Three-Dimensional Modeling of Coronal Magnetic Fields

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Abstract. The coronal magnetic field evolves in response to motions of the photospheric footpoints of coronal field lines. These motions include the effects of the solar differential rotation, meridional flows, and supergranular diffusion. Simulations have shown that these flows can lead to the formation of highly sheared or helical magnetic fields in the corona. Here we review recent modeling of such non-potential magnetic structures, and we discuss the relevance of these models to observations of filaments and prominences.

1. Introduction

Solar polarimetry provides measurements of vector magnetic fields in the photosphere (Zeeman effect) and in quiescent prominences (Hanle effect). Photospheric measurements show that in some active regions the vector field \( \mathbf{B}(r) \) deviates significantly from the potential field \( \mathbf{B}_p(r) \) derived by extrapolating the observed line-of-sight component. This indicates the presence of large-scale electric currents in the corona above such regions. The magnetic shear is often localized in areas close to the polarity inversion line (PIL) (Schmieder et al. 1996). The existence of magnetic shear is confirmed by observations of S-shaped and inverse S-shaped coronal X-ray structures (Acton et al. 1992; Rust and Kumar 1996; Leka et al. 1996). Pevtsov, Canfield, & McClymont (1997) inferred values of the linear force-free parameter \( \lambda_0 \) for 140 active regions (\( \nabla \times \mathbf{B} \approx \lambda_0 \mathbf{B} \) with \( \lambda_0 \) constant within an active region). There is a tendency for active regions in the northern hemisphere to have negative \( \lambda_0 \), while those in the southern hemisphere tend to have positive \( \lambda_0 \) (also see Seehafer 1990; Pevtsov, Canfield, & Metcalf 1994, 1995).

Strongly sheared magnetic fields are also thought to be important for the support of filaments and prominences on the quiet sun. Filaments are located in filament channels, i.e., corridors along PILs where the chromospheric magnetic field as outlined by chromospheric \( \text{H} \alpha \) fibrils is parallel to the inversion line (Gaizauskas 1998; Harvey & Gaizauskas 1998). Martin, Bilimoria, & Tracadas (1994) classified filament channels as either dextral or sinistral depending on the direction of the axial field as seen by an observer standing on positive-polarity side of the channel. Martin & Echols (1994) proposed an empirical model of a filament in which the cool plasma is located in a set of highly sheared magnetic arches, but the question how this material is supported against gravity is not addressed. Others have proposed that quiescent filaments are located in helical
flux ropes (e.g., Kuperus & Raadu 1974; Pneuman 1983; Priest, Hood, & Anzer 1989; van Ballegooijen & Martens 1989; Low & Hundhausen 1995; Rust & Kumar 1995). According to these models, the filament plasma is supported against gravity in the dips of the helical field lines (see Fig. 1). An important feature of such models is the existence of an overlying coronal arcade, which prevents the helical field from expanding radially outward. In section 2 we review numerical models of the formation of such helical fields.

Figure 1. Helical flux rope model (Priest, Hood, & Anzer 1989)

Martin, Bilimoria, & Tracadas (1994) found that quiescent filaments in the northern hemisphere are mostly dextral, while those in the southern hemisphere are mostly sinistral, independent of the solar cycle (also see Zirker et al. 1997; Martin 1998). This result is consistent with earlier measurements of prominence magnetic fields using the Hanle effect (see Leroy 1978, 1989; Leroy, Bommier, & Sahal-Bréchot 1983, 1984; Bommier & Leroy 1998). Figure 2 shows the observed pattern of prominence axial fields (Fig. 2a) together with predictions from two models in which the axial fields are produced by the solar differential rotation (Figs. 2b and 2c). In Fig. 2b the differential rotation is assumed to act on the coronal field overlying the nearly east-west oriented PILs; this yields sinistral fields in the North and dextral in the South, contrary to observations. However, if the two areas on either side of a PIL are magnetically connected below the solar surface, then the differential rotation acting on this subsurface magnetic field produces dextral fields in the North and sinistral fields in the South (see Fig. 2c), which is the correct chirality compared to the prominence observations. Therefore, it is reasonable to suggest that the axial magnetic fields observed in solar prominences are created below the solar surface and subsequently emerge into the solar atmosphere (van Ballegooijen & Martens 1990; Priest, van Ballegooijen, & Mackay 1996).

In the following we describe results from recent modeling of the formation of strongly sheared and helical fields in the corona, and then we discuss the
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Figure 2. (a) Axial fields in prominences as measured by Leroy et al. (1983); (b) differential rotation acting on coronal field; (c) differential rotation acting on the subsurface field.

relevance of these results for understanding the origin of prominence magnetic fields.

2. Models

In the following we review recent work in modeling the formation of non-potential magnetic structures in the solar corona. The coronal field is subject to random and systematic footpoint motions, which cause a build-up of magnetic shear and may lead to the formation of helical fields. We have developed numerical models for simulating the evolution of the coronal field in three dimensions. In these models the small-scale random motions of the photospheric footpoints are described in terms of diffusion of a mean magnetic field (i.e., the magnetic field averaged over length scales larger than those of the random footpoint mo-
tions). The response of this mean field to solar differential rotation, meridional flows, and diffusion is considered. The first such model was presented by van Ballegooijen, Cartledge, & Priest (1998) as an extension of the well-known surface diffusion model (Leighton 1964; Wang, Nash, & Sheeley 1989). In this case the initial conditions for the simulations are potential fields derived from measurements of the radial magnetic field at the photosphere. The model was used for studying the formation of filament channels in different phases of the solar cycle. The modeling approach did not allow for a proper treatment of the force balance of the coronal plasma, and therefore the predicted fields have some unrealistic features.

More recently we developed mean field models in which the corona is allowed to relax to a force free state (van Ballegooijen 1999; van Ballegooijen, Priest, & Mackay 2000, hereafter paper 1). This relaxation is done using the magneto-frictional method (e.g., Yang, Sturrock, & Antiochos 1986) in which the plasma velocity $\mathbf{v}(r, t)$ is proportional to the Lorentz force:

$$\mathbf{v} \propto (\nabla \times \mathbf{B}) \times \mathbf{B}/B^2.$$  

(1)

Coronal magnetic diffusion is neglected, so the coronal field evolves according to ideal MHD:

$$\frac{\partial \mathbf{A}}{\partial t} = \mathbf{v} \times \mathbf{B},$$  

(2)

where $\mathbf{A}(r, t)$ is the vector potential ($\mathbf{B} \equiv \nabla \times \mathbf{A}$). The random footpoint motions are described in term of diffusion at the photosphere ($r = R_\odot$):

$$\frac{\partial A_{0,\theta}}{\partial t} = +u_\phi B_{0,r} - \frac{D}{r \sin \theta} \frac{\partial B_{0,r}}{\partial \phi} - E_\theta,$$  

(3)

$$\frac{\partial A_{0,\phi}}{\partial t} = -u_\theta B_{0,r} + \frac{D}{r} \frac{\partial B_{0,r}}{\partial \theta} - E_\phi,$$  

(4)

where $(r, \theta, \phi)$ are spherical coordinates on the sun, $u_\phi(\theta)$ the velocity of the solar differential rotation, $u_\theta(\theta)$ is the meridional flow velocity, and $D$ is the photospheric diffusion constant (Leighton 1964). The electric field $(E_\theta, E_\phi)$ can be used to simulate the emergence of axial fields through the photosphere. In principle reconnection occurs only at the photospheric boundary, but in practice there is some numerical diffusion throughout the computational domain.

The surface diffusion leads to cancellation of opposite-polarity fields at the PIL, i.e., there is a net transport of magnetic fields to the PIL from both sides. This produces an enhancement of any magnetic shear that exists in the corona above the PIL. The boundary conditions (3) and (4) imply that the axial component of magnetic field (i.e., the component along the PIL) cannot submerge below the photosphere. Hence, there is a build-up of axial field in the corona. Furthermore, the cancellation of the photospheric flux at this PIL involves reconnection of magnetic field lines, and leads to the formation of helical fields in the corona (also see van Ballegooijen & Martens 1989; Amari et al. 1999). We identify these helical fields with the observed filament channels.

In paper 1 we present simulations of the interaction of an east-west oriented bipole with an existing coronal field emanating from higher latitude (polar field). Different polarities of the polar field are considered, corresponding to the rising
and declining phases of the solar activity cycle (the polar field changes sign at cycle maximum). The evolution of the field depends in part on the details of how the bipole is inserted into the model; the best approach is to allow the bipole to emerge into the overlying field, so that initially the old and new fluxes are not connected to each other. In this case there is a current layer separating the old and new magnetic flux systems, i.e., the initial state is not a potential field. We found that bipoles in the northern hemisphere naturally develop dextral fields along the PIL separating the leader and follower fluxes (see Fig. 3). This is due to the effect of the differential rotation acting on the magnetic field overlying the initially north-south oriented PIL inside the bipole.

![Diagram](a) ![Diagram](b)

**Figure 3.** Differential rotation acting on an east-west oriented bipole produces dextral fields in the northern hemisphere: (a) after 27 days of evolution; (b) after 54 days (same as Figs. 2b and 2c of paper I). The sign of the polar field corresponds to the rising phase of the activity cycle.

Mackay, Gaizauskas, & van Ballegooijen (2000) compared results from mean field simulations with filament observations taken at Ottawa River Solar Observatory in 1982 July - October (Carrington Rotations 1724-1727). The initial states for these simulations are potential fields derived from photospheric magnetic field measurements taken at the Kitt Peak Vacuum Tower telescope. We found that the mean field model correctly predicts the dextral and sinistral orientation of the observed filaments, although the predicted timescale for formation of these filaments is somewhat longer than observed.

The above results suggest that we are getting closer to understanding the formation of filament channels on the sun. However, one problem is that the model does not yield the correct orientation of axial fields over the polar crowns. The problem is illustrated in Fig. 4, which shows two different views of a bipole interacting with the polar field in the declining phase of the activity cycle (paper I). Note that there is a large right-helical flux rope which parallels the east-west portion of the PIL, but lies at somewhat lower latitude. Its magnetic field emerges from the leading part of the active region (positive polarity) and
then goes toward the left (east), i.e., this field has sinistral orientation. This is opposite to what one would expect based on the prominence observation by Leroy et al. (1983) and others. In our model this flux rope forms by differential rotation acting on the coronal arcade overlying the east-west portion of the PIL. This magnetic arcade did not exist initially, but was soon formed by numerical diffusion in the current layer separating the two magnetic flux systems (bipole and polar field). Therefore, with the present code it is not easy to obtain dextral fields along the high-latitude, east-west portion of the PIL, especially during the declining phase of the cycle.

![Figure 4](image_url)

Figure 4. Magnetic bipole interacting with polar field: (a) view from latitude $-45^\circ$; (b) view from latitude $+25^\circ$ (same model as in Fig. 2f of paper I, but different set of field lines).

Another problem is that the present model is not self-consistent: the model includes diffusion at the photosphere, but not in the corona. Random motions of the photospheric footpoints lead to twisting and braiding of coronal field lines (Parker 1972), which is likely to cause diffusion both in the corona and in the photosphere. Parker (1988) proposed that the magnetic free energy of the twisted field is released via small-scale reconnection events (nanoflares). Such reconnections not only contribute to coronal heating, but also cause gradual changes in the structure of the large-scale magnetic field. Furthermore, new flux continually emerges in the photosphere, and old flux is removed from the photosphere, causing changes in the structure of the overlying fields. Therefore, when considering the evolution of the corona on time scales of days to months, the effects of coronal reconnection must be taken into account. In paper I we argued that coronal diffusion occurs only in a boundary layer at the corona base, but we now realize that the $\alpha$-effect described in that paper includes the effect of coronal diffusion.

Figure 5 shows preliminary results from a model that includes coronal diffusion. The coronal diffusion coefficient $\beta(r, t)$ is a function of position (inversely proportional to the local field strength $B$), and the value of $\beta$ at the coronal base
is comparable to the photospheric diffusion constant. The results for the model with both coronal and surface diffusion is shown in Fig. 5a, and the corresponding model with surface diffusion only is shown in Fig. 5b. Note that a helical field is present in Fig. 5b but not in Fig. 5a. Nevertheless, the field is highly sheared, so coronal diffusion can prevent the formation of helical fields, but does not prevent the formation of strongly sheared fields inside active regions.

3. Discussion

We have shown that it is difficult to explain the observed global pattern of prominence magnetic fields with models that only include surface flux transport (Fig. 2b). It is unlikely that incorporating coronal diffusion into such models will help resolve this problem, because this would further promote the formation of a coronal arcade over the high-latitude, east-west portion of the PIL, allowing the differential rotation to have an even stronger effect on these arcades. Therefore, we now return to the alternative possibility that the axial field is generated by
differential rotation acting below the solar surface, followed by the emergence of this field into the corona (Fig. 2c). The latter would likely manifest itself as the emergence of a series of small bipolar (ephemeral regions and/or intranetwork fields) with a preferred orientation along the PIL. A problem with this scenario is that such preferred orientation of bipolar has not been observed; the orientations of small bipolar on the sun appear to be more or less random. Based on the observed rates and time scales for small-scale flux emergence, we estimate that the rate of axial flux emergence is less than $10^{19}$ Mx/day. This puts severe constraints on models of prominence magnetic fields in which the axial fields originate below the photosphere.

Although flux rope models have received considerable attention for explaining the magnetic support of prominences, it is not clear that they are appropriate for high-altitude, east-west oriented prominences such as those on the polar crown. The reason is that the axial flux required in such models is rather high, making it difficult to understand how such helical fields can be held down by the overlying arcades (if any). The prominence height $\sim 50$ Mm provides a lower limit on the radius of the flux rope, and combining this with the observed axial fields in prominences ($\sim 5$ G) we obtain an axial flux $\Phi > 4 \times 10^{20}$ Mx; this is comparable to the flux of the arcades. Furthermore, once a flux rope erupts, most of the axial flux is removed from the corona, and for the flux rope to reform would require the emergence of new axial fields from below the photosphere. Since the axial flux emergence rate is less than $10^{19}$ Mx/day, it would take at least 40 days for a new flux rope to form. However, new filaments are often found to reform within days of a filament eruption. This suggests that it takes much less than $4 \times 10^{20}$ Mx of axial flux to form a filament. Perhaps the axial field is located in a vertical sheet with a thickness not much larger than the filament itself (a few Mm). The details of such a model remain to be worked out.

We conclude that the magnetic structure of solar prominences is still unclear. Improved Hanle measurements of prominence magnetic fields are needed. Hopefully, future Stokes polarimeters will be able to provide such measurements.

References

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