Non-Linear Evolution of Self-Gravitational MHD Instabilities in Rotating Disks
Chou, W.1, Matsumoto, R.2, Tajima, T.3, Umekawa, M.2, and Shibata, K.4 1RIST, Japan, 2Chiba University, Japan, 3University of Texas at Austin, USA, 4Kyoto University, Japan
Email (W.C.), chouwc@chiba-u.ac.jp

Abstract
We carried out linear analysis and non-linear 3D simulations of self-gravitational magnetohydrodynamics of gas layer in a local co-rotating coordinates. We studied the evolution of Parker-Jeans instability in several cases which are different in factors such as rotational speed, magnetic field, external pressure, external gravitational force, and directions of perturbations. We found that the growth rate of the instabilities and the shape of the dense blobs aggregated in the nonlinear stage sensitively depend on these factors. This study provides us hints on how interstellar/intergalactic gas aggregates to form dense blobs/filaments, which correspond to molecular clouds, under the circumstance of rotation and magnetic field.

1 Introduction
The process of star formation begins by the collapse and fragmentation of interstellar clouds. Observations of star-forming regions suggest that many dark clouds have filamentary shapes. It is thought that the filaments are formed through the instability caused by the combination of self-gravity and magnetic field, which is called Parker-Jeans instability. Elmegreen & Elmegreen (1978), Hanawa et al. (1992) and Nagai et al. (1998) have carried out linear analysis of Parker-Jeans instability in several aspects. There are also efforts to simulate the collapse and fragmentation process of magnetized filaments/cylindrical clouds (INUTSUKA & MIYAMA 1997; Tominaka & Nakamura et al. 1993). However, the nonlinear evolution of Parker-Jeans instability in a sheet-like gas, i.e. the process which form a filament from a sheet-like gas, has not yet been investigated.

We can compare the time scale of Jeans instability (t\text{\text{Jeans}}) and that of Parker instability (t\text{\text{Parker}}) to see which of these two plays a more important role. The ratio of these two time scales is:

\[ \frac{t_{\text{Jeans}}}{t_{\text{Parker}}} = \frac{(C_s/v_A)(r_0/v_{K,\text{Kepler}})}{H/C_s} = \frac{2\pi \rho_0 v_A^3 C_s}{M v_A} \]

where \( M \) is the mass of central object, \( H \) is the scale height, \( C_s \) is the sound speed, \( v_A \) is the Alfvén velocity, \( \rho_0 \) is the local density, and \( r_0 \) is the distance to the central object. For typical values of parameters in interstellar gas, this ratio is on the order of unity. This means that these two processes are comparably important in bringing interstellar gas to form molecular clouds. Here we discuss the parameter regime when \( \rho_0 v_A^2 / M \geq 1 \) (the case that self-gravity is slightly stronger than magnetic force, but the latter is still important).

The governing equations are the MHD equations with Coriolis force plus the Poisson equation for the self-gravitational force. We normalize the equation by the sound speed \( C_s \), the unperturbed density at the mid-plane of the disk \( \rho_0 \), and the scale height which is defined by \( H = C_s / \sqrt{2\pi G \rho_0} \). We approximate the equilbrium gas layer by a uniformly rotating disk and we consider a local co-rotating sheet. In the unperturbed state the density, pressure, and magnetic field are uniform in the \( x \) and \( y \) directions and varies in the vertical \( z \) direction. The gravity in the horizontal plane is canceled by the centrifugal force. The equilibrium magnetic field aligns in the \( z \) direction. The ratio of gas pressure to the magnetic pressure \( \beta = 8\pi p_e B^2 / B^2 \) is assumed to be constant inside the disk.

2 Results of Linear Analysis
We reproduce the results of Nagai et al. (1998) when we assume there is no rotation (\( \Omega = 0 \)). Here we further investigate the modes when there is rotation. Figure 1 shows the maximum growth rate as a function of plasma beta and angular velocity \( \Omega \). The asymptotic value of the curve \( \Omega = 0 \) toward \( \beta = \infty \) corresponds to the growth rate of pure jeans instability. The increase of the growth rate when \( B \neq 0 \) is attributed to the Parker instability. For the cases of non-zero \( \Omega \), the Coriolis force suppresses the growth of Jeans instability because the paths of attracting particles are bent by the Coriolis force.

Figure 1: Maximum growth rate as a function of plasma beta and rotation

In this case the magnetic field contributes to the growth rate not only through Parker instability, but also by helping the gas to move along the field line and hence reducing the effect of the Coriolis force. From this figure we see that rotation could not change the maximum growth much when magnetic field is strong. When there is no magnetic field, however, there is a critical angular velocity \( \Omega_{\text{critical}} \) above which the disk is stabilized. One important implication of this result is that the molecular clouds could not be formed solely by self-gravity without the help of magnetic field because the galactic disk is rotating.

3 3D Simulation
We impose an initial velocity perturbation in the horizontal plane (i.e. \( v_x \) and \( v_y \)) on the equilibrium gas, where \( v_x \) and \( v_y \) are random numbers between (-0.05, 0.05). Figure 2 shows the iso-contours of density at the equatorial plane at the end of the simulations, for (a) \( \beta = 0.1 \), (b) \( \beta = 10 \), and (c) \( \beta = 1000 \), respectively. The contours are in logarithmic scale. The solid curves indicate positive contours (\( \rho > 1 \)) and dashed curves indicate negative contours. The magnetic field, which is not shown here, is mainly in the \( z \) direction. In figure 2(a) (strong magnetic field) we find that the final shape is long filaments perpendicular to the field line. This is because the gas is difficult to move across the field line. On the other hand, the shape of the final gas condensation becomes shorter filaments.
as the strength of the magnetic field decreases (figures 2(b) and 2(c)). The vertical cross section (cut at $y = 0$ plane, which is not shown here) reveals that when $\beta = 0.1$ the filament becomes elongated in the vertical direction, while the filaments have the capsular shape when $\beta$ is larger than unity.

Figure 3 shows the 3D view of the result when the effect of rotation (the Coriolis force) is included, here we adopt $\beta = 1$ and the half-thickness of the disk is one scale height. The surface is the isosurface of density $\rho = 2.7$ at the end of the simulation. Curves indicate the magnetic field lines. The filaments align coherently with their axes perpendicular to the magnetic field. The distance between two filaments (about 10H) agrees with the most unstable wavelength obtained in the linear theory.

Figure 4 shows the evolution history of the density perturbation. Figure 4(a) shows the result when we adopt the eigenfunction of most unstable wavelength as the initial perturbation and the parameters are $\beta = 1$, $\Omega = 0$, $a = 0$, and $z_{eq} = 5$. We choose the center of the equatorial plane as the sample point, and plot the density perturbation (in nature logarithmic scale) of this point as a function of time. The slope of the line in the linear growth stage is 0.83, which agrees well with that obtained in the linear theory. In the nonlinear stage the growth rate increases and the density grows without saturation. Figure 4(b) shows the evolution history of perturbed density when a random perturbation was imposed initially. We choose the position (1.2H, 2.4H, 0) as the sampling point, where the density has its maximum value at the end of the simulation. The dashed line in (b) shows the linear growth of the most unstable mode obtained in the linear analysis. The system spent long time after the random perturbation is imposed. Finally it pick up the most unstable mode and the perturbation grows exponentially. Figure 4(c) is the result from the same simulation but the position (0, 0, 0) is used as the sample point. In the linear growing period the perturbed density also grows exponentially as in the other position. In the later time, however, the perturbed density decreased rapidly because the gas starts to fall into its neighbor point where the density becomes maximum at the end. This figure demonstrates the steepening of perturbation in the nonlinear stage.

### Discussion and Conclusion

Here we plug in the typical values for the gas in a molecular cloud and compare our results with observations. Adopting sound speed $C_s = 5 \times 10^3$ cm/s (corresponds to temperature $T=30K$) and density $\rho = 2 \times 10^{-23}$ g cm$^{-3}$ (corresponds to about 10$^5$ particles per cubic centimeter), the scale height is then $H = C_s / \sqrt{2 \pi G \rho} = 1.7 \times 10^{18}$ cm = 0.55 pc, and the time unit is $[t] = [H / C_s] = 3.5 \times 10^{13}$ s = 1 x 10$^5$ year. The plasma beta $\beta = 1$ corresponds to the strength of magnetic field of 10 $\mu_G$. The most unstable mode obtained in our calculation has a wavelength $\lambda = 10H$, which is about 5pc. That is, our simulation result (figure 3) shows that filaments of molecular clouds align coherently with their major axes separated at a distance about 5pc. This result agrees well with the wild field view of the Taurus molecular cloud (Onishi et al. 1998) and of the Ophiuchus molecular complex (Loren 1989).

We summarize our results as follows. In our linear analysis we include the effect of rotation and strength of magnetic field. If the magnetic field is weak, the growth rate sensitively depends on the rotational angular velocity. When magnetic field is absent, rotation stabilizes all modes. This implies that molecular clouds must be formed with the help of magnetic field. From the simulation we find that the gas form long filamentary structure perpendicular to the magnetic field, and becomes coherent if rotation is included. The disk with low plasma beta forms long filaments with elongated vertical size, while the disk with high plasma beta forms short, capsular structures. The geometry (coherent long filaments) and the distances between filaments obtained from the linear theory and nonlinear results (about 5 pc) agree with observations of molecular clouds.

### References

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Proceedings of Star Formation 1999

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