Stellar Dynamos: A Modeling Perspective

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Abstract. Hydromagnetic dynamo action is the most likely origin for the magnetic fields observed or inferred in the Sun and most stars. After discussing two current competing dynamo models for the solar cycle, based respectively on mean-field electrodynamics and the Babcock-Leighton mechanism, we examine the degree to which extant activity cycle period determinations in late-type stars can help discriminating between variants of these two classes of models. We then briefly review recent modeling work pertaining to dynamo action in the convective core of early-type main-sequence stars, where the primarily difficulty is to carry the dynamo-generated magnetic field to the stellar surface.

1. Introduction

Fundamentally, a dynamo is a physical system that converts mechanical into electromagnetic energy, specifically electrical currents and their associated magnetic fields. Stars have two vast reservoirs of mechanical energy: rotation and convection. As both of these are ubiquitous across the HR diagram, stellar dynamo action should be the rule rather than the exception.

While most late-type stars are relatively slow rotators, from the standpoint of dynamo theory nearly all stars rotate “rapidly”\(^1\). The relevant quantity is the inverse Rossby number \(R_\theta^{-1} = 2\Omega\tau_c\), where \(\tau_c\) is the the convective turnover time and \(\Omega\) is the rotational frequency; if \(R_\theta^{-1} \geq 1\), the turbulence is affected by rotation. For the Sun, mixing length estimates for \(\tau_c\) yield \(R_\theta^{-1}(\odot) \sim 1\).

Stellar dynamo modeling is a risky proposition, considering that the solar dynamo is still not satisfactorily understood despite decades of modeling efforts and new helioseismic measurements of two key dynamo “ingredients”: internal differential rotation (=DR) and meridional circulation. The various dynamo models discussed below make use of drastic simplifications, especially with regards to the small-scale components of the magnetic field and flow. Nonetheless,

\(^1\)The more slowly rotating Ap stars, with \(P_{\text{rot}} \gg 100\) days, are the most notable exceptions; see Moss, and Rüdiger, in this volume, for alternate viewpoints on the origin of \(B\) in Ap stars.

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experience with the Sun indicates that such models provide useful frameworks in describing the large-scale field, which is the only stellar magnetic field component currently measurable. We thus proceed with these simple models.

In the MHD limit, the amplification of a magnetic field $\mathbf{B}$ by a flow $\mathbf{U}$ is governed by the induction equation. Both $\mathbf{U}$ and $\mathbf{B}$ include a small-scale component associated with thermally-driven convective turbulence. Mean-field electrodynamics provides a practical framework within which to deal with these troublesome small scales (for the full frontal assault alternative, see Cattaneo, this volume). Both $\mathbf{U}$ and $\mathbf{B}$ are separated into (mean) large-scale and (fluctuating) small-scale components, averaging is carried out over a suitable intermediate scale, and the few terms remaining involving small-scales are expressed in terms of the mean field (e.g., Moffatt 1978). In doing so, the induction equation picks up two additional terms on its RHS, a turbulent magnetic diffusivity $\eta_T$ and the crucial source term known as the "$\alpha$-effect":

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times \left( \mathbf{U} \times \mathbf{B} + \alpha \mathbf{B} - (\eta + \eta_T) \nabla \times \mathbf{B} \right). \quad (1)$$

For axisymmetric systems, there are two source terms on the RHS of the $\phi$-component of eq. (1), one involving the $\alpha$-effect\(^2\) and the other shearing of the poloidal field by differential rotation $\Omega(r, \theta)$. When the latter term dominates, one has an "$\alpha \Omega$ dynamo"; in the converse case, a "$\alpha^2$ dynamo". The general case, with both source terms present, is (what else?) an "$\alpha^2 \Omega$ dynamo". The large-scale flow $\mathbf{U}$ can include a meridional circulation $\mathbf{u}_m$, which can contribute significantly to the evolution of the magnetic field. A dynamo model requires that one specifies functional forms for $\Omega, \mathbf{u}_m, \alpha$, and $\eta_T$; (in the linear, kinematic regime), and their dependence on $|\mathbf{B}|$ for nonlinear models.

2. Dynamo action in the Sun

Solar dynamo modeling has become a much tougher game since helioseismology revealed the form of the Sun’s internal DR (Brown et al. 1989; Tomczyk et al. 1995; and references therein). Figure 1A shows a parametrized solar-like DR profile, used in dynamo models presented below, which catches the essential solar characteristics: latitudinal DR in the convective envelope, matched smoothly onto a rigidly rotating radiative core across a thin shear layer (the tachocline) roughly at the core-envelope interface. Because the tachocline is characterized by strong radial shear and lies partly in the subadiabatic outer radiative core, it is a prime location to store and amplify magnetic fields, and is currently seen as the preferred site of dynamo action (see Schüssler 1996).

The observed form of $\Omega(r, \theta)$ for the Sun turns out to be problematic from the dynamo point of view. Because of the strong radial shear present in the polar part of the tachocline (Fig. 1A), with the “minimal” $\cos \theta$ latitudinal dependency for the $\alpha$-effect the dynamo finds itself concentrated at high latitudes, contrary to observations (see Fig. 2A). Only by artificially concentrating the $\alpha$-effect in

\(^2\)The $\alpha$-effect is the only source term in the poloidal component of eq. (1) and is thus essential to evade Cowling’s antidynamo theorem.
the equatorial regions can a solar-like butterfly diagram be recovered (Fig. 2B). A related problem is the sign of the $\alpha$-effect; theory and simulations indicate that it should be positive in the N-hemisphere, except perhaps at the very base of the envelope. Yet a negative $\alpha$-effect is required in conjunction with the positive radial shear in the equatorial part of the tachocline to yield equatorward propagation of the dynamo wave.

Meridional circulation can offer a way out of this quandary. If the equatorward flow at the base of the envelope is fast enough (i.e., if the associated local magnetic Reynolds number $u_mR/\eta_T \gg 1$), circulation can produce apparent equatorial propagation of the dynamo wave even with a positive $\alpha$-effect (Choudhuri, Schüssler & Dikpati 1995). However, models of this type recently computed by Küker, Rüdiger & Schultz (2001) and one of us (PC; as yet unpublished) using a solar-like DR indicate that the situation is likely more complex than suggested by the Choudhuri et al. model, which has $\Omega = \Omega(r)$ only$^3$.

Alternatively, the solar dynamo may be of the Babcock-Leighton type, making the $\alpha$-effect irrelevant. These models rely on the decay of bipolar active regions to release poloidal flux in the surface layers, where meridional circulation carries it first poleward, then down to the base of the envelope, where it is sheared into a toroidal component (see Dikpati & Charbonneau 1999, hereafter DC99, and references therein). Figure 2C is a solar-like Babcock-Leighton

$^3$Specifically, a steady mode feeding on the latitudinal shear dominates in the nonlinear regime within a vast portion of parameter space, a difficulty that also characterizes other types of mean-field models with solar-like DR (see, e.g., Markiel & Thomas 1999).
Figure 2. Time-latitude diagrams of the N-hemisphere toroidal field evaluated at the core-envelope interface $r/R_\odot = 0.7$. If flux ropes giving rise to sunspots form in regions of strongest toroidal fields and rise radially to the surface, then such plots are equivalent to the sunspot butterfly diagram. All solutions use the solar-like DR and $u_m$ of Fig. 1. Solution (A) is an $\alpha\Omega$ mean-field model with the “minimal” $\cos(\theta)$ latitudinal dependency for the $\alpha$-effect, while solution (B) has the $\alpha$-effect artificially concentrated in the equatorial regions. Solution (C) is a Babcock-Leighton solar-like solution from DC99.
butterfly diagram (from DC99). This model reproduces well many features of
the solar cycle, including the oft-troublesome phase relationship between the
poloidal and toroidal magnetic components.

Thus two distinct classes of solar dynamo models emerge, relying on ei-
ther small-scale turbulence or surface decay of active regions to regenerate
the poloidal field. To which class does the Sun belong? Observations of mag-
netic activity cycles in cool stars may be useful in probing this question.

3. Dynamo action in solar-type stars

We focus on how the cycle periods \(P_{\text{cyc}}\) predicted by the solar-like mean-field
and Babcock-Leighton models of Fig. 2B and 2C vary with \(\Omega\). In linear mean-
field models with spatially coincident \(\alpha\)-effect and shear,

\[
P_{\text{cyc}} \propto (\alpha_0 (\Delta \Omega) R / \eta_T)^{-1/2}, \quad \text{[Linear Mean \dash Field]},
\]

where \(\alpha_0\) and \(\Delta \Omega\) measure the magnitude of the \(\alpha\)-effect and differential ro-
tation. In general, this relation cannot be expected to hold in nonlinear models.
Introducing classical \(\alpha\)-quenching in the model of Fig. 2B, i.e., \(\alpha \to \alpha(|B|) \propto
(1 + |B|^2)^{-1}\), leads to:

\[
P_{\text{cyc}} \propto \alpha_0^{-0.54} (\Delta \Omega)^{-0.57}, \quad \text{[Nonlinear Mean \dash Field]},
\]

similar to the linear regime, and in good agreement with the results of Tobias
(1998; hereafter To98) for his comparable \(\alpha\)-quenched 2D global model without
diffusivity quenching. The cycle period scaling for such models stands in marked
contrast to \(\alpha\)-quenched dynamo wave solutions computed in semi-infinite Car-
tesian slabs, for which \(P_{\text{cyc}}\) shows very little dependence on \(\alpha_0\) or \(\Delta \Omega\) (as in, e.g.,
the model of Rüdiger & Arlt 1996; see also To98, Fig. 2). Both mean-field mod-
els only have a very weak dependence on \(u_m\) in the parameter range considered
\(u_m(R_c) R / \eta_T \lesssim 1\). For solar Babcock-Leighton models, DC99 find:

\[
P_{\text{cyc}} \propto s_0^{-0.13} \eta_T^{0.22} u_m^{-0.89}, \quad \text{[Babcock \dash Leighton]}. \quad (4)
\]

Note that \(u_m\) is now the primary determinant of \(P_{\text{cyc}}\). In particular, \(P_{\text{cyc}}\) is
expected to show only weak dependence on \(\Delta \Omega\), although the strength of the
dynamo-generated field grows significantly with increasing DR and decreasing
\(\eta_T\) (see Dikpati et al., this volume).

To study the variation of \(P_{\text{cyc}}\) with rotation we must specify how the various
physical parameters on the RHS of the above expressions scale with \(\Omega\). Many
reasonable possibilities exist, leading to a variety of “theoretical” \(P_{\text{cyc}}\dash\Omega\ rela-
tionships. For solar-type stars Kitchatinov & Rüdiger (1999) find \(\Delta \Omega \propto \Omega^{-0.15}
\) and \(|u_m| \approx 4.3 \log(\Omega / \Omega_\odot) + 5\) at the base of the envelope (see their Figs. 1
and 2). Alternately, Donahue et al. (1996) infer surface \(\Delta \Omega \propto \Omega^{-0.67}\) from sea-
sonal \(P_{\text{rot}}\) variations in the Ca II HK time series, while the rotating turbulence
simulations of Brummell et al. (1998) suggest that \(u_m\) should increase with \(\Omega\)
in the laminar, low rotation regime. For the \(\alpha\)-effect, mean-field theory predicts
\(\alpha_0 \propto \Omega\) for slow rotation \(\left(R_{\odot}^{-1} \lesssim 1\right)\), but for higher rotation the situation is less
clear; \(\alpha\) may decrease with increasing \(\Omega\) (“rotational \(\alpha\)-quenching”).

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Stellar $P_{\text{cyc}}$ can be determined via analysis of timeseries of plage (Ca II HK core emission; e.g., Baliunas et al. 1995) or spot proxies (photometry; e.g., Olah et al. 2000). The pioneering study of Noyes et al. (1984) demonstrated the existence of a relatively well-defined $P_{\text{cyc}}-\Omega$ relationship in (relatively) inactive stars. In this spirit, Figure 3 uses the data collected by Saar & Brandenburg (1999, and this volume) for solar-type dwarfs ($0.56 \leq B-V \leq 0.76$; F9-G8), plotted as a function of $\Omega$. The various curves correspond to eqs. (2)—(4) above, with different assumptions for the dependence of $\alpha$, $\Delta \Omega$ and $u_m$ on $\Omega$ (see Table).

Table 1. Ingredients for $P_{\text{cyc}}-\Omega$ theoretical relationships of Fig. 3.

<table>
<thead>
<tr>
<th>Curve</th>
<th>$P_{\text{cyc}}$ relation</th>
<th>$\alpha$-scaling</th>
<th>$\Delta \Omega$-scaling</th>
<th>$u_m$-scaling</th>
</tr>
</thead>
<tbody>
<tr>
<td>LMF1</td>
<td>Eq. (2)</td>
<td>$\alpha_0 \propto \Omega$</td>
<td>$\Delta \Omega \propto \Omega^{+0.67}$</td>
<td>none</td>
</tr>
<tr>
<td>NMF1</td>
<td>Eq. (3)</td>
<td>$\alpha_0 \propto \Omega$</td>
<td>$\Delta \Omega \propto \Omega^{+0.67}$</td>
<td>none</td>
</tr>
<tr>
<td>NMF2</td>
<td>Eq. (3)</td>
<td>$\alpha_0 \propto \Omega$</td>
<td>$\Delta \Omega \propto \Omega^{-0.15}$</td>
<td>none</td>
</tr>
<tr>
<td>NMF3</td>
<td>Eq. (3)</td>
<td>none</td>
<td>$\Delta \Omega \propto \Omega^{+0.67}$</td>
<td>none</td>
</tr>
<tr>
<td>NMF4</td>
<td>Eq. (3)</td>
<td>none</td>
<td>$\Delta \Omega \propto \Omega^{-0.15}$</td>
<td>none</td>
</tr>
<tr>
<td>BL1</td>
<td>Eq. (4)</td>
<td>N/A</td>
<td>none</td>
<td>$u_m \propto \log(\Omega)$</td>
</tr>
<tr>
<td>BL2</td>
<td>Eq. (4)</td>
<td>N/A</td>
<td>none</td>
<td>$u_m \propto \Omega$</td>
</tr>
</tbody>
</table>

Saar & Brandenburg (this volume) find that in single (or wide binary) FGK dwarfs, $P_{\text{cyc}}$ shows two branches: $P_{\text{cyc}} \propto \Omega^{-1.15}$ and $P_{\text{cyc}} \propto \Omega^{-0.80}$ for inactive and active stars, respectively. Among the options studied (Table 1), we find that these power-law exponents are only approximately achievable with (i) mean-field models having $\alpha \propto \Omega$, and $\Delta \Omega \propto \Omega^{0.67}$ (LMF1, NMF1); or (ii) Babcock-Leighton models with $u_m \propto \Omega$ (BL2). Thus, available stellar cycle data appears to restrict models to those with relatively strong dependence of DR, $\alpha$, and/or $u_m$ on $\Omega$. If on the other hand one attempts to fit all the stellar data with a single $P_{\text{cyc}}-\Omega$ relationship, then models NMF4 and BL1 look the most promising. It should be noted, though, that these fits, which show $P_{\text{cyc}} \propto \Omega^{0.1}$, have significantly higher rms scatter than the “multi-branched” variety; see Saar & Brandenburg (1999).

Two nonlinear mechanisms were not considered here. The first is magnetic quenching of the turbulent diffusivity (Rüdiger et al. 1994). The modeling results of To98 for 2D global mean-field models similar to that used here indicate that $\eta$-quenching has only a moderate influence on the $P_{\text{cyc}}-\Omega$ relationship (see his Fig. 4 and accompanying discussion). This stands again in contrast to semi-infinite 1D or Cartesian slab models, where $\eta$-quenching has a strong influence on the cycle period (see Rüdiger & Arlt 1996). Since 2D global models are likely more relevant to the Sun, we tentatively conclude that $\eta$-quenching is not a primary determinant of the cycle period. The second mechanism not explored here is large-scale magnetic back-reaction on the DR. Its influence on $P_{\text{cyc}}$ can be quite complex, and has received comparatively little attention in the literature\(^4\).

\(^4\)Based on a small set of such solutions, obtained for a single value of the magnetic Prandtl number, and covering only a small range in dynamo number, To98 cautiously reports a weaker scaling of $P_{\text{cyc}}$ on the dynamo number than in the case of $\alpha$-quenching.
Figure 3. \( P_{\text{cyc}} \) vs. \( \Omega \) in solar-type dwarfs (F9-G8; *, plus secondary \( P_{\text{cyc}} \) given by + and connected with a dotted line), plus various curves constructed using relationships indicated in Table 1, and scaled to the Sun (⊙). Active (A) and inactive (I) star dynamo “branches” for single FGK dwarfs are shown for comparison (heavy solid lines; from Saar & Brandenburg, this volume).

4. Dynamo action in early-type stars

There is mounting evidence for the existence of magnetic fields in the atmosphere of early-type stars (see, e.g., reviews by Wade; Henrichs; Cassinelli; all in this volume). The rapid rotation and vigorous core convection in O and B-stars leaves little doubt that dynamo action is possible in their core. The challenge, however, is to bring the field to the stellar surface. Because the convective core is surrounded by a thick, stable envelope that is a good electrical conductor, the magnetic field is trapped in the deep interior (Schüssler & Pähler 1978).

Figure 4 illustrates this difficulty. Panel (A) shows an \( \alpha^2 \) solution computed for constant \( \eta \), taken from Charbonneau & MacGregor (2001; hereafter CM01). The magnetic field has no difficulty threading the surrounding radiative envelope. If a diffusivity contrast between core and envelope is introduced, though, the field is trapped close to the core-envelope interface (panel B). The situation is even worse with oscillatory \( \alpha \Omega \) or \( \alpha^2 \Omega \) type solutions, since the electromagnetic skin depth then further limits the outward spread of \( B \).

Thermally-driven meridional circulation is expected in the radiative envelope of a rotating, early-type stars (e.g., Tassoul 1978, and references therein). For near-solid body rotation, the flow is equatorward in the bulk of the envelope, with the poleward return flow confined to a thin boundary layer adjacent to the core-envelope interface (Tassoul & Tassoul 1982, Figs. 3, 4). Interestingly, this is where the dynamo eigenfunctions have their peak amplitude in the \( \eta_e/\eta_c \ll 1 \)
regime, the most relevant here (see Fig. 4B). Can circulation drag the field up to the stellar surface? CM01 have explored this, and found that as one increases $u_m$, dynamo action is impeded before significant surface fields materialize. This is a robust conclusion, independent of the details of the circulation flow profile, or on the mode of dynamo action.

MacGregor & Cassinelli (2001; hereafter MC01) have investigated an alternate, more promising mechanism. They start with the hypothesis that (as in the Sun) conditions at the core-envelope interface of an early-type star support the formation of toroidal magnetic flux ropes. Horizontal pressure equilibrium forces a lower gas density inside the rope, which then feels an upward-directed buoyant force. Using the thin flux tube approximation, MC01 follow the rising path of the rope under the combined influence of buoyancy, magnetic tension, Coriolis force, and aerodynamic drag. Their results indicate that flux ropes that are sufficiently strong ($\sim 10^6$ Gauss; $\approx$ in equipartition with core convection) and sufficiently thin (diameter $\lesssim 10^{-4}$ scale height) can rise to $r/R_* \simeq 0.95$ in a small fraction of the main-sequence lifetime$^5$

If the dynamo-generated B does reach the stellar surface in the form of flux ropes, the solar analogy suggests one should expect variable flare-like X-ray emission due to magnetic reconnection taking place between the emerging flux ropes and the ambient magnetic field. It might be fruitful to monitor the X-

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$^5$At first sight this conclusion is at odds with that reached by Moss (1989); this is due to the fact that MC01 have assumed much thinner flux tubes, in line with current solar inferences.
rays of early-type stars with known surface fields; non-variable emission would be difficult to reconcile with the flux rope hypothesis. Moreover, since little if any convection is expected in the outer stellar layers, the emerged magnetic flux ropes might take much longer to decay than they do in the Sun, leading to long-lived ($\gg P_{\text{rot}}$) rotational modulation in photospheric and wind properties.

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Discussion

MOSS: If you use buoyancy to carry field from the core of a massive star across its envelope, would this not reduce the efficiency of the dynamo, in the same way that efficient flux removal by meridional circulation does?
CHARBONNEAU: It might reduce it, without altogether shutting off the dynamo. This is what seems to be happening in the Sun, where according to most models the erupted magnetic flux is a small fraction of the dynamo-generated flux. This certainly still needs to be looked into in the case of massive stars.
PRIEST: So, for the Sun, does an interface \( \alpha \Omega \) dynamo or a Babcock-Leighton dynamo better explain the observations?
CHARBONNEAU: In the present state of modeling both types of models do more or less as well, with a slight edge for Babcock-Leighton perhaps. However the Babcock-Leighton models seem more “robust” with respect to nonlinearities and imposed stochastic fluctuations, so at this writing I am inclined to favor them over mean-field models.
S. VAUCLAIR: We expect stellar winds in hot stars. How would this affect your results? Also, the meridional circulation may not be steady; can you comment about this?
CHARBONNEAU: Mass loss might help “expose” magnetic field having risen through the bulk of the envelope, so if anything mass loss would help. As for non-steady meridional circulation, having experimented with a variety of steady circulation patterns I see no \textit{a priori} reason why non-steady circulation, or multicell circulation patterns for that matter, could lead to significant field dredge-up without killing the dynamo.
REISENEGGER: Do you see a way, in your models, to explain why Ap and Bp stars show strong surface magnetic fields, but most early-type stars don’t?
CHARBONNEAU: I see no natural, straightforward way to absorb that dichotomy entirely within the dynamos I discussed here. At this point I remain inclined to favor the fossil field hypothesis for Ap and Bp stars.
JOHNS-KRULL: What can you say about the fully convective stars? Do we expect activity cycles or other particular types of variation?
CHARBONNEAU: Hard to guess at this stage. In the related case of core dynamos, nothing dramatic happens as you gradually turn on differential rotation at the core-envelope interface and transit from \( \alpha^2 \) to \( \alpha^2 \Omega \) dynamo modes, other than the solution going from steady to oscillatory. You certainly still get dynamo action, though you might well lose cyclic activity. The central question in this context is whether fully convective stars support significant levels of differential rotation. If so, cyclic activity definitely remains a possibility.
PISKUNOV: What parameter(s) determine the direction of dynamo wave propagation in late-type stars?
CHARBONNEAU: In classical mean-field models it is the sign of the product between the radial shear and \( \alpha \)-effect, unless meridional circulation in the dynamo region is faster than the phase speed of the dynamo wave. In Babcock-Leighton models, propagation of the dynamo wave (which is then no longer really a dynamo wave in the usual sense) is controlled by the meridional flow at the base of the convective envelope.