CORRELATIONS BETWEEN PROPERTIES OF TYPE II BURSTS AND ASSOCIATED FLARES

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Abstract. Relations between characteristics of metric type II bursts and the associated microwave bursts are investigated to test the model of the shock wave formation via a flare ignited blast. It is found that the time delay of the type II burst start after the onset of the microwave burst is shorter for a more powerful microwave burst. A considerably lower correlation coefficient is found for the microwave burst impulsiveness. The starting frequency of a type II burst increases with the microwave burst impulsiveness. The instantaneous relative bandwidths and the propagation velocities are positively correlated, and are larger when associated with stronger microwave bursts. All of the obtained correlations are fully consistent with the flare ignited blast wave scenario for the shock wave formation.

Key words: flares - shock waves - type II bursts

1. Introduction

Metric type II bursts reveal propagation of fast mode MHD shock waves through the solar corona (Uchida, 1974). These bursts are often associated with solar flares, indicating that coronal shock waves are ignited by an explosive heating associated with the energy release. A back-extrapolation of the emission lanes of high-frequency type II bursts that are starting in the dm-m wavelength range indicate that the shocks are launched during the fast growth phase of the associated microwave burst (Vršnak et al., 1995).
Presuming that the coronal shock waves are generated by flares, a relation between the type II burst characteristics and the associated flare properties can be expected. Vršnak and Lulić (2000a) proposed a simple 1-D model for the formation of perpendicular MHD shock wave by steepening a large amplitude perturbation generated by an abrupt expansion of the source region. In the next paper Vršnak and Lulić (2000b) considered the process of the source region expansion caused by an impulsive heat input. The results were elaborated in a form explicitly relating the properties of the heating process and the characteristics of the shock formation process. Here, the empirical relations between the characteristics of type II bursts and the associated microwave bursts (further on MW burst) will be investigated to confront the model with observations.

2. The Model

The considered model treats a perfectly conducting low $\beta$ plasma. This means that the ”frozen-in” condition is fulfilled, and that the sound velocity is much smaller than the Alfvén velocity ($c_S \ll v_A$). The second condition implies that the fast magnetosonic velocity $v_{fm} = \sqrt{v_A^2 + c_S^2}$ can be approximately taken as $v_{fm} \approx v_A$. Only the simplest case of a perturbation propagating perpendicular to the ambient magnetic field is considered in the model (to model the formation of oblique shocks, the set of equations treated by Mann (1995) should be taken as a starting point). It is assumed that the magnetic field, plasma density, temperature, and pressure are uniform in the unperturbed medium.

In the model, it is assumed that at $t = 0$ an impulsive heating of the region between $x = 0$ and $x = x_0$ (source region) starts. The heating causes a pressure increase that drives an abrupt expansion of the source region. In the external region ($x > x_0$) a large amplitude MHD perturbation is created, spreading outwards through the ambient magnetoplasma. The leading edge of the perturbation forms during the time interval $0 < t < t_m$ in which the expansion velocity $u$ increases from $u = 0$ to the maximum value $u_m$. After $t = t_m$ it is assumed that the source region continues to expand for some time at the constant velocity $u_m$. 

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In the case of a large amplitude perturbation the velocity \( v \) of an element of the perturbation depends on the associated plasma flow velocity \( u \). The velocity \( v \) can be expressed as \( v = v_0 + u/2 \), where \( v_0 \approx v_A \) is the fast magnetosonic velocity in the unperturbed plasma (Vršnak and Lulić, 2000a). This means that each point of the wave is propagating at its own rest frame speed. Denoting the rest frame velocity of the signal as \( w \), it can be written:

\[
 w = v + u = v_0 + \frac{3}{2} u, \tag{1}
\]

(Vršnak and Lulić, 2000a). Equation (1) shows that as the magnetoplasma is pushed outwards faster, the associated perturbation spreads at an accordingly higher velocity. After some time \( t_s \) a faster element reaches a slower one, and a discontinuity in the perturbation profile is created. The discontinuity begins to develop at the segment where the initial perturbation profile has the steepest gradient, i.e. where \( \partial^2 u/\partial x^2 = 0 \) (Vršnak and Lulić, 2000a).

The time delay of the shock appearance after the onset of the source region expansion can be expressed in a normalized form as:

\[
 T_s = T_0 + \frac{2 + U T_0}{3 \left( \frac{\partial U(T)}{\partial T} \right)_{T_0}}, \tag{2}
\]

where \( U = u/v_0 \), \( T = t/t_m \), and \( T_s = t_s/t_m \). The normalized time \( 0 < T_0 < 1 \) determines the element of the perturbation which will first steepen into the shock.

The distance \( d_s \) at which the shock formation starts, normalized with respect to the distance \( d_0 = v_A t_m \) (the distance to which a small amplitude perturbation spreads during \( t_m \)) can be expressed as:

\[
 D_s = \frac{d_s}{d_0} = \int_0^{T_0} U dT + \left( \frac{2 + 4 U + \frac{3}{2} U^2}{3 \left( \frac{\partial U(T)}{\partial T} \right)_{T_0}} \right)_{T_0}. \tag{3}
\]

Equations (2) and (3) are quite general and can be applied to all other processes characterized by a fast volume expansion, including also fast CMEs. So, the equations have to be applied to the specific case of a flare generated process.
In the paper by Vršnak and Lulić (2000b) the relations that predict a number of correlations between the type II burst characteristics and properties of the associated flares were derived. It was shown that the time profile of the normalized expansion velocity $U(T)$ is determined by the evolution of a heat input rate $q$ normalized with respect to the initial magnetic field energy density ($Q(T) = 2\mu_0 q/B_0^2 = q/p_{B0}$). In general, the highest value of the expansion velocity ($U = U_m$) is a monotonously increasing function of the maximum heat input rate $Q_{max}$ and can approximately be represented by $U_m \approx (Q_{max})^\alpha$, where $\alpha \approx 1/2$. On the other hand, the time $t_{max}$ when the heat input rate attains the maximum can be approximately taken as $t_{max} \approx t_m$, i.e. $T_{max} = t_{max}/t_m \approx 1$.

A key assumption used in the further statistical investigation is that the peak flux $PF$ of the flare associated MW burst is related with the heat input, $Q_{max} \propto PF$, and that the growth time of the heat input ($t_{max} \approx t_m$) is related to the flare rise time $t_F$, i.e. $t_F \propto t_m$.

Since the expansion velocity attains the maximum value $U_m$ at $T \approx 1$, Equations (2) and (3) can be modified into an approximate form substituting $\partial U/\partial T \approx U_m$. Thus, Equation (3) can be expressed as:

$$t_s \approx \left( C_1 + \frac{C_2}{\sqrt{PF}} \right) \cdot t_F, \quad (4)$$

where $\alpha \approx 0.5$ was used. The parameters $C_1$ and $C_2$ depend on the form of the time profile of the expansion velocity $U(t)$, whereas $C_2$ depends also on the coronal Alfvén velocity $v_A$. In most cases one can take $C_1 \approx 0.5$ (Vršnak and Lulić, 2000a). Equation (3) predicts a positive correlation between the time delay ($t_s$) of the type II burst onset and the flare rise time $t_F$. Furthermore, dividing Equation (3) by $t_F$ one finds that a negative correlation should be expected between the normalized time delay $T_s = t_s/t_F$ and the flare peak flux $PF$.

Analogously, the distance at which the shock occurs (i.e. the height in the corona) can be related to $PF$ and $t_F$ as:

$$d_s \approx \left( C_1 + \frac{C_2}{\sqrt{PF}} + C_3 \sqrt{PF} \right) v_A t_F. \quad (5)$$

The parameter $C_3$ depends on the coronal Alfvén velocity $v_A$. Equation
(5) shows that the starting frequency \( f_s \) of the type II burst should be smaller for a larger flare rise time since the coronal density decreases with the height. On the other hand, \( f_s \) should not depend much on \( PF \). Furthermore, Equation (5) indicates that due to \( d_s \propto t_F \) an indirect positive correlation of the starting frequency \( f_s \) and the flare impulsiveness \( IMP = PF/t_F \) (Pearson et al., 1989) can appear due to \( IMP \propto 1/t_F \).

Finally, let us consider an intrinsic property of the fast magneto-sonic shock waves. The factor of compression \( \Gamma \) behind the shock front (downflow region) is related with the plasma flow velocity as \( \Gamma = (1 + U/2)^2 \) (Vršnak and Lulić, 2000a). Assuming that the band splitting of type II emission lanes (or the instantaneous type II burst bandwidth) is related to the density jump across the shock front (Mann et al. 1995), one can write for the relative bandwidth:

\[
BDW = \frac{\delta f}{f} = \sqrt{\Gamma} - 1 = \frac{U_m}{2} \propto \sqrt{PF}.
\]  

Equation (6) indicates that a good correlation between the instantaneous relative bandwidth and the peak flux should be expected. Furthermore, since the shock Mach number is an increasing function of \( U_m \), the relative bandwidth should be correlated with the velocity inferred from the frequency drift of the type II burst.

In the following, the implications of Equations (4), (5) and (6) will be confronted with the observations, bearing in mind that the starting frequency \( f_s \) of a type II burst is a decreasing function of \( d_s \) since the plasma density decreases with height.

3. Observations

In order to investigate the correlations between the properties of type II bursts and the associated flares a data set based on the reports in Solar Geophysical Data (SGD further on) is used. The sample contains 75 metric types II bursts and the associated MW bursts observed at \( \approx 3 \) GHz in the period from December 1997 to June 1999. Only the events associated with \( H\alpha \) flares on the visible solar hemisphere are considered.
Table I: Basic Quantities and Expected Correlations.

<table>
<thead>
<tr>
<th>FLARES</th>
<th>rise time $(t_F)$</th>
<th>peak flux $(PF)$</th>
<th>impulsiveness $(IMP = PF/t_F)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>TYPE II</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>type II delay $(t_s)$</td>
<td>$\rightarrow$</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>norm. delay $(T = t_s/t_F)$</td>
<td>-</td>
<td>$\nearrow$</td>
<td>0</td>
</tr>
<tr>
<td>starting freq $(f_s)$</td>
<td>$\searrow$</td>
<td>0</td>
<td>($\nearrow$)</td>
</tr>
<tr>
<td>inferred velocity $(v)$</td>
<td>0</td>
<td>$\nearrow$</td>
<td>($\nearrow$)</td>
</tr>
</tbody>
</table>

The starting frequency, the onset time and the inferred shock velocity, (SGD – Spectral Observations) will be used to specify the type II burst properties. The time of beginning and maximum of the associated MW burst and its peak flux (SGD – Solar Radio Emission - Outstanding Occurrences) will be used to describe the flare properties.

In addition, 25 type II bursts studied by Mann et al. (1995) are used as an independent sample. The reported data set contains the data on the instantaneous relative bandwidths and drift rates of type II bursts.

Quantities that will be investigated in the further statistical analysis are summarized in Table I. The correlations predicted by the model are shown as "$\rightarrow$" – positive correlation; "$\searrow$" – negative correlation; "$0$" – no correlation; "$-$" means that the correlation has no sense. The symbols in brackets indicate that only a weak or indirect correlation is expected.

4. Results

4.1. Time Delays

In Figure 1 the time delays of type II bursts after the MW burst onsets $(t_s)$ are shown versus the rise times of MW bursts $(t_F = t_{max} - t_{beg})$. As expected from Equation (4) one finds that flares of longer rise times cause type II bursts of longer delays. The sample is split into two subsets: L – ”large flares” with peak fluxes $PF \geq 100$ sfu (crosses) and S – ”small flares” with peak fluxes $PF < 100$ sfu (squares). Small flares, having an average rise time of $\bar{t}_F = 3.5 \pm 5.6$ min are associated
Figure 1: Time delays \( t_s \) versus MW burst rise times \( t_F \). Squares (crosses) – flares with peak fluxes smaller (larger) than 100 sfu. The linear least squares fit of the form \( y = kx \) embracing the entire sample is shown in the inset.

with the type II bursts of an average delay of \( \bar{t}_s = 10.4 \pm 7.2 \) min, which gives \( \bar{T}_s = t_s/t_F = 3.0 \). For large flares one finds \( \bar{t}_F = 4.0 \pm 4.1 \), \( \bar{t}_s = 7.0 \pm 6.7 \) and \( \bar{T}_s = 1.7 \) showing that in average small flares cause type II bursts of longer delays, as predicted by Equation (4). In the inset the linear least squares fit of the form \( y = kx \) for the entire sample is shown, indicating a good correlation (correlation coefficient \( C = 0.68 \)).

The relation between the normalized time delays of type II bursts \( T_s = t_s/t_F \) and the peak fluxes of associated MW bursts \( (PF) \) is shown in Figure 2a. One finds a rather good correlation (power law correlation coefficient \( C = 0.44 \)) showing that the normalized time delays are shorter for more powerful flares, as expected from Equation (4).
Figure 2: Normalized time delay $T_s$ versus peak flux (a) and impulsiveness (b). Power law fits with the correlation coefficients $C$ are shown in the insets.
Figure 3: Starting frequency $f_s$ versus the MW burst rise time $t_F$.

The normalized time delays $T_s$ are confronted with the MW burst impulsiveness defined as $IMP = PF/t_F$ in Figure 2b. As expected, no correlation was found: the least squares fit gives for the power law exponent the value of $\approx 0.05$ (implying $T_s \approx const.$) and the correlation coefficient is only $C = 0.06$.

4.2. Starting Frequencies

In Figure 3 the correlation between the type II bursts’ starting frequencies $f_s$ and rise times of the associated MW bursts $t_F$ is shown. The starting frequencies are lower when the MW bursts rise times are longer. This is consistent with Equation (5) since a lower starting frequency implies that the shock occurs higher in the corona (a larger $d_s$). The correlation coefficient amounts to $C = 0.23$. 
Figure 4: Starting frequency $f_s$ versus: a) MW burst impulsiveness $IMP$  
b) MW burst peak flux $PF$.
The starting frequencies of type II bursts $f_s$ are shown versus the impulsiveness $IMP$ of the associated MW bursts in Figure 4a. A correlation similar to the one established by Pearson et al. (1989) using the hard X-ray bursts is found: the starting frequency increases with the flare impulsiveness. Let us note that the exponent of the power law fit amounts to $<0.09$ (see the inset in Figure 4a) indicating that the two parameters are not much related. The correlation coefficient amounts to $C = 0.23$. A weak positive correlation between $f_s$ and $IMP$ is probably a reflection of the negative correlation between $f_s$ and $t_F$ (see also Equation (5)) combined with the definition of impulsiveness ($IMP \propto 1/t_F$).

The starting frequencies of type II bursts $f_s$ are shown versus peak fluxes of the associated MW bursts $PF$ in Figure 4b. The correlation is even weaker than the one with the impulsiveness (the least squares fit gives for the power law exponent the value of $\approx 0.06$ and the correlation coefficient is only $C = 0.17$). This is consistent with Equation (5): the first term on the right hand side is proportional to $PF^\alpha$, whereas the second one is proportional to $IMP^{-\alpha} \propto PF^{-\alpha}$.

Figures 3 and 4 indicate that the starting frequency of type II bursts depends primarily on the rise time of the flare, i.e. the duration of the related source expansion time. As expected, $f_s$ does not depend much on the flare peak flux ($PF$). A weak positive correlation of $f_s$ with the impulsiveness $IMP = PF/t_F$ can be interpreted as a reflection of the negative $f_s(t_F)$ correlation.

4.3. Velocities and Bandwidths

The correlation between the shock velocities $v_s$ inferred from the drift rates of type II bursts and the MW peak fluxes $PF$ is shown in Figure 5a. The values for velocities are taken from SGD and we do not know which coronal model was used in each individual case. The positive correlation revealed by Figure 5a is consistent with the model, since $U_m$ is an increasing function of $Q^{max}$, and the shock Mach number $M = v_s/v_A$ is an increasing function of $U_m = u_m/v_A$. Assuming that the variations of $v_A$ are not too large from one to another active region, a positive correlation between the shock velocity and the $PF$ can be
Figure 5: Inferred shock velocity $v_s$ versus: a) MW burst peak flux $PF$
b) MW burst impulsiveness $IMP$. 
Figure 6: Instantaneous relative bandwidth of type II burst BDW versus the inferred shock velocity $v_s$.

The inferred type II burst shock velocities $v_s$ are shown versus impulsiveness IMP in Figure 5b. The correlation is of the same type but weaker then with the $PF$ (see the correlation coefficients in the insets in Figure 5). This is consistent with the model, since there is no reason to expect any correlation between $v_s$ and $t_F$.

The correlation between the inferred shock velocities and the instantaneous relative bandwidths $BDW = \delta f/f$ of type II bursts is shown in Figure 6. Here the ”small-sample” is used since it provides the values of $BDW$ (see Section 3). Consistent with Equation (6), one finds a high correlation coefficient ($C = 0.72$). Note that the equation for the linear least square fit exposed in the inset of Figure 6 shows $v \approx 0$ for $BDW = 0$. 

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Table II: The Correlation Coefficients.

<table>
<thead>
<tr>
<th>TYPE II</th>
<th>FLARES</th>
<th>rise time $(t_F)$</th>
<th>peak flux $(PF)$</th>
<th>impulsiveness $(IMP = PF/t_F)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>type II delay $(t_a)$</td>
<td>+0.68</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>norm. delay $(T = t_s/t_F)$</td>
<td>-</td>
<td>-0.44</td>
<td>≈ 0</td>
<td></td>
</tr>
<tr>
<td>starting freq $(f_s)$</td>
<td>-0.23</td>
<td>+0.17</td>
<td>+0.23</td>
<td></td>
</tr>
<tr>
<td>inferred shock velocity $(v_s)$</td>
<td>≈ 0</td>
<td>+0.47</td>
<td>+0.39</td>
<td></td>
</tr>
</tbody>
</table>

5. Discussion and Conclusions

The presented analysis reveals a number of correlations between the characteristics of metric type II bursts and the associated flares. In Table II the correlations are summarized showing the correlation coefficients $(C > 0 – \text{positive correlations ”} ^{\uparrow} \text{”}, C < 0 – \text{negative correlations ”} \downarrow \text{”})$. Comparing Table II with Table I, a very good agreement is found, indicating that the characteristics of a shock wave are related in the predicted manner with the properties of the flare energy release. So, the observations confirm that the model by Vršnak and Lulić (2000a,b) provides a plausible description of the coronal shock wave formation.

The scatter in correlations may be caused by several factors which can be summarized as follows.

a) Almost all of the correlations depend on the preflare Alfvén velocity, which can be significantly different in different active regions. For example $U_m = u_m/v_A$, and $U_m \propto (Q^{\text{max}})^{\alpha}$, and almost all quantities depend on $Q^{\text{max}}$.

b) All of the studied characteristics of type II bursts depend on a number of flare characteristics, which are not necessarily related. For example, the time delay $t_s(Q^{\text{max}}, t_F)$, but $Q^{\text{max}}$ itself is not related to $t_F$.

c) When the starting frequencies $f_s$ are considered, the scatter can be explained simply by a different density and Alfvén velocity coronal scale heights for different events.

Finally, there is another aspect which has to be taken into account. Mass ejecta in general do not develop fast enough to generate
type II bursts starting in the dm/m wavelength range. The eruption growth rates are usually not higher than \( \omega = 10^{-3} \text{s}^{-1} \) for CMEs and eruptive prominences (Vršnak, 2000), corresponding to a time scale of an hour. So, CMEs can be sources of type II bursts starting in the deka/hectometric wavelength range. However, a fraction of type II bursts starting at \( f_s < 100 \text{ MHz} \) could be caused by fast eruptive ejecta such as flare-sprays, since their growth rate can be as fast as \( \omega = 10^{-2} \text{s}^{-1} \) (Vršnak, 2000), corresponding to a time scale of ten minutes. Using \( v_{FS} \approx 1000 \text{ km s}^{-1} \) one obtains \( d = 600000 \text{ km} \) i.e. \( f_s^H \approx 80 \text{ MHz} \) for the harmonic frequency in the 2xNewkirk coronal density model (Newkirk, 1961). Using the flare energy release indicators, which in fact are not related to the type II burst formation in such cases, one introduces an additional scatter in the considered correlations.

References

KORELACIJE SVOJSTAVA PROVALA RADIO ZRAČENJA TIPA II I PRIDRUŽENIH BLJESKOVA

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izlaganje sa znanstvenog skupa
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Sažetak. Istražuje se povezanost svojstava metarskih provala radio zračenja i pridruženih provala mikrovalnog zračenja zbog provjere modela koji opisuje nastanak udarnog vala procesom ustrmljivanja poremećaja velike amplitude izazvanog Sunčevim bljeskom. Ustanovljeno je da je kašnjenje pojave provale radio zračenja tipa II za početkom provale mikrovalnog zračenja kraće pri snažnijoj mikrovalnoj provali. Značajno je manji koeficijent korelacije za impulzivnost mikrovalne provale. Početna frekvencija provala tipa II raste s impulzivnošću mikrovalne provale. Snažnije mikrovalne provale povezane su s provalama tipa II veće relativne spektralne širine i veće brzine prostiranja, te su i oba ova parametra korelirana. Sve korelacije su u potpunoj suglasnosti s modelom stvaranja udarnog vala eksplozivnim širenjem područja bljeska.

Ključne riječi: bljeskovi - udarni valovi - provale radio zračenja