & Ruiz Cobo 2000) significantly improves the accuracy, but it is still inferior to the Feautrier on sparse grids.

- The Hermitian method uses 4th order piecewise approximation to the Stokes vector instead of the source function. We find that it has problems at high optical depths ($\tau_\lambda > 8$).

Figure 2 summarized these results and the magnetic Feautrier method was our choice for the calculations presented in this paper.

Another important issue is the regularization function. In conventional DI two types of regularization are used: Tikhonov and Maximum Entropy. In the case when a problem has a unique solution (that is there are sufficient observational data with small errors) the role of regularization is reduced to prevent the minimization procedure from leaving the sensible parameter range and thus the exact form of the regularization function is not important. Although it was never proven that the DI or MDI problem has a unique solution if a perfect data set is available, extensive experiments carried by many authors suggest that it is the case. When only partial data is available (e.g. only Stokes $I$ and $V$ in MDI) the solution is clearly non-unique and the regularization must compensate for the missing data. In this case the exact form of the regularization function is critical. The best solution would be to use a theoretical model of the field, for example, a combination of low order multipoles, which seems to be appropriate to CP stars. On the other hand, restricting the field to low order multipoles is too restrictive. We will not be able to tell how consistent with the data the multipolar model is. Instead we have introduced the multipolar regularization
which is the difference between the current solution and the best multipolar fit to this solution. Such approach has three advantages: we do not have to make any assumptions about the orientation of the multipoles, we get the best multipolar solution (together with the real one) and we get to know how significant are the deviations from the multipolar field model. We should stress again that multipolar regularization is only required when the solution is not unique, that is when we do not have a complete set of observations.

3. Numerical experiments

Numerical experiments consist of setting up an artificial model of magnetic field and abundance distribution, generating a realistic set of “observations” and solving the inverse problem (1). This allows us to test the stability and convergence properties of the method as well as its sensitivity to typical problems (e.g. phase gap, errors in rotational velocity, inclination etc.). Figure 3 shows one such experiment using the full set of Stokes parameters. Tikhonov regularization was applied to map the magnetic field and the abundances. The reconstruction is very similar to the original model. Among surprises, we found that MDI is capable of reproducing the maps, including abundance, for even very slow rotators ($v \sin i \leq 10 \, \text{km} \, \text{s}^{-1}$). This is related to the fact the rotational modulation of the field orientation is more important than Doppler shifts while the polarization profiles provide additional information for the abundance distribution.
Figure 4. Numerical experiment with $I$ and $V$ Stokes parameters. The original model (panel I) consists of dipolar magnetic field. The reconstructions with Tikhonov and multipolar regularizations are shown in panels II and III.

Figure 4 illustrates the application of multipolar regularization in a case where only $I$ and $V$ Stokes data are available.

4. $\alpha^2$ CVn

The new code was used to reconstruct the field geometry and abundance distribution for a well known magnetic Ap star, $\alpha^2$ CVn. The results are presented in a poster (Kochukhov and Piskunov, this volume). The consistency of the field geometry obtained from the lines of different chemical elements confirms the reliability of the reconstruction using INVERS10.

References

Discussion

WADE: Your Cr map of $\alpha^2$ CVn shows $\approx 3$ dex abundance contrast. In this case the disk-integrated Stokes profiles are strongly weighted toward high-abundance spots and hence sample the disk in a non-uniform way. One would expect a rather spotty reconstruction of the field.

PISKUNOV: Although the central depth of the local line profiles of Cr changes by a factor of 8, the maximum relative polarization $V/I$ is nearly constant which is also clearly visible in the co-variance matrix where the components related to the abundance have a patchy structure while those related to the field do not.

SCHMIDT: $\alpha^2$ CVn yielded quadrupole to dipole ratio of about 0.25 while all stars in the sample of Bagulo showed this ratio to be larger than 1.

PISKUNOV: $\alpha^2$ CVn was extensively studied by different techniques including broad-band linear polarization and Hydrogen line polarization measurements. These methods are relatively insensitive to the abundance inhomogeneities and they show that the field is dominated by the dipolar component similar to what we get with MDI.

DONATI: I strongly disagree with the fact that data sets with all 4 Stokes parameters contain enough spectral information on the field to recover accurately the magnetic distribution of organized field even in cases as simple as those of dipolar fields. I showed that the inversion can exhibit a very different field distribution that reproduces perfectly the input synthetic 4 Stokes data set, but does not in any ways resemble the dipole used to compute the synthetic data set. This proves that the solution to this problem is not unique contrary to what you stated in your talk. You therefore had to make some assumptions about the field structure and the multipolar regularization you are using is undoubtedly one such assumption (since you are selecting on this criterion one image out of all possible). I personally find that this regularization is far from satisfactory as it is purely arbitrary. Assuming local constraints on the field structure (e.g. potential, force free field) is certainly much more physical than the multipolar regularization, while allowing at the same time a perfect reconstruction of organized field structures from all Stokes data set, and even from $I$ and $V$ data only.

PISKUNOV: We could not find a configuration of an organized field that could not be reconstructed from 4 Stokes parameters. We interpret this as an indication that MDI has a unique solution if sufficient data is available. As for multipolar regularization, we only use it for the partial data set (e.g. $I$ and $V$ data only). With 4 Stokes parameters we use Tikhonov regularization, which (by the time the convergence is achieved) has a minor influence on the final solution, very similar to the conventional DI.

DEL TORO INIESTA: How do the errors in your study of different radiative transfer schemes compare with the authors of the Hermitian method? I would also like to see an octupolar field reconstructed with a dipolar model.

PISKUNOV: I am also surprised that the Hermitian method did not perform up to the expectation. I intend to contact the authors to verify if my implementation of the method is optimal before publishing the final conclusions. The A&A publication describing the numerical experiments with INVERS10 will be submitted by mid-2001. It will contain the experiments with high-order multipoles.

RUEDIGER: Only linear theories provide a simple mode such as dipole (aligned or orthogonal). Non-linear are always yielding spectra of modes, never a simple one. It makes only restricted (?) sense to fight against the results of linear MHD-theories.

PISKUNOV: First, we learned to reconstruct reliably non-dipolar fields. Now we are trying to understand at least the most important systematic effects and only then we will be able to confront the non-linear theory predictions.