Dynamics of Local Isolated Magnetic Flux Tubes in a Rapidly Rotating Stellar Atmosphere

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Abstract

The dynamics of magnetic flux tubes in a rapidly rotating stellar atmosphere is considered. We focus on the effects and signatures of the instability of the flux tube emergence influenced by the Coriolis force. We present the results from a linear stability analysis and discuss its possible signatures during the course of the evolution of G-type and M-type stars. We carried out three-dimensional magnetohydrodynamical simulations of local isolated magnetic flux tubes under a magnetic buoyancy instability in co-rotating Cartesian coordinates. We have found that the combination of the buoyancy instability and the Coriolis effect gives rise to a mechanism to twist the emerging magnetic flux tube into a helical structure. The tilt angle, east-west asymmetry, and magnetic helicity of the twisted flux tubes in the simulations have been studied in detail. The linear and nonlinear analyses provide hints as to what kind of pattern of large spots in young M-type main-sequence stars might be observed. We have found that young and old G-type stars may have different distributions of spots, while M-type stars may always have low-latitude spots. The size of stellar spots may decrease when a star becomes older, due to a decrease in the magnetic field. A qualitative comparison with solar observations is also presented.

Key words: Magnetohydrodynamics — Stars: activity — Stars: rotation

1. Introduction

The introduction (Vogt, Penrod 1983) and use of the Doppler-imaging techniques allow the reconstruction of surface-luminosity patterns of rapidly rotating stars. A surprising result provided by these Doppler images is the presence of large, dark starspots appearing at high latitudes. In contrast, sunspots are restricted to a latitude band within 30° of the solar equator, and are relatively small in size. The Zeeman and magnetic Doppler images suggest that stellar spots are magnetic in nature, just like sunspots (Donati, Catala 1993; Saar et al. 1994). One might naturally ask why sunspots are different from some starspots, both in size and distribution. Two possible explanations for the large high-latitude starspots have been proposed. One is by Schüssler and Solanki (1992): the Coriolis force deflects a magnetic flux tube rising through the stellar convection zone towards higher stellar latitudes. The other is by Chou et al. (1997): the magnetic buoyancy of flux tubes at low latitudes is suppressed by the Coriolis force, while flux tubes at high latitudes remain unstable and their longer wavelength perturbations grow faster, forming large active regions. Schüssler et al. (1996) made a further analysis on the emergence of flux tubes in stars of 1 $M_\odot$ ranging over different ages and at different rotational speeds. They concluded that the young Sun may have had high latitude spots. However, the shape and distribution of starspots for other types of young stars, which may have magnetic fields and rotational periods different from those of the Sun, remain unknown.

Lower main-sequence stars (spectral types G, K, and M) have deep convection zones. The magnetic activity in these cool stars is believed to be associated with the combination of convection and rotation of the stars (Weiss 1994) and is similar in essence to, though possibly different in details from, that found in the Sun. It has been

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known for many years that Ca II emission is closely correlated with magnetic fields on the Sun. Skumanich et al. (1975) deduced a linear proportionality of Ca II emission to the magnetic field strength at the surface. It therefore provides a good measure to estimate the surface magnetic field. On the other hand, Rosner and Weiss (1985) studied the rotational history of the Sun (or G type stars), and concluded that the Sun had a rapid acceleration of rotation due to a decrease in the moment of inertia while it gravitationally collapsed onto the main sequence, followed by a spin-down to its current rotational speed through magnetic braking, owing to torque exerted by the stellar (solar) wind. The maximum angular velocity occurs at the age of $3 \times 10^7$ yr, and may have reached a speed 100-times faster than its current value. Moreover, it has become apparent that there is a strong positive correlation between the stellar rotation rates and the overall chromospheric activities (the Ca II emission levels). Skumanich (1972) studied the relations of the Ca II emission luminosity and the rotation of Sun-like stars to their age, and found that the rotational temporal decay curve follows the same rule as that of the Ca II emission. This means that the surface magnetic field and the rotation of sun-like stars decay at the same temporal rate. For stars of other spectral types Noyes et al. (1984) studied the relation of magnetic activity to the rotation and to the spectral type. They found that the mean chromospheric emission is rather well characterized by a single parameter, the Rossby number. The Rossby number in rotational hydrodynamics is the ratio of the rotation period to the convective overturn time, characterizing the thermal instability of the rotating fluid. The Rossby number is defined as

$$ Ro = \frac{P}{\tau_c}, $$

where $P$ is the rotation period and $\tau_c$ is the convective overturn time. In Noyes et al. (1984) they used the observed rotation period $P$ and the estimated convective overturn time $\tau_c$ near the bottom of the convection zone. Their study indicates that $\tau_c$ is a function of the spectral type alone. The mixing-length theory predicts an increasing influence of rotation on convection by decreasing the Rossby number, which may in turn lead to a greater $\alpha$-effect for late-type stars (Schüssler 1983). The magnetic field generated by this dynamo process is determined by the Rossby number. For stars of the same rotational periods $P$ (and hence, the same age), later spectral type stars have deeper convection zones, and thus have a larger $\tau_c$ and stronger magnetic fields, which may result in higher Ca II emission.

Parker (1955, 1966) was the first to propose that the magnetic activities observed in the Sun originate from the emergence of magnetic flux embedded in the convection zone. The sunspots are thought to be the foot points of emerging magnetic flux tubes. The physics of emerging flux tubes in the Sun has been discussed in detail in the recent literature using the so-called thin flux tube approximation (see review by Moreno-Insertis 1993; also Caligari et al. 1995; Moreno-Insertis et al. 1994; Fan et al. 1993, 1994; D'Silva, Chaudhuri 1993; Wang, Sheeley 1991; Chou, Fisher 1989; Chaudhuri 1989; Chaudhuri, Gilman 1987). Compared with the current Sun, young G-type stars, at the age of $3 \times 10^7$ yr, rotate much faster (100 times faster) and possess much stronger magnetic fields. Young M-type stars have even stronger magnetic fields than young G-type stars. There is evidence showing that stars with strong magnetic fields have large starspots [up to 60% of the surface area (Byrne 1992)]. Therefore, the thin flux tube approximation may not apply to these stars, and flux tubes with finite dimensions must be considered in order to understand the formation of large spots. Recently much effort has been devoted to understanding the behavior of a fat flux tube based on threedimensional magnetohydrodynamical (3D MHD) simulations (Dorch, Nordlund 1998; Longcope et al. 1998; Ziegler, Ulmschneider 1997a, 1997b; Matsumoto et al. 1998; Fan et al. 1998). In what follows we present a linear theoretical analysis and a nonlinear MHD simulation of this problem. The advantage of using a fat flux tube is that it has a radius instead of a thread-like ring, so that we can study its internal structure, and it more likely corresponds to large spots of stars with strong magnetic fields. In this paper, we consider the dynamics of local isolated emergent flux tubes (Matthews et al. 1995), particularly in its nonlinear stage, under the influence of the Coriolis force. We set the tube in a local co-rotating Cartesian frame, neglect the magnetic tension force produced by the curvature of the ring, and assume an isothermal background. This work provides general ideas about how the Coriolis force affects a fat buoyant flux tube, while providing a theoretical understanding of the features of active regions in a young M-type star; it also gives a comparison with solar observations and simulations using the thin flux tube approximation.

2. Basic Equations and Linear Stability Analysis

The basic equations governing the plasmas in a rotating stellar atmosphere are:

$$ \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{V}) = 0, \quad (2) $$

$$ \frac{\partial (\rho \mathbf{V})}{\partial t} + \nabla \cdot \left( \rho \mathbf{V} \mathbf{V} + \rho \mathbf{I} - \frac{BB}{4\pi} + \frac{B^2}{8\pi} \mathbf{I} \right) + \rho g + 2\rho \mathbf{\Omega} \times \mathbf{V} = 0, \quad (3) $$

$$ \frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{V} \times \mathbf{B}), \quad (4) $$
\[
\frac{\partial}{\partial t} \left( \rho U + \frac{1}{2} \rho V^2 + \frac{B^2}{8\pi} \right) + \nabla \cdot \left[ \left( \rho U + p + \frac{1}{2} \rho V^2 \right) V + \frac{c}{4\pi} E \times B \right] = -\rho g \cdot V,
\]
(5)

where

\[
U = \frac{1}{\gamma - 1} \frac{p}{\rho},
\]
(6)

\[
E = -\frac{1}{c} V \times B,
\]
(7)

\[\gamma\] is the adiabatic gas constant, \(\Omega\) is the rotational angular frequency, and the other letters have their usual meanings in magnetohydrodynamics. Compared with the MHD equations in the inertial frame, the only change in the current set is that the momentum equation (3) has an additional Coriolis-force term, \(\Omega \times V\). The centrifugal-force term is neglected, since in the stellar case it is small compared with the gravity term. The resistivity, viscosity, and thermal diffusivity are neglected.

A simplified linear analysis considering the stabilization effect on buoyancy (Parker) instability by the Coriolis force was carried out by Chou et al. (1997). Here, we briefly summarize the analyses and the key results. We adopted isothermal hydrostatic equilibrium with a constant gravitational field \(g\) in the negative \(z\)-direction and a magnetic field of constant \(\beta\) in the \(x\)-direction. The perturbation is assumed to be of the form

\[
\delta p = \delta p_0(z) \exp(i\omega t - ik_xx - ik_yy).
\]
(8)

After appropriate manipulation similar to what has been carried out by Parker (1979), we obtained a linear dispersion relation by a determinant of a \(8 \times 8\) matrix. From this dispersion relation we found that the 3D unstable buoyancy mode (mixed mode of interchange and undular modes, see Matsumoto et al. 1993) at low latitudes is stabilized when the rotational angular velocity \(\Omega\) is large enough (see Acheson 1978, 1979 for the rotational stabilization of buoyancy modes). Furthermore, it is found that the critical rotational angular velocity \(\Omega_c\) that stabilizes the buoyancy of the equatorial toroidal magnetic fields depends on the field strength. The solid thick line in figure 1 shows the critical \(\Omega_c\) as a function of the field strength, or the plasma \(\beta\). The region above the line (upper-right) is a stable region, where low-latitude spots are prohibited and high-latitude spots are expected. The region below the line is where equatorial magnetic fields are unstable. It is interesting to compare this critical curve with that estimated by Schüssler et al. (1996). They suggested a minimum field strength of \(B_{\text{min}}\) for a significant influence of the buoyancy force. When buoyancy dominates the Coriolis effect (i.e. \(B > B_{\text{min}}\)), a radial eruption occurs instead of poleward motion. In terms of solar values (see the next paragraph), they have

\[
B_{\text{min}} = 10^5 \left( \frac{H}{H_\odot} \right) \left( \frac{\rho}{\rho_\odot} \right)^{1/2} \left( \frac{\Omega}{\Omega_\odot} \right) \beta.
\]
(9)

We find in figure 1 that this equation approximately fits with the critical curve \(\Omega_c\) which was numerically derived by our linear analysis. In their conclusion, a flux tube in the upper-right region of figure 1 will move poleward and generate high-latitude spots. This is the same assertion of our linear analysis. Although our calculation deals with a continuous distribution of magnetic fields and analyzes the linear stability, and Schüssler et al. (1996) adopted thin flux tubes and carried out non-linear stability analysis, these two calculations reach the same result.

Adopting the values from Spruit’s solar convection zone model (Spruit 1974), at the bottom of the convection zone the temperature \(T\) is \(2.2 \times 10^6\) K, the sound speed \(C_s\) \(2.2 \times 10^7\) cm s\(^{-1}\), the density \(\rho_0\) 0.2 g cm\(^{-3}\), the pressure \(P\) \(6 \times 10^{13}\) dyn cm\(^{-2}\), and the scale height \(H\) 5.6 \(\times\) 10\(^9\) cm. The magnetic field in this region is
estimated to be $10^5$ G (D’Silva, Choudhuri 1993). The angular velocity $\Omega$ is then $7 \times 10^{-4}$ [$H/C_\alpha$], and the plasma $\beta$ is $1.6 \times 10^5$. A young G-type star at an age of $3 \times 10^7$ yr (a “zero-age” G-type star) rotates 100 times faster than the current Sun, and the magnetic field (at the bottom of the convection zone) is estimated to be 100 times stronger. Using the values calculated by stellar models adopted by Schüssler et al. (1996), we estimate that the pressure at the bottom of the convection zone in the zero-age G-type star is about 10 times larger than that in the current Sun. The corresponding values for the current Sun and the zero-age G-type star are marked in figure 1, and an estimated evolution trace is drawn by a dotted line. The positions of the Sun at ages of $10^8$ and $10^9$ yr are characterized by its rotational speeds relative to the current Sun (Rosner, Weiss 1985), and are also marked on this trace. Extrapolating from the relation $\tau_c (B - V)$ in Noyes et al. (1984) to stars of spectral type M, assuming that young M stars rotate as fast as young G stars of the same age, and making another extrapolation to estimate $B(R_0)$, we estimate that the strength of the magnetic field in an M-type star is one order of magnitude greater than that in a G-type star. Thus, the zero-age M-type star has parameters indicated by the dark triangle in figure 1 ($\beta \sim 1$, $\Omega \sim 0.1$); these are the values we adopted in our simulation. An estimated trace of the evolution for an M-type star is drawn by another dotted line, assuming that it is parallel to the evolution of a G-type star. We find that M-type stars stay in the unstable region during their evolution, meaning that magnetic fields embedded in an M-type star erupt radially at low latitudes, regardless of its age. In contrast, the evolution path of a G-type star crosses the critical $\Omega_c$ curve so that a zero-age G-type star (in which low latitude spots are prohibited) and the Sun have different spot distributions. Caution should be exercised because we have made a rough extrapolation and assumption about the magnetic fields and rotational speed of M-type stars. However, since the M-star evolution curve (the dotted line) is away from the critical $\Omega_c$ curve (the thick line) at least one and a half orders smaller in $\beta$, the conclusion that M-type stars always stay in the unstable region probably remains valid even if we had underestimated $\beta$ to one order of magnitude. The dashed line in figure 1 corresponds to the relation $B \propto \Omega$, as if the stellar structure (pressure, scale height, etc.) were not changed during the evolution. In other words, this line corresponds to the quantity $\Omega H/V_\Lambda = 0.25$ remaining the same value as in the Sun, where $V_\Lambda = B/\sqrt{4\pi \rho}$ is the Alfvén speed. The physical interpretation is that every point in this line has the same ratio of the Coriolis force to the magnetic buoyancy force.

3. Nonlinear Simulation

3.1. Initial and Boundary Conditions

We adopt the normalization $\rho (z = 0) = C_\alpha = H = 1$ in the following analysis, where $C_\alpha$ is the sound speed, $H$ is the scale height, and $z = 0$ is the central layer of the initial flux tube with $+z$ pointing upwards. In the simulation, we assume an ideal gas. Cartesian coordinates are used with $z$ in the vertical direction and a cylindrical flux tube lying along the $x$ direction. The gravitational acceleration is $g = -ge_z$, where $g$ is assumed to be constant. The equilibrium density and pressure distribution without a magnetic field is derived from the equation of hydrostatic balance, assuming isothermal atmosphere. The parameters adopted in our simulation are the angular velocity $\Omega = 0.1$ and the plasma $\beta = 1$, which are indicated as the triangle in figure 1. As explained in the previous section, this set of parameters corresponds to a flux tube embedded in the atmosphere of a young M-type star.

The magnetic field is embedded inside a cylinder of radius $R$ and parallel to the $x$-direction with the distribution

$$B_x = \left[ \frac{8\pi p(z)}{\beta + 1} \right]^{1/2},$$

and $B = 0$ outside the tube. Inside the tube, the magnetic pressure, $B^2/8\pi$, is subtracted from the pressure $p(z)$, so that this equilibrium state satisfies the force balance, and hence the temperature inside the tube is lower than that of the surroundings. Another choice of the initial condition could be that the temperature is constant while the density inside the tube is reduced. This initial condition is not in equilibrium, and the tube would buoyantly rise. In our arrangement, on the other hand, we start from a flux tube which is in (unstable) equilibrium and rises because of the growth of the undular mode of the magnetic buoyancy instability. In this paper, we refer to a ‘buoyant tube’ as being an initial equilibrium tube, but rising due to the buoyancy instability. At the edge of the tube, we smooth the edge by using a hyperbolic tangent function,

$$B'(r) = B(r) \times \frac{1}{2} \left[ 1 - \tanh \left( \frac{r - R}{\Delta} \right) \right],$$

in order to avoid an abrupt change of the field and thus numerical errors. Here, $r = \sqrt{y^2 + z^2}$ is the distance from the center of the tube, $R$ is the radius of the tube, and $\Delta$ is the grid size.

The code uses the modified Lax–Wendroff scheme with artificial viscosity to avoid a numerical instability. The accuracy of such an MHD code is described by Shibata (1983). Some 3D tests using this code are described by Matsumoto and Shibata (1992) and Matsumoto et al. (1993). The number of grid points in our simulation is
Fig. 2. Isosurface of magnetic field energy density $|B|^2 = 0.5 \left[ \rho_0 C_s^2 \right]$, for (a) model 1 (a flux tube at the north pole) at time $= 29.09 \left[ H/C_s \right]$, (b) model 2 (a toroidal flux tube at north $45^\circ$) at time $= 28.37 \left[ H/C_s \right]$, and (c) test model, no rotation, at time $= 25.79 \left[ H/C_s \right]$.

$N_x \times N_y \times N_z = 42 \times 42 \times 72$. A periodic boundary condition is imposed at $y = y_{\text{min}} = -4H$, $y = y_{\text{max}} = 4H$, $x = 0$, and $z = L_z = 20H$ (where $L_x$ is the length of the flux tube), while the symmetric boundary condition at $z = z_{\text{min}} = -4.32H$, and the free boundary at $z = z_{\text{max}} = 8.64H$ so that waves transmit through the top boundary. The simulation starts with a straight cylindrical magnetic flux tube with radius $R = H$. To initiate the dynamics, a small velocity perturbation is added inside the tube,

$$V_x = 0.05 \sin[k_x(x - L_x)] \cos(k_y y),$$  \hspace{5cm} (12)

where the perturbation wavelengths are $k_x = 4\pi/L_x$ and $k_y = 2\pi/L_y$. The wavenumber $k_x$ is chosen to have the maximum growth rate (in the linear stage) of the Parker instability. If a random perturbation is imposed, a small wavelength interchange mode has a larger growth rate during the linear stage and dominates the undular mode. As a result, the tube would not rise. However, if a large-scale perturbation exists, the (large-scale) undular mode will turn out to be important in the nonlinear stage, and the top portion of the undulating tube may arise (Matsumoto et al. 1998). We do not adopt a random perturbation because our goal is not to study all of the modes. We are interested only in the mode which makes the flux tube arise. Matthews et al. (1995) proposed a new mechanism by which one can form arched magnetic flux tubes from an initially uniform field by the interaction of vortices. In that case, no explicit perturbation, such as equation (12), is needed.

We adopt a nearly isothermal value of the adiabatic index $\gamma$ of 1.05. In an isothermal atmosphere, which is adopted in our simulation to keep the analysis simple, a larger $\gamma$ has a stabilizing effect on the Parker instability. In reality, in the boundary between the convection zone and the radiative zone where we expect that magnetic twists are accumulated, the background atmosphere is not isothermal, but has a temperature gradient slightly below the marginally stable value for convection. In such a case, we can use a larger $\gamma$ to study the growth of a magnetic buoyancy instability. The combined effect of the temperature gradient and Coriolis twisting on buoyant flux tubes will be studied in future work.

3.2. Simulation Results

We show the results from three models plus one test case: in model 1, the angular velocity $\Omega$ is in the $x$-direction, which corresponds to the flux tube sitting at the north pole. In model 2, the $x$-direction is stellar west and the $y$-direction is north, and $\Omega$ lies in the $y$–$z$ plane having an angle of $45^\circ$ with the $z$-axis. This corresponds to the case of a toroidal magnetic flux tube sitting at the latitude of north $45^\circ$. Model 3 will be discussed later. We also have a testing model to check our code by setting $\Omega = 0$. We compared the result with the existing studies and have successfully reproduced simulations of magnetic buoyancy in a static inertial frame, similar to the results by Matsumoto et al. (1993)

Figure 2 shows isosurfaces of magnetic field energy density, $|B|^2 = 0.5[\rho_0 C_s^2]$. In model 1 (figure 2a), we can see that the tube is twisted by the Coriolis force. It looks like a rubber tube wrung from two ends. The reason for such a twist is that the buoyant flux tube expands at a higher level so that the rising tube has velocity compo-
ponents of $\pm V_y$ on two different sides of the tube; hence, the directions of Coriolis forces are different on these two sides and cause the twist. This is the same mechanism as the "alpha effect", which was first proposed by Parker (1955): the expanding tube rotates anticyclonically as it expands due to the conservation of angular momentum. When the flux tube is in the southern hemisphere, the tube is twisted in the opposite direction and hence the tilt angle and the helicity have opposite signs compared to those in the northern hemisphere. On the other hand, when the rotational angular frequency $\Omega$ does not have a $z$-component, which corresponds to the tube sitting at the equator, we do not find twisting. We conclude that it is the $z$-component (anti-parallel to gravity) of $\Omega$ that causes the twisting of the flux tube. In figure 2b, since $\Omega$ has both $z$ and $y$ components, the Coriolis force not only serves as a twisting agent, but also makes the emerging portion and foot point develop east-west asymmetry because the rising portion of the flux tube is shifted toward the $x$-direction. Figure 2c is the result of the non-rotating model ($\Omega = 0$). We show this figure for the purpose of a comparison.

Figures 3 and 4 are top views of the flux tube at a specific altitude ($z = 4.86$). Here, we show (a) the current in the $z$ direction $J_z$, (b) the magnetic field in the $z$ direction $B_z$, (c) the current field, and (d) the local helicity pattern, defined by $\alpha = J_z/B_z$. Figure 3 is for model 1 (tube at the north pole) and figure 4 is for model 2 (tube at north 45°), respectively. These top views give us a hint of what we can expect if we have sufficient power to resolve a stellar surface. The thin line in each $J_z$ figure indicates the tilting of the flux tube viewed from above. Note that the flux tube is twisted in such a way that its foot points (corresponding to large $B_z$ areas, denoted by "preceding" and "following" in figures of $B_z$) align in the direction of upper-left to lower-right if the tube is in the northern hemisphere. This result shows that the alignment of the stellar spots is essentially the same as that of the sun, in which case the thin flux tube simulation has obtained a result consistent with the solar observation. In figure 3, we see that the preceding and following foot points have the same pattern and are symmetric to each other. However, in figure 4 we lose this symmetry. Instead, the field in the preceding spot is slightly greater than that in the following one. We conclude that this east-west asymmetry is due to the presence of the $y$-component of rotational angular velocity $\Omega_y$. This is because the Coriolis force, the cross-product of the rising velocity $V_y$ with $\Omega_y$, is in the negative $x$-direction. The preceding spot, which is located on the right hand side (positive $x$) of the following one, has a lower gas pressure, since the plasma has been pushed away towards the following spot. Thus, the preceding spot has a stronger magnetic field. From figure 4 we also note that there is no noticeable poleward (y-direction) motion of the flux tube upon its emergence to the surface.

In order to compare the influence of plasma $\beta$ on the flux-tube dynamics, we arrange model 3, whose configuration is identical to model 1, except for the plasma $\beta$ value ($\beta = 10$ for model 3 and $\beta = 1$ for model 1). In figure 2a and 2b as well we note that the flux tube does not become deformed into smaller filaments, but
Fig. 4. $XY$-cross section for model 2 (a toroidal tube at north 45°) at time=28.42 [$H/C_s$]. The meanings of figures (a) through (d) are the same as those in figure 3.

Fig. 5. $YZ$ cross section at $x = 10$ plane (middle of the flux tube) for model 1 ($\beta = 1$) at $t = 29.09$ and model 3 ($\beta = 10$) at $t = 88.8$, showing (a) the velocity field for $\beta = 1$, (b) isocontour of the magnetic field $B_x$ for $\beta = 1$, (c) isocontour of $B_x$ for $\beta = 10$. Note that the flux tube becomes filament-like in (c) compared with (b).

rather rises as a whole. Figures 5a (velocity field) and 5b (isocontour of $B_x$) show the $YZ$ cross section at the center of the flux tube ($x = 10$) for model 1 at $t = 29.09$. The flux tube expands itself into a mushroom shape as it rises toward a lower density layer. Since the radius of the flux tube is relatively large (one scale height), we conclude that this flux tube should produce large spots on the stellar surface. Figure 5c shows the isocontour of $B_x$ for model 3 ($\beta = 10$) at the same cross section as in 5a and 5b at $t = 88.8$. The velocity field for model 3 is not shown because the buoyant motion has ceased and the velocity becomes small. In contrast to figure 5b, figure 5c shows that the flux tube has split into smaller filaments. Tajima and Shibata (1997) pointed out that the general morphology of plasmas with high $\beta$ tends to be filamentary, while that of low-$\beta$ plasmas is cellular or sheet-like. The simulation result shown here agrees with this tendency: filamentary for the high-$\beta$ case (model 3) and cellular for low $\beta$ (model 1). The value of $\beta$ in the overshoot region of the current Sun is large ($\beta = 10^5$); hence, we can surmise that the thin flux tube simulation applies. However, for zero-age M-stars whose $\beta$ at the bottom of the convection is estimated to be on the order of unity, it is possible that the flux tubes are fat. This is consistent with the observation that large spots are inferred in stars with strong magnetic fields.
4. Discussion

Since the Sun is the only star with surface magnetic features that we can observe in detail, it is of interest to qualitatively compare the simulation result with the solar observation. The solar soft X-ray pictures taken by the Yohkoh satellite show a sequence of regularly spaced, S-shaped active regions at low latitudes of the northern and southern hemispheres (Acton et al. 1992; Shibata 1994; Chou et al. 1996; Matsumoto et al. 1998). In general, sunspots come in pairs with opposite polarities. In the absence of rotation it is expected that the axis of the pair lines up parallel to the equator, and that the strength of each spot in the pair is approximately equal. However, the optical images and magnetograms show that the sunspots have a definite arrangement: the preceding polarity of an active region tends to be in the form of large well-formed sunspots with a stronger magnetic field near to the equator, whereas the following polarity tends to appear less organized and more fragmented with a weaker magnetic field farther from the equator (Moreno-Insertis 1993). Thus, actual sunspots show a tilt and asymmetry, where tilt refers to the axis of the pair that veers away from a latitudinal line, and asymmetry refers to the differences of appearance (including the shape, and the field magnitude) in the preceding spot and the following one.

There have been two schools of thought to study the tilt and asymmetry features of sunspots. The first one is based on the thin flux tube approximation (e.g., Moreno-Insertis 1993). These authors claim that this rotation, more specifically its Coriolis force, causes the east-west asymmetry and the tilting of magnetic flux tubes. Another set of authors (Rust, Kumar 1996; Moreno-Insertis, Emonet 1996; Matsumoto et al. 1998) studied magnetic flux tubes helically kinked prior to their emergence to the surface of the Sun. Ideally one would like to simulate the full evolution of a rising magnetic flux tube with off-axis structures. However, such a real 3D fat flux tube simulation adopting solar parameters is still beyond current computing power. The results shown in this paper simulate the dynamics of a real 3D fat flux tube, though it does not adopt solar parameters. Still, it is interesting to make a qualitative comparison of our results with the solar observation. We compare the latitudinal dependence of the tilt angle and that of the east-west asymmetry in our “fat” tube simulation with those from solar observations. The magnetic helicity, which cannot be studied by the thin-tube approximation, is also discussed.

In order to investigate the effect of the Coriolis force on emerging flux tubes located at different latitudes, we carried out simulations at latitude \(0^\circ\) (the equator), \(\pm10^\circ, \pm20^\circ, \pm30^\circ, \pm45^\circ, \pm60^\circ, \pm75^\circ,\) and \(\pm90^\circ\) (the poles). Figure 6 shows the tilt angle of the emergent portion of flux tubes as a function of the latitude in both northern and southern hemispheres. The tilt angle is defined as the angle of the thin line in figure 3 or figure 4 to the latitudinal line (x-direction). Positive values mean that such a thin line is rotated counterclockwise from the horizontal orientation. The tilt angle has a different sign in each hemisphere. In the northern hemisphere the angle is negative, meaning that a pair of spots are positioned at the upper left and lower right; the southern hemisphere has the opposite arrangement. The absolute value of the tilt angle ranges from zero at the equator to about ten degrees at the north and south 45° latitude. These results suggest that the correlation of the tilting angle and the latitude of stellar spots in a young M-type star are similar to that in the Sun.

Our simulation results show that the ratio of maximum \(|B_z|\) in the preceding foot to that in the following one is slightly greater than unity at low latitudes, ranging from 1.0 to 1.25, indicating that the field strength is just slightly greater in the preceding foot. Two reasons may explain why the ratio is near unity. First, our simulation does not consider the effects that the flux tube would encounter while it goes through the convection zone. Second, Moreno-Insertis (1993) and Fan et al. (1993) point out that if the flux tube has a magnetic field as large as \(1 \times 10^5\) G or larger, the ratio of the field strength between the preceding and following wings would be close to unity. Because our simulation employs a magnetic field larger than this value, the east-west asymmetry is
not pronounced. However, we do have the correct tendency of the asymmetry expected by the argument of the previous section.

Pevtsov et al. (1995) studied the average magnetic helicity of the Sun using a 1988–1994 data set of vector magnetograms. They concluded that statistically the average magnetic helicity of the Sun has a sign difference in opposite hemispheres. Intuition might also attribute the sign difference in the north and south hemispheres to rotation. We would like to analyze the helicity patterns caused by the Coriolis force, which cannot be seen from the thin flux tube simulation. The magnetic helicity density is usually defined by $B \cdot A$, where $A$ is the vector potential with Coulomb gauge and $\nabla \times A = B$. However, it is difficult to obtain this quantity from observations. On the other hand, if we conveniently define $\alpha$ as

$$\alpha = J_z/B_z,$$

(13)

the sign of $\alpha$ would be the same as that of the local magnetic helicity density $(B \cdot A)$, and the quantities $J_z$ and $B_z$ could be easily deduced from observations. Note that the magnetic field is not force-free in the photosphere where it is observed. However, we are analyzing the same $\alpha$ as that in the observation (Pevtsov et al. 1995), regarding it as an indication of helicity because of its simplicity. Figure 7 shows the dependence of $\alpha$ (integrated on $z = 4.86$ plane) on the latitude. The absolute value of $\alpha$ indicates how strongly the magnetic field lines are twisted. The fact that the absolute value of $\alpha$ is larger at a higher latitude could be expected, since the flux tube at a higher latitude is twisted more than it is near the equator where $\alpha = 0$ (no twisting). From our simulation, the sign of $\alpha$ is positive in the north and negative in the south. This is, however, in opposition to the observation by Pevtsov et al. (1995). It does not help if we reverse the magnetic field in the simulation, because the patterns, the asymmetry, and especially the helicity $\alpha$ remain the same, which follows directly from the basic MHD equations that a sign reversal of B keeps the equation unchanged. This suggests that the helicity observed by Pevtsov et al. (1995) may not be produced by Coriolis twisting of rising-expanding flux tubes, but instead may be the result of the dynamo action with a convection column (Kageyama et al. 1995; Yoshizawa, Yokoi 1996). Another possibility is that adopting real solar parameters in the simulation would generate the same helicity results; but this is highly improbable.

One way to check the accuracy of the code is to compare the linear growth rate with that calculated by linear analysis. Figure 8 shows the time history of $\ln |V_x|$ in model 1, where $V_x$ is the $x$-component of velocity at a fixed grid point (chosen at the center of the flux tube). There is a linear growth stage during $7 < t < 15$, and a nonlinear (saturation) stage thereafter. The linear growth rate, which is the slope of the straight line, in our simulation is 0.42. The current existing linear analysis in the literature calculates the growth rate of a ‘thin’ flux tube (e.g., Spruit, van Ballegooijen 1982, which does not include the effect of rotation; Ferriz-Mas, Schüssler 1994, which includes rotation), an isolated flux sheet (Matsumoto et al. 1993), or continuous field distributions (Chou et al. 1997, and the references cited), but the analysis of a ‘fat’ flux tube is absent. Comparing the value in our simulation to these linear analyses, we find that the growth rate is identical to that calculated by Matsumoto et al. (1993) and Chou et al. (1997) with wavelengths $k_x = 0.6$ and $k_y = 3$, which were adopted as the parameters in our simulation. Therefore, we confirm the accuracy of our MHD code. This growth rate is, however, greater than the value in Spruit and van
Ballegooijen (1982). The reason may be that the growth rate of the Parker instability is enhanced by interchange modes; but, this effect is not included in the thin flux tube approximation. On the other hand, there have been several authors who have recently questioned the validity of the thin flux tube approximation. For example, Hughes et al. (1998) questioned that the thin tube approximation could not address the issue of how tubes manage to preserve their identity as they rise.

5. Summary

Young stars, which rotate much faster and possess much stronger magnetic fields than old stars, like the Sun, have different spots from sunspots in their sizes and distributions. The large size of stellar spots is considered to be an outcome of a strong magnetic field that is, stars with smaller $\beta$ at the bottom of their convection zone may form a fatter flux tube and produce larger spots when the fat tube emerges to the surface. The distribution of stellar spots depends on the ratio of the buoyancy force to the Coriolis force, that is, on the position in the plane of $\beta$ and $\Omega$ in figure 1. Possible evolution traces for G- and M-type stars are also shown in figure 1. The trace for G-type stars crosses the critical $\Omega_c$ threshold so that the distribution of stellar spots for young and old G-type stars are different. On the other hand, M-type stars stay on the unstable side of the threshold, and we expect that starspots at low latitudes may be observed in M-type stars of all ages. However, because the value of plasma $\beta$ increases during the evolution, the size of starspots is expected to be large and cellular in young stars (of M-type, and probably of G-type as well), but to become small and filamentary in old stars.

In a rotating system, the Coriolis force twists the buoyant magnetic flux tube to form a helical structure. The $z$-component of the angular velocity, which is antiparallel to the direction of the gravitational force, causes this twisting. Except for the differences in size and the latitudinal location, the simulation shown in this paper indicates that stellar spots are qualitatively similar to sunspots in two aspects: the helical structure, and the latitudinal dependence of the tilt angle of the emergent twisted flux tube. On the other hand, we expect the $y$-component of $\Omega$, which is perpendicular to the direction of the magnetic field, to make the Coriolis force acquire a negative $x$-component upon the emerging toroidal flux tube in the northern hemisphere, and thus to produce E-W asymmetry. Our simulation, adopting a flux tube with a strong magnetic field (plasma $\beta = 1$), shows this tendency, but it is not significant. Our 3-D fat tube simulation shows the current field from top view. This result is new and can not be obtained by the thin flux tube simulation. We are looking for an observational signature that corresponds to this current field.

The signs of magnetic helicity $\alpha$ obtained in our simulation are opposite in each hemisphere, indicating that the direction of twisting is opposite in each hemisphere. However, these signs are opposite to the existing solar observations. This discrepancy suggests that the observed helicity patterns in the solar surface may not be due to the Coriolis twisting of the emerging magnetic flux tubes, and hence other models such as the dynamo action may be needed.

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