Magnetic Fields across the Hertzsprung-Russell Diagram
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Magnetic Field Diagnostic Techniques Based on the Zeeman and Hanle Effects

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Abstract. A brief review of the methods that are commonly employed in the determination of magnetic fields in the solar and in stellar atmospheres is presented. Such methods can be classified into two broad categories according to the physical mechanism (the Zeeman effect or the Hanle effect) responsible for the appearance of the spectro-polarimetric signal employed in the diagnostic of the magnetic field. The basic differences between the two categories are analyzed and a comparison is made in order to clarify the main features inherent to either effect when used as a diagnostic tool for the measurement of the magnetic field. It is concluded that the methods based on the Zeeman effect are in general more robust and that they require a simpler mathematical formalism for the interpretation. On the other hand, the Hanle effect provides the unique possibility of determining the full magnetic field vector in the weak intensity regime where $B$ is of the order of a few Gauss or less.

1. Introduction

When facing the problem of measuring (or of deducing the presence of) a magnetic field in an astronomical object, the scientist generally proceeds to obtain a detailed spectrum of the object and then looks for the possible presence of characteristic spectral signatures induced by the magnetic field. With the remarkable exception of a typical broadening of spectral lines, which is at the basis of an important diagnostic method mostly used for cool stars, the spectral signatures due to the magnetic field show up most prominently in polarization. For this reason, spectropolarimetry is the technique that has contributed the most to our present understanding of magnetism in extra-terrestrial objects.

It is well known since more than a century that most spectral lines split up into three or more components in the presence of a magnetic field. The principal property of this phenomenon, usually called the Zeeman effect, is that the splitting is linear in the magnetic field intensity, and that the different components have different polarization signatures which sharply depend on the direction of the magnetic field vector. The Zeeman effect thus provides a very important diagnostic tool for the magnetic field and the large majority of observations of this physical quantity in stars and other astrophysical objects heavily rely on this effect.
Less known by the astronomical scientific community is the Hanle effect, which can be considered as a kind of by-product of the Zeeman effect because its action is ultimately due to the same phenomenon of splitting induced by the magnetic field. The Hanle effect, which consists of a modification of resonance polarization which takes place when a magnetic field is present in the scattering plasma, has been mainly used for the diagnostic of weak magnetic fields in solar prominences. Nowadays, it is also employed for the diagnostic of chromospheric fields and it is not to be excluded that it will be used in the future for measuring magnetic fields in the solar corona and in extended stellar envelopes and in stellar winds.

This review is mainly intended to describe the various methods that have been developed to measure magnetic fields in stellar atmospheres -with particular emphasis on the solar case- and to outline the differences between the Zeeman and the Hanle effects from the point of view of their diagnostic contents.

2. Measurements of magnetic fields via the Zeeman effect

The traditional methods that have been employed for the diagnostic of magnetic fields are all based on the Zeeman effect. Out of spectropolarimetry, a new discipline, magnetometry, has emerged through the years. Magnetometry can be defined as the "art" of measuring magnetic fields from the analysis of polarimetric observations performed in one or more spectral lines and its birthday can be fixed at 1908. By means of a Fresnel rhomb (acting as a quarter-wave plate) and a Nicol prism (acting as a polarimeter), George Ellery Hale succeeds in taking two separate spectra of a sunspot in right and left circular polarization. The line shifts observed by comparison of the two spectra lead him to to the discovery of the existence of magnetic fields in extra-terrestrial objects (Hale 1908).

Magnetometry has enormously evolved since the pioneering work of Hale and is now a mature discipline which employs sophisticated technologies and modern astronomical instrumentation. The photographic plate used by Hale has been replaced, during the years, by photomultipliers, diode arrays, and CCD cameras. Similarly, the Fresnel rhomb and the Nicol prism have been substituted by new polarimeters with higher and higher sensitivity and accuracy. Finally, the advent of techniques of image stabilization and the construction of new telescopes in proper sites has opened the way to observations of higher and higher spatial and temporal resolutions.

On the other hand, all these technological improvements have been accompanied by a parallel effort aimed at a better understanding of the theoretical aspects of the physics of the generation and transfer of polarized radiation in stellar atmosphere. As a consequence, many practical methods are nowadays available to extract from polarimetric measurements the main information concerning the magnetic field vector.

Without pretending of being exhaustive, we can just quote here the following techniques, especially developed for the interpretation of solar observations and all based on the Zeeman effect: a) the longitudinal magnetograph technique (Babcock 1953); b) the vector magnetograph technique; c) the line-bisector technique (Rayrole 1967); d) the center-of-gravity technique (Semel 1967); e) the line-ratio technique (Stenflo 1973); f) the fitting technique to the
Unno-Rachkovsky solution (Skumanich & Lites 1987); g) the Stokes inversion based on response functions, or SIR, technique (Ruiz Cobo & del Toro Iniesta, 1992), and h) the broad-band linear polarization technique (Leroy 1962). For stellar observations, several of these techniques have been generalized and original diagnostic methods have also been developed. Among these we can quote: i) the technique based on the moments of the Stokes profiles (Mathys 1995a, b); j) the line-broadening technique (Robinson 1980), and k) the Zeeman-Doppler imaging technique (Donati & Semel 1990).

Each of these techniques has its own advantages and disadvantages. Moreover, each of them is more suitable for the interpretation of particular sets of data. Just to make an example, and referring for the sake of simplicity to the solar case, the longitudinal magnetograph technique is based on the interpretation of signals of the form

\[ S_V = \frac{\int V(\lambda) p(\lambda) \, d\lambda}{\int I(\lambda) p(\lambda) \, d\lambda}, \]

where \( I(\lambda) \) and \( V(\lambda) \) are the Stokes parameters of the radiation coming from a resolution element of the solar atmosphere and where \( p(\lambda) \) is a wavelength profile which depends on the particular magnetograph employed and which typically represents the transmission function of a narrow-band filter centered on the wing of a Zeeman sensitive spectral line. It is well known that, as a first order approximation, the signal \( S_V \) is proportional to the line-of-sight component, \( B_\parallel \), of the magnetic field vector, or, in formulae

\[ S_V = C_\parallel B_\parallel, \]

where \( C_\parallel \) is the calibration constant of the magnetograph. However, it is also known that this approximate equation breaks down for sufficiently large magnetic field intensities (when the Zeeman splitting is comparable or larger than the typical width of the line, as an order of magnitude), which points to a severe limitation of the longitudinal magnetograph technique.

As a further example, we can consider the fitting technique to the Unno-Rachkovsky solution of the radiative transfer equation. This technique, which has been brought to a high degree of sophistication by the polarimetry group of the High Altitude Observatory working on the ASP (Advanced Solar Polarimeter) data, is particularly suitable to recover the magnetic field vector from the Stokes parameters profiles observed in a wide spectral interval typically covering one or more magnetically sensitive spectral lines. The basic idea which lays behind the method is the minimization, in the parameters’ space, of a merit function of the form

\[ M (\{\zeta_i\}) = \sum_{j=1}^{4} \sum_{\alpha=1}^{N} W_{j\alpha} \left[ S_{j\alpha}^{(\text{obs})} - S_{j\alpha}^{(\text{thr})} (\{\zeta_i\}) \right]^2, \]

where \( S_{j\alpha}^{(\text{obs})} \) is the \( j \)-th Stokes parameter \((j = 1, \ldots, 4)\) observed at spectral pixel \( i \), \((i = 1, \ldots, N)\), \( S_{j\alpha}^{(\text{thr})} (\{\zeta_i\}) \) is the corresponding value of the theoretical Stokes parameter depending on the set of parameters \( \{\zeta_i\} \), and \( W_{j\alpha} \) are suitable weights that can be adjusted to give more emphasis, in the fitting routine, to particular
Stokes parameters or to particular wavelength-points. The theoretical Stokes parameters are given by the Milne-Eddington solution of the transfer equation for polarized radiation (usually referred to as the Unno-Rachkovsky solution) and depend on several parameters: the intensity, inclination and azimuth of the magnetic field vector, the Doppler broadening, damping constant, and relative strength of the line(s) (defined as the ratio between line integrated and continuous absorption coefficients), and the thermodynamical parameters $B_0$ and $\beta$ which define the linear behavior of the Planck function with continuum optical depth: $B(\tau) = B_0(1 + \beta \tau)$. All these parameters are supposed to be constant with depth in the atmosphere and a non-linear least squares routine allows to recover their best-fit values thus providing a “measurement” of the magnetic field vector and of the other thermodynamical parameters. This technique has brought important results in establishing the geometry of the magnetic field vector in sunspots and active regions. Its main limitations are, from one hand, the Milne-Eddington assumption on the thermodynamics of the atmosphere, and, from the other hand, the further assumption of the constancy with depth of the magnetic field. More recently, both these limitations have been overcome by a novel approach based on response functions (the SIR technique) which appears today to be one of the more promising tools of solar magnetometry.

All the technological improvements reached in solar spectro-polarimetry, joined with the specific inversion techniques developed for magnetometry have obviously led to a better understanding of solar magnetism but, at the same time, have also raised new questions about the meaning itself of the process of measuring a physical quantity, the magnetic field, which is pervading, often with an intermittent behavior, a highly structured plasma such as the solar atmosphere. Today, there is strong evidence that the solar magnetic field outside sunspot is concentrated in tiny structures having lateral dimensions which are at the limit, or even beyond the limit, of the resolving power that can be attained by the best instruments. Even more intriguing is the scenario emerging from some new ideas which point to the existence in the solar atmosphere of magnetic fields organized in fibrils of some tens of km in diameter (the MISMA hypothesis, Sánchez Almeida et al. 1996) and of ‘turbulent’ fields of a few Gauss permeating the full solar atmosphere outside active regions (Stenflo 1994). Although these ideas have not yet been fully confirmed by observations, they undoubtedly suggest the difficulties inherent in the measurement of a quantity that may show sharp variations over length scales smaller than the resolution element of the telescope and of the scale height of formation of spectral lines.

Most of the methods and the techniques that have been developed for solar magnetometry ignore, however, this kind of difficulties and are based on the simple assumption that the magnetic field is uniform across the resolution element, though, in some cases, the “filling factor” hypothesis is introduced. This is a severe limitation that hopefully will be overcome in the near future.

3. Measurements of magnetic fields via the Hanle effect

Though less known by the astrophysical community, the Hanle effect has been shown to be a very powerful diagnostic tool for inferring the magnetic field vector in the outer layers of the solar atmosphere and, particularly, in prominences
(Bommier 1977). As stated in the introduction, the Hanle effect consists in a modification of the linear polarization of scattered radiation with respect to its value in the absence of magnetic fields. In most cases, such a modification turns out to be a depolarization and a rotation of the plane of linear polarization, although, in other cases, the modification may just consist of the opposite phenomenon, namely an increase of the value of polarization.

The full theory of the Hanle effect is rather complicated and it implies the use of a heavy formalism involving a description of the atomic system in terms of the density-matrix operator and a description of the radiation field in terms of suitable polarization tensors. The theoretical framework makes full use of some tools that are rather unfamiliar to astronomers, such as the Racah algebra and the algebra of irreducible spherical tensors (see, for instance, Landi Degl’Innocenti 1982).

Without entering into a detailed description of the theory, it is convenient to illustrate the Hanle effect by means of a schematic experiment where an unpolarized, unidirectional beam of radiation is resonantly scattered by a simple model-atom consisting of a lower level with total angular momentum quantum number \( J_L = 0 \), and an upper level with \( J_u = 1 \). In a 90° scattering experiment, the frequency integrated radiation scattered by the atom is totally linearly polarized, the direction of linear polarization being perpendicular to the scattering plane. When a magnetic field is introduced in the scenario, the polarization is deeply modified in a way that depends on the intensity of the magnetic field and on its direction. Considering for instance the case of a magnetic field aligned with the scattering direction, the fractional linear polarization is given by the expression

\[
pl = \frac{1}{\sqrt{1 + 4H^2}} ,
\]

and the direction of linear polarization is rotated –with respect to the zero-field direction– by an angle \( \alpha \) given by (positive rotation means counterclockwise looking against the scattered beam)

\[
\alpha = \frac{1}{2} \arctan(2H) .
\]

The parameter \( H \) entering these expressions is the ratio between the Larmor (angular) frequency associated with the magnetic field and the Einstein coefficient for spontaneous de-excitation of the atom, namely \( H = 2\pi \nu_L / A \). Expressing \( B \) in Gauss and \( A \) in units of \( 10^7 \) s\(^{-1} \), one has, numerically

\[
H = 0.88 \frac{B}{A} .
\]  

(1)

Considering on the contrary a magnetic field directed perpendicularly to the scattering plane, the direction of linear polarization remains unchanged with respect to the zero-field situation, whereas the fractional linear polarization decreases according to the expression

\[
pl = \frac{1 + 2H^2}{1 + 6H^2} ,
\]
which shows that even for very large magnetic fields the fractional polarization
does not go to zero but reaches the asymptotic value $p_L = 1/3$.

A different case which illustrates the polarizing role that may played by
the magnetic field is the case of forward scattering. Here, the scattered beam
is unpolarized in the absence of a magnetic field. On the contrary, introducing
a magnetic field perpendicular to the beam’s direction, the scattered beam gets
linearly polarized, the direction of polarization being aligned with the magnetic
field, and the fractional polarization being given by

$$p_L = \frac{H^2}{1 + 3H^2}.$$ 

The illustrative examples presented here are just intended to give the reader
an intuitive grasp of the Hanle effect, but the interpretation of observations has
to be performed by means of the general theory outlined above. Most of the
astrophysical applications of the Hanle effect have concerned solar prominences.
This is because for this type of objects the scattering geometry is relatively
well defined and because the radiation field scattered by the prominence plasma
is also well known (in as far as the prominence is optically thin), being the
solar photospheric spectrum. In practice, one has just to solve the statistical
equilibrium equations for the density-matrix elements of the model-atom consid-
ered. The solution can then be directly substituted into the relevant equations
which relate the (polarized) emission coefficient in the line of interest with such
density-matrix elements. The emissivity in the four Stokes parameters can then
be compared with observations in order to retrieve the magnetic field vector
providing a best-fit with observations. More recently, a theoretical effort has
been devoted to the interpretation, in terms of resonance polarization and the
Hanle effect, of polarimetric observations performed over the solar disk. Here
the situation is much more complex because the statistical equilibrium equations
for the density-matrix elements have to be solved self-consistently with the
radiative transfer equation for polarized radiation. These calculations can
be considered as a natural generalization to the polarized case of the classical
non-LTE calculations developed over the past decades for the diagnostic of stel-
lar atmospheres and the overall problem of the self-consistent solution is now
referred to as non-LTE of the 2\textsuperscript{nd} kind. The solution of this problem has been
made possible by the generalization to the polarized case of highly convergent
iterative methods that had been previously developed for the standard, non-LTE
multilevel problem (Trujillo Bueno & Landi Degl’Innocenti 1997; Trujillo Bueno

It has also to be remarked that we have here spoken only about the upper
level Hanle effect and that, for transitions different from the simple transition
considered above, $J_L = 0, J_u = 1$, the theory also predicts the existence of
a ground level Hanle effect connected with the phenomenon of depopulation
pumping. The relevant parameter which controls this effect is given by
$H' = 2\pi\nu L/(B\mathcal{J})$, where $\mathcal{B}$ is the Einstein coefficient for absorption and $\mathcal{J}$ is the solid
angle average of the radiation field at the frequency of the transition. Since
in most astrophysical plasmas the quantity $B\mathcal{J}$ is smaller (often of orders of
magnitude) than $\mathcal{A}$, through the lower level Hanle effect it is possible, at least
in principle, to diagnose magnetic fields of the order of fractions of 1 Gauss or even smaller.

4. **Comparison between the Zeeman effect and the Hanle effect**

Both the Zeeman and the Hanle effect can be used as diagnostic tools for the measurement of magnetic fields in astrophysical objects. Each of the two effects has its own peculiarities, and this Section is devoted to make a direct comparison in order to better clarify the advantages and disadvantages of the one with respect to the other in different physical situations.

First of all, it has to be remarked that the Hanle effect is a typical Non-LTE phenomenon; an indispensible ingredient for its existence is the presence of an asymmetry in the excitation conditions of the atom, like, for instance, in the case where the atom is illuminated by an external source of radiation or where it is excited by collisions with a collimated beam of particles. The Hanle effect does not exist in LTE. On the contrary, the Zeeman effect exists both in LTE and in Non-LTE.

Second, the relevant parameter that controls the Hanle effect (we speak here of the usual, or upper level, Hanle effect) is the quantity $H$ defined in Eq.(1). On the contrary, the Zeeman effect is basically controlled by the ratio $\nu_L/\Delta \nu_D$, where $\Delta \nu_D$ is the line Doppler broadening expressed in frequency units. For typical astrophysical conditions, the sensitivity of the two effects to the magnetic field intensity is schematically represented in Figure 1. The figure shows that the diagnostic content of the Hanle effect is restricted to a small interval of magnetic field intensities (between approximately 1 and 10 Gauss for the case of the figure). In this interval, the Hanle effect provides a more sensitive diagnostic tool than the Zeeman effect.

As a third point it has to be remarked that the Hanle effect, in order to be used as a diagnostic tool, requires the knowledge of the “zero-field” polarimetric signal. Without this knowledge (which often can be derived only from theoretical arguments), there is no possibility of extracting the value of the magnetic field from observations. In the case of the Zeeman effect, on the contrary, the “zero-field” polarimetric signal is known a priori, just being zero.

Finally, it should be pointed out that there are different physical phenomena which do not have anything to do with the magnetic field and which nevertheless introduce on scattering polarization some characteristic signatures mimiking the Hanle effect. Let us consider for instance the case of a plasma having density inhomogeneities from point to point. An increase in density implies a proportional increase in collisions and, if these collisions are capable of introducing a depolarizing effect on the scattered radiation, it would result that the polarization signal emerging from a region with higher particle density would be weaker than the corresponding signal from low-density regions. As a result, a density increase may be erroneously interpreted as a magnetic field. Similarly, the Doppler effect may lead to important modifications of the scattered polarization (increase or decrease of fractional polarization and rotation of the polarization angle) when the spectrum of the incident radiation presents an absorption or an emission line across the frequency interval corresponding to the atomic transition. In other words, an atom moving with velocity $\vec{v}$ “sees” a radiation field different from...
the field "seen" by an atom at rest and the scattered radiation has, in general, a polarization signature which depends on the velocity of the scattering atom. As a result, a velocity field may be erroneously interpreted as a magnetic field. On the contrary, there is no physical phenomenon that is capable of mimicking the Zeeman effect.

5. Conclusions

The Zeeman effect can still be considered the most reliable method for the diagnostic of magnetic fields in astrophysical plasmas. The interpretation of the polarimetric signal due to the Zeeman effect is rather direct and can be performed without the introduction of complicated mathematical formalisms. On the other hand, the Hanle effect is of invaluable help in the weak intensity regime where the magnetic field modulus is of the order of a few gauss or less. In this regime, the Zeeman effect generally produces a very weak circular polarization signal proportional to the longitudinal component of the magnetic field vector and totally insensitive to its transversal component.

Notwithstanding the mathematical difficulties involved in the interpretation of the Hanle signals, and notwithstanding the confusing effect that alternative phenomena may introduce in the interpretation, it has to be expected that the diagnostic techniques based on the Hanle effect will end up to be more widely used in the future. In any case, such techniques have already shown, at least for the weak fields typically met in solar prominences, to be not just a complement,
but an important and more powerful alternative to the traditional techniques based on the Zeeman effect.

References

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Discussion

SÁNCHEZ ALMEIDA: One of your figures shows that Zeeman signals are larger than Hanle signals for magnetic field strengths larger than some 10 G. Then, why are solar, turbulent magnetic fields studied using Hanle polarization signals? Zeeman signals tend to cancel for complex magnetic topologies, but this cancelation is never perfect.

LANDI DEGL’INNOCENTI: Indeed the figure that I presented in my talk (Figure 1) is only schematic and the value of the magnetic field at which the Zeeman signal gets larger that the Hanle signal depends on the line considered, on the geometry of the magnetic field, etc.. In any case, the depolarizing effect of a turbulent magnetic field on resonance polarization does not depend, to a large extent, on the details of distribution, $f(\vec{B})$, which gives the probability of finding a given value of $\vec{B}$ within the resolution element. On the contrary, the Zeeman signal depends very critically on the distribution $f(\vec{B})$ and, more particularly, on how this distribution differs from an isotropic distribution.

FAUROBERT: A turbulent magnetic field with mixed polarities on the resolution element of the telescope should not give rise to detectable Zeeman polarization because of cancellation effects.

SÁNCHEZ ALMEIDA: The cancellation cannot be perfect. You need exactly the same amount of positive and negative polarities in the resolution element, having exactly the same speed. This is improbable. Deviations from the ideal cancellation of the Zeeman signals should be used to diagnose the turbulent solar magnetic fields.

JUDGE: Can you comment on the practical application of the Hanle effect to measure magnetic fields in prominences, especially given the potentially confusing effect of Doppler shifts in spectral lines.

LANDI DEGL’INNOCENTI: Fortunately, the He lines that are traditionally used to diagnose magnetic fields in prominences through the Hanle effect are not sensitive to effects induced by Doppler shifts because the He spectrum is practically absent in the solar photospheric radiation that is scattered by the He atoms in prominences. Obviously, this statement does not hold, in general, for spectral lines of different species.