Magnetic Cycles and Activity in FGK Stars in the Framework of Babcock-Leighton Dynamos

M. Dikpati

HAO/NCAR, 3450 Mitchell Lane, Boulder, CO 80301, USA

S.H. Saar

CFA, MS-58, 60 Garden St., Cambridge, MA 02138, USA

N. Brummell

JILA, University of Colorado, Boulder, CO 80309, USA

P. Charbonneau

HAO/NCAR, 3450 Mitchell Lane, Boulder, CO 80301, USA

Abstract. Motivated by the data on long-term stellar magnetic activity variations, and by the recent success of Babcock-Leighton type flux-transport dynamos in explaining many large-scale solar cycle features, we explore the viability of a similar dynamo in G stars over a wide range of rotation rates ($\Omega_c \leq \Omega \leq 100\Omega_c$). We show that with suitable scaling of the differential rotation, meridional circulation, surface poloidal source term, and turbulent diffusion, this class of model correctly predicts that faster rotators should produce more magnetic flux and have shorter cycle periods than slower rotators, as observed. We also explore how the cycle period in such models may vary in stars cooler than the Sun.

1. Introduction

All cool dwarf stars show magnetic activity, and those with significant convective envelopes (> early F) have activity levels which increase with rotation rate. Long-term monitoring of Ca II HK and photometry show that: (i) F stars are less active at the same rotation frequency ($\Omega$) and tend to show less cyclic activity variation compared to G stars; (ii) K stars are more active at the same $\Omega$ and show activity cycles more often (compared to G stars); (iii) many stars (all masses) with intermediate activity levels show evidence for two cycle periodicities. Magnetic dynamos are the likely cause of all these activity features.

The Babcock-Leighton flux-transport solar dynamo has recently been quite successful in reproducing many large-scale solar cycle features (Wang, Sheeley & Nash 1991; Choudhuri, Schüssler & Dikpati 1995; Durney 1995; Dikpati & Charbonneau 1999; Charbonneau & Dikpati 2000). The major ingredients in this class of dynamo are differential rotation (DR), meridional circulation, a

235
surface poloidal source term (the result of the decay of tilted active regions), and turbulent diffusion. We simulate a Babcock-Leighton flux-transport stellar dynamo in G stars over a wide range of rotation rates (Ω_⊙ ≤ Ω ≤ 100Ω_⊙), and study the strength of the toroidal field B_φ generated in the shear layer as a function of (1) DR, (2) shear layer thickness d_s, (3) turbulent diffusivity η_T and (4) meridional flow speed u_m. We then take DR ∝ Ω^{0.67} (Donahue, Saar & Baliunas 1996), and assume u_m ∝ Ω to obtain B_φ as a function of Ω. The assumption that the Ca HK activity index (a plage proxy) and photometric variation (a starspot proxy) scale with B_φ then allows comparison with observation.

2. Dynamo model calculation

Formulating the Babcock-Leighton type flux-transport stellar dynamo, we solve the following two kinematic equations for toroidal and poloidal field (under the assumption of axisymmetry) in spherical polar coordinates:

\[
\frac{\partial A}{\partial t} + \frac{1}{ω} (u_m \cdot \nabla)(ωA) = η_T (\nabla^2 - \frac{1}{ω^2})A + S(r, θ, B_φ),
\]

\[
\frac{\partial B_φ}{\partial t} + ω \nabla \cdot \left( \frac{u_m B_φ}{ω} \right) = ω (B_p \cdot \nabla)Ω(r, θ) - \nabla η_T \times (\nabla \times B_φ e_φ) + η_T (\nabla^2 - \frac{1}{ω^2})B_φ,
\]

where ω = r sin θ, B_p = ∇ × A e_φ is the poloidal field, and S a non-local poloidal field source-term nonlinear in B_φ (see Dikpati & Charbonneau 1999).

3. Results

Our main results are shown in Figures 1B-D (1A shows the schematic diagram of the model). By far the largest effect on the toroidal field B_φ(r_E) comes from a decrease η_T, a consequence of the fact that the poloidal field suffers less resistive decay as it is carried by meridional circulation to the thin shear layer beneath the core-envelope interface. A decrease in d_s leads to an increase in B_φ(r_E) (see Figure 3). There are two competing effects at play here (i) with the decrease in d_s, the radial shear is enhanced, increasing B_φ; (ii) the dissipation rate also increases with decreasing d_s, which tends to reduce B_φ. Here the first effect dominates. The toroidal field increases more significantly with Ω, a consequence of our assumption DR ∝ Ω^{0.67} (Figure 1D). In fact B_φ scales almost linearly with DR, since shear is the only B_φ-amplification mechanism in the model. The value of u_m hardly affects B_φ, although it is the primary determinant of the cycle period (see below).

4. Variation of cycle period with spectral type

Although Dikpati & Charbonneau (1999) only presented dynamo solutions for a solar model, their results make it possible to estimate how the cycle period P_cycle may vary with spectral type. The crucial result is the scaling law:

\[ P_{cycle} \propto u_m^{-0.89} s_0^{-0.13} η_T^{0.22}, \]

where s_0 is the amplitude of the source term and u_m.
the meridional flow speed. In Babcock-Leighton models, $P_{\text{cyc}}$ is set primarily by the turnover time of the circulation. Since $u_m$ is slowest at the core-envelope interface (see Figure 2), flow along $L$ dominates the turnover time, and one may write $P_{\text{cyc}} \sim L/u_m$. A K-star has an envelope that extends deeper in fractional depth $r_E/R$, and the star is also smaller so that $L \approx 0.7L_\odot$. The results of Kitchatinov & Rüdiger (1999) indicate that $u_m$ at the base of the envelope is also smaller in K-stars, which partly cancels the effect of the smaller $L$. In both models $L/u_m$ then differ only by about 15%, leading to $P_{\text{cyc}}$ differing by about 10% as per eq. (3). Even if the turbulent velocities at the base of the envelope differ in G and K stars, possibly leading to different $\eta_T$, this has little bearing on the model’s $P_{\text{cyc}}$ since the dependence on $\eta_T$ is quite weak.

A weak dependency on spectral type, with the primary dependency on $\Omega$, is consistent with current $P_{\text{cyc}}$ data. In particular, there is no clear distinction between G and K stars in the active and inactive branches identified by Saar & Brandenburg (1999; also this volume) in the observed $P_{\text{cyc}}-P_{\text{rot}}$ diagram.
Figure 2. An estimate of Babcock-Leighton cycle period in G and K stars. The dashed arc is the core-envelope interface (radius $r_E$). The solar model has $R/R_\odot = 1$, $r_E/R = 0.7$, $L/R_\odot = 1.1$, and $u_m = 500$ cm s$^{-1}$; the K-star model has $R/R_\odot = 0.8$, $r_E/R = 0.6$, $L/R_\odot = 0.76$, and $u_m = 300$ cm s$^{-1}$. $P_{cyc} \sim L/u_m$ is almost the same in both models.

5. Concluding comments

Two robust features of observed activity cycles materialize naturally within the Babcock-Leighton framework: (1) increase of magnetic activity with $\Omega$; and (2) weak dependence of $P_{cyc}$ on spectral type. As with mean-field based models, these results are predicated on modeling assumptions regarding the variations of $u_m$ and DR with spectral type and rotation rate. Parity issues notwithstanding (see Dikpati & Gilman, this volume), Babcock-Leighton dynamos offer an explanatory framework for stellar dynamos that is evidently worth pursuing.

Acknowledgments. We thank Keith MacGregor for his helpful review of the manuscript. This work is partially supported by NASA grants W-19752, S-10145-X, and NSF grants AST-9528563 and AST-9731652. The National Center for Atmospheric Research is sponsored by the National Science Foundation.

References