The Physics of Low-mass Stars and Substellar Objects. Galactic Implications

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Abstract. We briefly discuss the main mechanical and thermal properties of low-mass stars and substellar objects, which determine their structure and evolution. Comparison is made with observations in various color-magnitude diagrams. We determine the stellar and substellar mass functions and then the contribution of these objects to the mass budget of the various Galactic components.

1. Introduction

The search for substellar objects has bloomed over the past few years with the unambiguous identification of several free floating brown dwarfs (BD) and with the discovery of several planets orbiting stars outside the solar system. Substellar objects therefore exist and ongoing and future observational projects are likely to reveal hundreds more of such objects and finally allow to evaluate precisely their contributions to the baryonic content of the Galaxy. These surveys will deliver large observational data bases of planets and brown dwarfs which will necessitate the best possible theoretical foundation.

The Lyon group has ambitious to derive a complete theory of the evolution and spectral signature of low-mass, dense objects, from Sun-like to Saturn-like masses, covering 3 orders of magnitude in mass and 9 in luminosity, and bridging the gap between stars and gaseous planets. We have incorporated the best possible physics aimed at describing the mechanical and thermal properties of these objects - equation of state, synthetic spectra, non-grey atmosphere models. The Tucson group first developed a theory for extra-solar planets (Saumon et al. 1996; Guillot et al. 1996) and extended it to larger masses (Burrows et al. 1997), while the Lyon group has proceeded downwards from the low-mass stellar regime to the planetary range (Baraffe et al. 1997; 1998; Chabrier et al. 2000). Both groups have “met in the middle” by successfully identifying the main spectral properties of the benchmark BD Gl229B, providing a determination of its mass and age (Marley et al. 1996; Allard et al. 1996). We refer to the afore-mentioned papers and the references therein for a complete description of
the physics entering these models. A complete review of the theory of low-mass stars and substellar objects will be published elsewhere (Chabrier & Baraffe, 2000). Only a brief outline of the characteristic properties of these objects is given below.

2. The physics of sub-stellar objects

2.1. Interior

Central conditions for substellar objects (SSOs) are typically $T_c \lesssim 10^5$ K and $\rho_c \sim 10^2 - 10^3$ g cm$^{-3}$. Under these conditions, the average ion electrostatic energy $(Ze)^2/a$, where $a = (\frac{3}{4\pi} \frac{V}{N_i})^{1/3}$ is the mean interionic distance, is several times the average kinetic energy $kT$, characterizing a strongly coupled plasma. The temperature is of the order of the electron Fermi temperature $kT_F$, so that the electron gas is only partially degenerate. The temperature in the envelope is $kT < 1$ Ryd, so electronic and atomic recombinations take place in this region. Finally, the electron binding energy is of the order of the Fermi energy $Ze^2/a_0 \sim E_F$ so that pressure-dissociation and ionization take place along the interior profile. Very recently laser-driven shock wave experiments at Livermore have achieved experiments on liquid D$_2$ up to 5 Mbar at high temperature (Collins et al. 1998), exploring for the first time the regime of pressure-dissociation and ionization, therefore probing the equation of state (EOS) under conditions characteristic of SSO interiors. As shown in Fig. 3 of Collins et al. (1998), the experimental hagioniot revealed the excellent behaviour of the EOS developed by Saumon and Chabrier (Saumon et al. 1995 and references therein). In particular the pronounced compressibility observed in the dissociation domain ($\rho/\rho_0 = 5.88$) agrees remarkably well with the theoretically predicted value. These experiments open a new window in physics and astrophysics by constraining the physics of the interior of SSOs in laboratory experiments.

2.2. Atmosphere

Atmosphere equilibrium condition $d\tau = \kappa dP/g$, where $g = Gm/R^2 \approx 10^3 - 10^5$ cm s$^{-2}$ is the characteristic surface gravity of SSOs, yields $P_{ph} \sim g/\kappa \approx 1 - 10$ bar and $\rho_{ph} \approx 10^{-4}-10^{-5}$ g cm$^{-3}$ at the photosphere. Collision effects are important under such conditions and induce new sources of absorption like e.g. the collision-induced absorption between H$_2$-H$_2$ or H$_2$-He below $\sim 5000$ K (see e.g. Linsky, 1969; Borysow et al. 1985). In the effective temperature range characteristic of low-mass stars (LMS) and SSOs ($T_{\text{eff}} \lesssim 5000$ K), numerous molecules form, in particular metal oxides and hydrides (TiO, VO, FeH, CaH), the major absorbers in the optical, and H$_2$, H$_2$O which dominate in the infrared. The situation becomes even more complicated for substellar objects, which exhibit very distinct spectral signatures due to the changes in molecular chemistry which occur across the atmospheric temperature range from 2000 K (the coolest stars) to 170 K (Jovian planetary conditions). At 2000 K, most of the carbon is locked into carbon monoxide CO, while the oxygen is found in titanium TiO and vanadium VO monoxides (dominating the optical aborption) and water vapor H$_2$O (shaping the infrared spectrum). Below $\sim 1800$ K, the dominant equilibrium form of carbon is no longer CO but CH$_4$ (Fegley & Lodders 1996). As confirmed
by the observation of Gl229B (Oppenheimer et al. 1995), methane features begin to appear in the infrared while titanium dioxide and silicate clouds form at the expense of TiO, modifying profoundly the thermal opacity of the atmosphere. For jovian-like atmospheres, the dominant equilibrium form of nitrogen N₂ is NH₃ (T eff ≤ 600 K) and below T eff ~ 200 K water condenses to clouds at or above the photosphere. As shown by Tsuji et al. (1996) and Jones & Tsuji (1997), there is evidence for condensation of metals and silicates into grains (e.g. TiO into CaTiO₃, Mg, Si into MgSiO₃) for T eff ≤ 3000 K, i.e. at the bottom of the main sequence.

![Figure 1](image_url)  
**Figure 1.** Near-IR K – (J – K) color-magnitude diagram. The data are from Leggett (1992) (dots), Dahn et al. (2000) (squares) and Reid et al. (1999) (filled triangles). Some identified BDs are indicated by diamonds: Kelu-1 (Ruiz et al. 1997); GD165B (Kirkpatrick et al. 1999); LHS102B (Goldman et al. 1999). Theoretical models correspond to various atmosphere models and metallicity: solid lines: DUSTY (right) and COND (left) models for 1 Gyr, solar metallicity; dash-dotted line: NextGen (grainless) models for 1 Gyr, solar metallicity (Baraffe et al. 1998); long-dashed line: NextGen models for 10 Gyr, [M/H] = −2.0 (Baraffe et al. 1997). Masses (in M☉) and T eff are indicated for the DUSTY and COND isochrones.
These grains affect the atmosphere in different ways. Grain formation depletes the corresponding gas-phase absorber in certain region of the atmosphere and modify the EOS itself and thus the atmospheric temperature-profile, but they also strongly modify the opacities and thus the emergent spectrum. At last they produce an increase of the temperature in the uppermost layers of the atmosphere, the backwarming effect, destroying otherwise stable molecules. A proper inclusion of this complex atmosphere chemistry represents the main challenge for a correct description of the spectral signature and the evolution of substellar objects. Recently, we have extendend our previous calculations of main sequence objects (Baraffe et al. 1998) by taking into account this grain formation process. We include explicitely all condensed species not only in the atmosphere EOS but also into the radiative transfer equations (Allard et al. 2000) and we conduct consistent evolutionary calculations for such “dusty” objects (Chabrier et al. 2000).

3. Color-magnitude diagram

The present evolutionary calculations have been conducted with three different atmosphere models: (i) for objects hot enough, i.e. massive or young enough, to preclude the formation of grains ($T_{\text{eff}} \gtrsim 2800$ K), we have used the grainless so-called NextGen (NEG) most recent atmosphere models (Hauschildt, Allard & Baron, 1999); (ii) for objects in the range 3000−1000 K, i.e. from the bottom of the MS down to G1229B-like objects, we have used complete atmosphere models which include the grain opacity sources in the transfer equations (Allard et al. 2000), the so-called DUSTY models; (iii) for objects below 3000 K down to Jupiter-like temperatures, we have also considered cases where the condensates settle rapidly below the photosphere and – although modifying the atmosphere EOS - do not participate to the opacity, the so-called COND models. These latter calculations are similar to the Burrows et al. (1997) calculations and are motivated by the absence of grain features in the atmosphere of objects below $T_{\text{eff}} \sim 1000$ K, i.e. G1229B-like objects.

We found the effect of grain opacity (DUSTY) to affect only moderately the H-burning minimum mass. Models with grainless atmosphere yield $m = 0.072 M_\odot$, $L = 5 \times 10^{-5} L_\odot$ and $T_{\text{eff}} = 1700$ K at the H-burning limit, whereas models with grain opacity give $m \approx 0.07 M_\odot$, $L \approx 4 \times 10^{-5} L_\odot$ and $T_{\text{eff}} \approx 1600$ K.

Figure 1 displays a $K$ vs $J-K$ color-magnitude diagram (CMD). In terms of colors there is a competing effect between grain and molecular sources of absorption for objects at the bottom of and just below the MS. As already identified for e.g. in G1229B (Allard et al. 1996; Marley et al. 1996) CH$_4$ and collision-induced absorption of H$_2$ in the infrared lead to a blueshift in the infrared for SSOs, as illustrated by the NEG and COND models, whereas grain opacity yields a severe redshift of the colors, a consequence of the afore-mentioned backwarming effect. Indeed the present DUSTY models reproduce the spectra of the DENIS objects (Tinney et al. 1998) and of the long-time puzzling object GD165B, which lies on the very edge of the H-burning limit, depending on its age (Kirkpatrick et al., 1999). As effective temperature decreases, most grains form or settle below the photosphere and the DUSTY track will merge with the
COND one. Brown dwarfs radiate nearly 90% of their energy at wavelengths longward of 1 μm and infrared broad-band colors are preferred to optical ones (at least for solar metal abundance), with $M_M \sim M_L \sim 10-11$, $M_K \sim M_H \sim M_J \sim 11-12$ at the H-burning limit, at 1 Gyr (see Chabrier et al. 2000).

![Figure 2](image)

Figure 2. Pre-main sequences isochrones. The mixing length parameter, which is consequential only for $m \geq 0.7 M_\odot$, is fixed arbitrarily to $L_{mix}/H_P = 1.0$. Calculations have also been done for a value 1.9, which reproduce the present Sun (see ftp-site).

Several BD surveys are been conducted in young clusters and it is important to develop accurate non-grey models for pre-MS stars and young BDs. The more massive of such objects will be hot enough so that grain formation does not occur. Figure 2 displays the present pre-MS models based on non-grey atmosphere models for several isochrones in a HR diagram (Baraffe et al. 2000).

4. Stellar and brown dwarf mass functions

4.1. Stellar mass function

It is now well established that visible stars are not numerous enough to account for the dynamics of our Galaxy, the so-called galactic dark matter problem. One
of the longstanding proposals for baryonic dark matter has been the possibility that brown dwarfs could be produced in large enough numbers to make up for this missing mass. A correct study of this problem requires the correct determination – slope and normalization – of the mass-function (MF) down to the hydrogen burning limit, in order to rely on robust grounds to extrapolate into the substellar domain. This issue, however, is not completely settled at present, and significant differences exist between various luminosity functions at faint magnitude \((M_V \gtrsim 12)\) (see e.g. Méra et al. 1998; Chabrier & Baraffe, 2000). It is relatively safe to say, however, that the MF keeps rising down to the H-burning limit, although with a slope shallower than a Salpeter MF, i.e. with \(\alpha \simeq 1.2\), where \(dN/dm \propto 1/m^\alpha\) (Reid et al. 1999; Chabrier & Baraffe, 2000). The LMS \((m \leq 0.6 \, M_\odot)\) mass density inferred from the integration of this MF yields \(\rho_{\text{LMS}} \approx (1.6 \pm 0.2) \times 10^{-2} \, M_\odot \, \text{pc}^{-3}\). Adding up the contribution from more massive stars and stellar remnants yields \(\rho_* \approx (3.7 \pm 0.3) \times 10^{-2} \, M_\odot \, \text{pc}^{-3}\) in the Galactic disk, i.e. a surface density \(\Sigma_* \approx 22 \pm 2 \, M_\odot \, \text{pc}^{-2}\).

For the Galactic spheroid the question is more settled with a MF with \(\alpha \lesssim 1\) and a stellar density \(\rho_* < 4.0 \times 10^{-5} \, M_\odot \, \text{pc}^{-3}\), i.e. less than 1% of the required dynamical mass (Graff & Freese, 1996; Chabrier & Méra 1997; Gould et al. 1998), i.e. a number density \(n_* < 10^{-3} \, \text{pc}^{-3}\). Extrapolating into the BD domain yields for the spheroid \(n_{\text{BD}} < 10^{-4} \, \text{pc}^{-3}\), \(\rho_{\text{BD}} < \rho_* / 10\), i.e. an optical depth \(\tau \sim 10^{-9}\), about 1% of the value measured toward the LMC. The puzzle remains complete for the dark halo MF, although both the HDF observations and the narrow-range of the observed time-distribution of the microlensing events towards the LMC strongly suggest an IMF different from a Salpeter one below \(\sim 1 \, M_\odot\) (see e.g. Chabrier, 1999).

### 4.2. Brown dwarf mass function

A proper census of the number of brown dwarfs \((m < 0.07 - 0.09 \, M_\odot)\), depending on the metallicity) has significant implications for our understanding of how stars and planets form. The determination of the BD MF is a complicated task. By definition BDs never reach thermal equilibrium and most of the BDs formed at the early stages of the Galaxy will have fainted to very low-luminosities \((L \propto 1/t)\), see e.g. Burrows & Liebert, 1993). Thus observations are biased towards young and/or massive BDs. The age-undetermination is circumvented when looking for the BD MFs in clusters since objects in that case are likely to be coeval. The Pleiades cluster has been extensively surveyed and several BDs have been identified down to \(\sim 25 \, M_J\) (Martín et al. 1997; Bouvier et al. 1998; Hambly et al. 1999). A single power-law function from \(\sim 0.4\) to \(0.04 \, M_\odot\) seem to adequately reproduce the observations with some remaining undetermination in the exponent \(\alpha \sim 0.6 - 1.0\). Mayor et al. (1997) find for the mass function of the companions of the G and K dwarf sample observed with CORAVEL a power-law function with \(\alpha \sim 0.4\) from 0.4 down to 0.005 \(M_\odot\). The determination of the MF in young clusters is potentially very interesting. Indeed young clusters did not have time to experience evaporation in the outer regions or mass segregation in the central regions and the present-day mass function should reflect relatively closely the initial mass function. Such a MF determination, however, is hampered by two observational and theoretical problems. From the observational point of view, young objects are immersed into dust and a proper
determination of the luminosity function requires a correct determination of the differential reddening in the cluster. From the theoretical point of view, all MF determined up to now in young clusters rely on grey atmosphere models and thus are of dubious validity. Moreover, the calculations of evolutionary tracks require a detailed analysis of the initial conditions and of the atmospheric gravity. For all these reasons, it is certainly premature to claim a robust determination of the MF in young clusters.

An independent, powerful information on the stellar and substellar MF comes from microlensing observations. Indeed, the time distribution of the events provides a (although model-dependent) determination of the mass distribution and thus of the minimum mass of the dark objects: \( dN_{ev}/dt_e = E \times \epsilon(t_e) \times d\tau/dt_e \propto P(m)/\sqrt{m} \), where \( E \) is the observed exposure, i.e. the number of star\times\text{years}, \( \epsilon \) is the experimental efficiency, \( \Gamma \) is the event rate and \( P(m) \) is the mass probability distribution. The analysis of the published 40 MACHO + 9 OGLE events towards the bulge is consistent with a rising mass function at the bottom of the MS with a minimum mass \( m_{\text{inf}} \sim 0.05 \, M_\odot \), although a decreasing MF below 0.2 \( M_\odot \), i.e. a Miller & Scalo IMF, is excluded at the 95\% level (Han & Gould, 1995; Méra et al. 1998). Although the time distribution might be affected by various biases (e.g. blending) and robust conclusions must await for larger statistics, the present results suggest that in order to explain both star counts and the microlensing experiments, a substantial amount of SSOs must be present in the galactic disk. Indeed, extrapolation of the stellar MF into the BD domain down to 0.01 \( M_\odot \) yields for the Galactic disk \( \rho_{BD} \sim 3.0 \times 10^{-3} \, M_\odot \, pc^{-3} \), i.e. \( \Sigma_{BD} \approx 2 \, M_\odot \, pc^{-2} \), i.e. a BD number-density comparable to the stellar one, \( n_{BD} \sim 0.1 \, pc^{-3} \sim n_* \).

For the spheroid, extrapolation of the afore-mentioned stellar MF yields \( \rho_{BD} \lesssim 10^{-5} \, M_\odot \, pc^{-3} \), \( n_{BD} \lesssim 10^{-3} \, pc^{-3} \), whereas for the dark halo the normalization is about 2 orders of magnitude smaller (Chabrier & Méra 1997).

### 5. Conclusion and perspectives

As mentioned in the introduction, accurate models for low-mass stars, brown dwarfs and giant planets are needed to shed light on the observable properties of these objects and to provide guidance to the ongoing and future surveys aimed at revealing their contribution to the Galactic population. Tremendous progress has been realized from this point of view within the recent years with the derivation on non-grey models which accurately reproduce the observed sequences of globular clusters, young clusters, field objects and of the benchmark BD Gl229B in various photometric passbands as well as the observational mass-magnitude relationships both in the optical and in the infrared (Baraffe et al. 1997, 1998; Delfosse et al. 2000)(see Chabrier & Baraffe, 2000 for a complete review). On the other hand the afore-mentioned LMS models have been shown to yield a consistent, coeval sequence for the quadruple system \( GG \) Tau, whose component masses extend from 1.2 \( M_\odot \) down to \( \sim 0.035 \, M_\odot \) (White et al., 1999), a formidable test for the theory. On the other hand, stringent constraints on the theory of dense/cool objects are now provided by laboratory high-pressure experiments, for the interior, and by various spectroscopic and photometric observations of LMS and SSOs. Any theory aimed at describing
the mechanical and thermal properties of these objects must be confronted to these experimental/observational constraints in order to assess its degree of validity. Improvement in the theory will proceed along with the discovery of many more SSOs, bridging the gaps on either side of Gl229B from the bottom of the MS to Jupiter-like objects.

The increasing number of observed LMS and SSOs, together with the derivation of accurate models, will allow eventually a robust determination of the stellar and substellar mass functions, and thus of the exact density of these objects in the Galaxy. As discussed in §4, present determinations in various Galactic regions point to a slowly rising MF near and below the H-burning limit, with a BD number density comparable to the stellar one. Whether this behaviour is universal (although we already know it is certainly not the case for the dark halo), whether it is consistent with a general log-normal form, must await confirmation from future observations.

Note: The various models presented in this paper are available from:
ftp ftp.ens-lyon.fr, username: anonymous
ftp > cd /pub/users/CRAL/ibaraffe
or upon request to I. Baraffe.

References

Burrows A., & Liebert, J., 1993, Rev. Mod. Phys., 65, 301
Delfosse, X., et al., in preparation
Hambly NC, Hodgkin ST, Cossburn MR, Jameson RF. 1999. MNRAS. 303, 835
Mayor, M, Queloz, D., Udry, S., & Halbwachs, J.-L., 1997, in *Rencontre de Blois*

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