ASYMETRIES ACROSS A CORONAL HOLE EXTENSION

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ABSTRACT

The magnetic topology of a prominent coronal hole extension, observed by SOHO during August 1996, is determined using a magnetofrictional code in a finite volume which provided magnetostatic field solutions. Beforehand, the global coronal magnetic field is found analytically, assuming a linear force-free field to constrain the normal field over the local boundary. The sensitivity of the coronal hole field evolution to spatial variations in helicity across its cross-section is tested. The results reveal asymmetries in the coronal hole opening angle, and the boundaries also differ in form being curtain-like on one side and smooth on the other. Supporting evidence for the asymmetry can be seen in measurements of temperature and density by the Coronal Diagnostic Spectrometer on SOHO, and of solar wind speed obtained from EISCAT IPS data.

Key words: magnetohydrodynamic; solar.

1. INTRODUCTION

The simplest model for the magnetic equilibrium of the solar corona is a potential field approximation, but it is more realistic to include currents. The coronal plasma is small so that the Lorentz force almost vanishes and the associated equilibrium can be modelled as "force-free". This requires that local currents are parallel to the magnetic field obtained from the superposition of all currents in the corona, that is a solution to the non-linear problem

$$\nabla \times \mathbf{B} = \alpha(\mathbf{B}) \mathbf{B}$$

and a numerical technique is required. The profile \(\alpha(\mathbf{B}) = j \cdot B / B^2\) is determined from the distribution of transverse magnetic field gradients over the solar surface, and each footpoint of the magnetic field is associated with a value of \(\alpha\) which remains constant along a fieldline as it passes through the corona, connecting to a point on the boundary \(S\) of the computational domain, a Cartesian box. The \(\alpha\) profile can sometimes be obtained from vector magnetogram measurements but these are not yet available for the weak field regions investigated here, that is, an extended coronal hole observed by SOHO during August 1996 (Figure 1).

For the coronal hole problem, the \(\alpha\) profile is input as a free variable, based on intuition about how magnetic helicity might be injected or equivalently where the coronal field is most likely to be sheared by the photospheric motions. The computational problem is then to find a force-free field with a specific profile of \(\alpha\) over the solar surface, associated with the transverse magnetic field there, and this choice must be consistent with conditions applied to the other surfaces of the box. In addition, the problem is constrained by the normal component of field over \(S\); this is obtained by first finding a linear force-free field (by analysis, using an appropriate uniform value of \(\alpha\)) for the coronal region of interest, but sensitive to boundary conditions over the whole sun (transformed from photospheric line-of-sight magnetograms). Although \(B_n\) across \(S\) should be distributed according to the unknown field from a global non-linear problem (the required computational boundary condition), it is at least an improvement on previous modelling to choose a non-zero \(B_n\) derived from a particular linear global field condition. For a given choice of \(B_n\), there is then a field for every \(\alpha\) profile considered (obtained by changing the transverse field) though it is important to realise that the fields remain very sensitive to \(B_n\). Of particular interest is the opening angle of the coronal hole boundary with height for differential shear on the solar surface.

Figure 1: SOHO EIT emission map (195Å) showing the extended coronal hole of 26 August 1996.


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It should be emphasised that the weak coronal hole magnetic field is strongly influenced by its global environment; this contrasts with a model of an active region where the system can be treated as approximately “isolated” (Figure 2).

Figure 2: Differences in boundary conditions between (a) an “isolated” active region system and (b) the open domain of a coronal hole.

In Figure 3, typical analytical global solutions are shown which illustrate the approximate dynamic evolution of the coronal hole under study, but it is the 26 August 1996 image (Figure 3b) that is used to provide an appropriate boundary condition for this numerical study.

Figure 3: Simple potential field solutions showing a sector of the Sun over four rotations starting July 1996. The images show the evolution of the extended coronal hole obtained using MDI SOHO magnetogram data.

2. NUMERICAL CODE

In this section the numerical algorithm to find solutions to the non-linear force-free field problem is described. For numerical work, the vector potential, \( A(x, y, z) \), is used to represent the coronal magnetic field \( B = \nabla \times A \), since this guarantees \( \nabla \cdot B = 0 \). The domain is a Cartesian box above the solar surface \( y = 0 \) centred on the coronal hole extension. A corresponding global solar magnetic field has been found from a linear force-free field analysis (Clegg et al., 1999a,b,c) and this provides an (initial) boundary condition in terms of \( A_x, A_y, A_z \) over the surface \( S \) of the local numerical box. Inside, the initial condition is that of a random (vector potential) field and the code is designed to “relax” to a force-free field (Section 2.2) appropriate to boundary conditions which evolve to be consistent with a chosen \( \alpha \) profile (Section 2.1). The code is similar to the two phase “stress and relax” algorithm following Roumeliotis (1996), but here all vector potential parameters are free to change on the boundary and so a new “correct phase” is necessary to maintain the normal component of field over the boundary. Also, iterations to different (open) boundary surfaces are made to improve the self-consistency.

2.1 “Stress” Phase

The objective is to examine the effect of changes to the transverse field \( B_t \) over \( S \), that is, the \( \alpha \) profile and non-linearity of the problem (1), without significant changes to the normal component of field \( B_n \) during the code evolution. Suppose a particular \( \alpha = \alpha^0 \) profile is required for \( y = 0 \), then necessary changes to \( A_x, A_y, A_z \) there (to attain \( \alpha \) should ideally not affect \( B_n = \frac{\partial A_t}{\partial \tau} - \frac{\partial A_x}{\partial \tau} \). To accomplish this the value of \( \alpha \) is found at each point on the boundary \( (x, y = 0, z) \) at iteration time \( n \), from \( \alpha = \mathbf{j} \cdot \mathbf{B} / \mathbf{B}^2 \), written

\[
\alpha^n = \left( \frac{\partial B_t^n}{\partial y} \frac{\partial B_y^n}{\partial x} - \frac{\partial B_y^n}{\partial x} \frac{\partial B_x^n}{\partial y} \right) \frac{B_x^n + \left( \frac{\partial B_y^n}{\partial x} - \frac{\partial B_x^n}{\partial y} \right) B_t^n}{\left( B_x^n \right)^2 + \left( B_y^n \right)^2 + \left( B_t^n \right)^2} \tag{2}
\]

where across \( (x, y = 0, z) \) transverse field components are \( B_t^n = \frac{\partial A_t}{\partial y} - \frac{\partial A_y}{\partial x} \) and \( B_y^n = \frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} \) \tag{3}

The intention is to progress to new boundary conditions such that \( \alpha^t - \alpha^n \) decreases with time. Perturbations of the current values of \( A_x, A_y, A_z \) are explored (with adjacent node values temporarily fixed) and the values which minimise \( \alpha^t - \alpha^n \) are adopted for each successive boundary point \((i, 0, k)\). However, this process also introduces an unwanted change \( B_t^n \) which has to be corrected. This is achieved by periodically restoring...
two gradients $g_1(i, 0, k) = \frac{\partial a^e}{\partial x}$ and $g_2(i, 0, k) = \frac{\partial a^e}{\partial x}$ (which together determine $B_y$) found from the initial conditions. A staggered mesh of the new values for $A_x(i, 0, k)$ together with $g_1(i, 0, k)$ is used to generate $A_x(i, 0, k + 1)$, whilst $A_x(i, 0, k)$ and $g_2(i, 0, k)$ is used to generate $A_x(i + 1, 0, k)$. The staggered mesh is phase shifted at alternate steps. However, at any given time the transverse field components $B_x^g$ and $B_y^g$ on $y = 0$ and thence $a^e$, also depend on the instantaneous values of $A_x^g$ and $A_y^g$ over the first interior points $(x, y = dy, z)$ to allow the $\partial / \partial y$ discretisation, and this is sensitive to the degree of relaxation attained at time $n$. Choices then have to be made about the time spent “stressing” the boundary conditions to coincide with $a^e$, “correcting” to restore a time invariance in $B_y$, and of forcing the interior points in the box to “relax” towards the corresponding force-free field (section 2.2), assuming this exists.

There is however a problem with the solution so far because the transverse field components imposed over $y = 0$, which correspond to $a^e$, are now inconsistent with the original transverse field components on the other 5 sides of the computational box. To remedy this the (relaxed) field lines are followed through the interior, following the stress phase on $y = 0$, noting that $\nabla a^e \cdot B = 0$, to find the mapping of $a^e$ onto the other surfaces. This is followed by successive “stress and correct” phases for each of the 5 surfaces, operating on the appropriate components of $A$ which determine the transverse or normal field there.

2.2 “Relax” phase

a. Analysis

The algorithm for the relaxation phase is described by Craig and Sneyd (1986) but a Eulerian formulation is used. The magnetofrictional approach is one in which the elliptical boundary problem (1) is changed into a parabolic initial value problem which converges to an equilibrium because it is arranged that fluid motions decrease the magnetic energy of the plasma, with a viscous force included to prevent an indefinite oscillation between kinetic and magnetic energy. In essence the stored magnetic energy always decreases (relaxing asymptotically to a force-free state) because the magnetic field lines which move in response to the Lorentz force are doing work in moving against viscous friction. Starting from a momentum equation, $\rho Dw/Dt = J \times B - \nu v$, the inertia is neglected, $\rho = 0$, so that a flow exists for as long as the Lorentz force remains non-zero, that is $v = (J \times B)/\nu$. Following Roumeliotis (1996), the artificial viscosity $\nu$ is designed to accelerate the approach to a force-free state in regions of weak magnetic field by choosing $\nu = \mu_\nu B^2$ where $\mu_\nu$ is a constant of proportionality. Now take the induction equation

$$\frac{\partial B}{\partial t} = \nabla \times (\nu \times B - \eta J)$$

(4)

with resistivity set by a (decreasing) volume average of the Lorentz force $\eta = \langle J \times B \rangle / (\mu_\nu B^2) = \eta^* / (\mu_\nu B^2)$ (where $\mu_\nu$ is another arbitrary constant) so that reconnection is encouraged far from a force-free field, and slows down as $\langle J \times B \rangle \to 0$ (Roumeliotis, 1996). Uncurling (4), the entire evolution reduces to

$$\frac{\partial A}{\partial t} = ((\nabla \times B) \times B) / (\mu_\nu B^2)$$

(5)

This equation then determines the evolution of interior points, expanded using $\nabla \times B = \nabla (\nabla \cdot A) - \nabla^2 A$. The first term does not vanish, that is, a Coulomb gauge $\nabla \cdot A = 0$ is not maintained because arbitrary modifications are made to $A$ as the code evolves. The component equations $(x_1, x_2, x_3)$ can be written more compactly with a Kronecker delta symbol as

$$\frac{\partial A_{xj}}{\partial t} = \sum_{j=1}^{3} S_{xj,1} \cdot \tilde{L}(A_{xj})$$

(6)

where

$$S_{xj,1} = \delta_{xj,2} (\eta^* \mu_\nu B^2 - \mu_\nu^2) - B_{xj} B_{xj} - \mu_\nu^2 \mu_\nu^2$$

$$\tilde{L}(A_{xj}) = (\tilde{L}_1 + \tilde{L}_2) A_{xj} = \nabla^2 A_{xj} - \nabla (\nabla \cdot A)_{xj}$$

(7)

(8)

where for example the operation on $A_{x1}$ becomes

$$\tilde{L}(A_{x1}) = [\Delta^2_{x1} A_{x1} + \Delta^2_{x2} A_{x1} + \Delta^2_{x3} A_{x1}]$$

$$- C \frac{\partial A_{x1}}{\partial x_1} + \frac{\partial A_{x1}}{\partial x_2} + \frac{\partial A_{x1}}{\partial x_3}$$

(9)

where $C = 1$. However it is found that convergence is slowed by the presence of mixed derivatives and so it is efficient to set $C = 0$ initially and reintroduce it only close to equilibrium. Standard discretisations are used.

b. ADI method

The Alternating direction implicit (ADI) method (Richtmyer and Morton, 1967) involves taking the three component equations $A_{x1}, A_{x2}, A_{x3}$, and splitting the operations into derivatives in respect of the three spatial directions $\partial / \partial x_1, \partial / \partial x_2, \partial / \partial x_3$, so that there are nine equations to solve each time step. According to Craig and Sneyd, 1986, it is sufficient to introduce a general function $\tilde{M}_{xj}$ written as

$$\tilde{M}_{xj} = \prod_{j=1}^{3} \tilde{M}_{xj,1} = \prod_{j=1}^{3} (1 - \theta S_{xj,1} \Delta t \cdot \Delta^2_{xj})$$

(10)

$$= (1 - \theta S_{xj,1} \Delta t \tilde{L}_1) + O(x^4)$$

where $0 \leq \theta \leq 1$ determines the degree of implicitness of the code, with explicit and implicit limits at $\theta = 0$ and $\theta = 1$ respectively (although mixed derivative terms are left explicit, $\tilde{L}_2(A_{xj}^{n+1}) \to \tilde{L}_2(A_{xj}^{n})$). The algorithm for (6) is
\[ \hat{M}_{\alpha} A^{n+1}_{x} = (\hat{M}_{\alpha} + K_{\alpha} \Delta S_{x} \hat{L}) A^{n}_{x} \]  
\[ + \Delta t \sum_{p=1}^{3} S_{n+1 \alpha} \hat{L}(A^{n}_{y}) + \Delta t \sum_{p=1}^{3} S_{n+1 \alpha} \hat{L}(A^{n+1}_{y}) \]  

where \( K_{\alpha} \) is another efficiency factor, set to \( K_{\alpha} = S_{x}^{1} \), to aid the initial convergence, but ultimately replaced by \( K_{x} = 1 \) to properly represent (6).

The ADI method involves intermediate level approximations \( A^{n+1}_{x} \) and \( A^{n+1}_{y} \), and the sequential operations are then

\[ \hat{M}_{\alpha x} A^{n+1}_{x} = (\hat{M}_{\alpha x} + K_{\alpha \tau} \Delta S_{x} \hat{L}) A^{n}_{x} \]  
\[ + \Delta t \sum_{p=1}^{3} S_{n+1 \alpha} \hat{L}(A^{n}_{y}) + \Delta t \sum_{p=1}^{3} S_{n+1 \alpha} \hat{L}(A^{n+1}_{y}) \]  

\[ \hat{M}_{\alpha y} A^{n+1}_{y} = (\hat{M}_{\alpha y} + K_{\alpha \tau} \Delta S_{y} \hat{L}) A^{n}_{y} \]  
\[ + \Delta t \sum_{p=1}^{3} S_{n+1 \alpha} \hat{L}(A^{n}_{x}) + \Delta t \sum_{p=1}^{3} S_{n+1 \alpha} \hat{L}(A^{n+1}_{x}) \]  

while the terms (7) have to be calculated explicitly because of non-linearity, for example \( B^2 = |\nabla \times A|^2 \) depends on squares of first order derivatives and it is not possible to incorporate implicitly terms like \((\Delta \alpha A^2_{x})^2\), so \((\Delta \alpha A^2_{x})^2\) has to be used.

Now each of the nine equations (12) has a tridiagonal form upon discretisation, for example the 1st equation is

\[ (1 - \theta S_{x} \Delta t \Delta \alpha^2) A^{n+1}_{x} = RH \]  
\[ RH = (\hat{M}_{\alpha x} + \Delta \alpha \hat{L}) A^{n}_{x} + \Delta S_{x} \Delta t \hat{L}(A^{n+1}_{y}) \]  

and the differentiated \( N \times N \) matrix for the LH of (13) is

\[ X = \begin{bmatrix} 1 & -\Delta \alpha^2 & -\Delta \alpha^2 & \cdots & -\Delta \alpha^2 \\ -\Delta \alpha^2 & 1 & -\Delta \alpha^2 & \cdots & -\Delta \alpha^2 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ -\Delta \alpha^2 & -\Delta \alpha^2 & \cdots & 1 & -\Delta \alpha^2 \\ -\Delta \alpha^2 & -\Delta \alpha^2 & \cdots & -\Delta \alpha^2 & 1 \end{bmatrix} \]

\[ Y = \begin{bmatrix} A^{n+1}_{0} & A^{n+1}_{2} & \cdots & A^{n+1}_{2} & A^{n+1}_{0} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ Z \end{bmatrix} \]

\[ R = \begin{bmatrix} \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & RH & RH & \cdots & RH \\ RH & RH & \cdots & RH & RH \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots \end{bmatrix} \]

where \( Z = \theta S_{x} \Delta t \Delta \alpha^2 \), the problem is to find \( Y \) from \( X \times Y = R \). The superscript \( Z \) denotes the sensitivity of \( B_{i,j,k}^2 \) to the index \( i \) (in the above matrix for each \( j,k \)). In \( R \), an exchange \( A^{n+1}_{0} = A^{n}_{0} \) and \( A^{n+1}_{\tau} = A^{n}_{\tau} \), and is made since these are fixed during the "relax" phase, that is they are boundary conditions. Finally the values of \( A^{n+1}_{x} \) through to \( A^{n+1}_{y} \) are found by standard inversion techniques for the tridiagonal matrix. This is repeated for each of the nine equations.

Figure 4: Example of the relaxation of an initially random field to the force-free field with boundary conditions from a coronal hole system. The field is shown for 0, 10, 200 and 1000 iterations. Notice in (d) the asymmetry in the opening angle of the boundary, and difference in form, being "curtain-like" to the east, and smooth to the west (right in the figure).

Useful qualitative results can be obtained at low resolution (say \( \geq 16^3 \)) and here large time steps can be used given the Courant-Friedrichs-Lewy (CFL) stability condition. The code converges best for large \( v \) and small \( \eta \). The ratio of iterations spent on the three phases "stress", "correct", and "relax" is problem dependent and has to be found from test runs, but the code does not need to fully relax at each intermediate stage.

2.3. Testing the code

The code is tested against the known solution to a twisted flux tube problem, generating nested flux tubes through the (cylindrical coordinate) vector potential

\[ A_{\phi}(r) = -\frac{e}{2} \ln(r^2 + \beta^2), \quad A_{\theta}(r) = \frac{e}{2} \cdot \ln(r^2 + \beta^2) \]  

(14)
where $\beta$ determines the degree of shear (of a uniformly twisted field), where a small value corresponds to more twist, that is a state further away from constant $a$. For the code, only values of (14) on $S$ are used, with random values in the interior, and the final output relaxes to the known solution.

![Figure 5. Test fields for the boundary conditions of a twisted flux tube, after 40 relaxation iterations from a random interior field. The twist factors are $\beta = 1, 0.1$](image)

3. RESULTS

In this section, numerical solutions to the coronal hole problem are presented for various $a$ profiles, with $B_n$ fixed. The distribution of $B_n$ on $S$ is obtained from a potential global field model, shown locally in Figure 6a. However, $B_t$ is adjusted, that is “stressed”, until either a uniform value of $a$ is obtained across $S$, Figure 6b,c, or a non-uniform $a$ profile, Figure 6d,e, with a different sense of $a$ either side of the coronal hole boundary, and zero in-between, Figure 6f. The effects of shear, imparted by photospheric motions, will modify the closed field regions adjacent to the coronal hole, possibly creating more open field, but in the coronal hole itself $a$ should stay close to zero since any twist there rapidly becomes sparsely distributed along the open magnetic field-lines. The opening angle and shape of the boundary is obvious upon an examination of the coronal hole cross-sections shown in the figures; notice that it is the sign of $a$ that determines whether the field lines are concave or convex with respect to a hole-centred radial vector.

![Figure 6: Relaxed fields, and cross-sections of the coronal hole boundary. (a) reference potential field solution, (b)-(e) stressed transverse B.Cs. providing (b) a positive uniform value of $a$ over $S$, (c) a negative uniform value of $a$ over $S$, and (d) and (e) $a$ profiles with a sign change across the CH boundary.](image)
4. OBSERVATIONS

The divergence of field-lines across the extended coronal hole of August 1996, and the asymmetry of its boundaries, are supported by observations:

The SOHO Coronal Diagnostic Spectrometer (CDS) instrument provides measurements at diﬀerent wavelengths which are sensitive to the temperature, density and plasma elemental abundances. To a ﬁrst approximation, a particular wavelength can be associated with a broad characteristic height because of a general increase in temperature with height. The situation is somewhat more complicated, but the coronal hole shows up as a minimum in emission at the wavelengths of interest, having a typical temperature of $8.10^5 K$ and electron density of $2.10^8 cm^{-3}$ thermal pressure is about $2.10^{14} cm^{-3} K$ in the coronal hole, some one half to one third of the values found in the surrounding quiet sun regions (Del Zanna, 1999). The coronal hole profile can then be estimated by a comparison of data from Hei, MgI, MgX, SiXII and FeXII which are sensitive to the plasma from the upper chromosphere through to the corona. There is a clear expansion of the hole with height, and over this range the coronal hole boundary is estimated to make a $37^\circ \pm 3^\circ$ angle with the radial direction (Bromage et al., in preparation), and this is supported by direct images from the Extreme Imaging telescope (EIT) instrument. However, careful examination shows that there is also a skew to the coronal hole boundary which has a steeper eastern side. The temperature asymmetry and divergence of the coronal hole is conﬁrmed by Yohkoh soft X-ray data (Bromage et al., in preparation). In addition, a skew in the coronal hole magnetic topology is expected from the inﬂuence of the strong active magnetic ﬁeld region.

An asymmetry is also found in measurements of the solar wind. EISCAT (Interplanetary scintillation) observations of the solar wind have been mapped back to the coronal hole (Linner et al., 1999). Of interest here is a differing wind speed across the coronal hole with a wind of some $350 km s^{-1}$ formed close to the (hotter) eastern boundary of the hole, but a faster latitude dependent “core” speed $500 - 750 km s^{-1}$. No wind was detected from the western boundary; this might be due to an insuﬃcient (temperature dependent) driving mechanism at the cooler western boundary, or further evidence for the magnetic topology asymmetry since a slow wind might be generated only by magnetic field-lines which dynamically interact with the heliospheric current sheet (Fisk et al., 1999).

5. CONCLUSIONS

There is a marked asymmetry between the boundaries of the extended coronal hole for all $a$ profiles considered. This is manifest in a steepened eastern coronal hole boundary, probably due to the presence of the active region and arcade channel alongside. There is also a change in form since the eastern ﬂank is “curtain-like” in its magnetic topology but appears smoother to the west. The kinks arise because of alternate concentrations of ﬂux along and across the eastern boundary as seen in SOHO MDI magnetograms.

Shear can be seen in $H_a$ images and is thought to be important to the evolution of the coronal hole and adjacent prominence structure. In the absence of measurements, a family of shear proﬁles have been tested, and the topology is found to be quite sensitive to diﬀerent transverse boundary conditions (for ﬁxed $B_z$). The opening angle has been determined across the boundary (as opposed to the angle of the ﬁeld-lines), and shown to be consistent with the spectroscopic estimates for most of the proﬁles considered. In fact, it is found that it is the sign of $a$ either side of the boundary that determines whether there are convex or concave ﬁeld-lines to a mid-coronal hole radial; the latter is suggestive of a conversion of closed into open ﬁeld lines, that is reconnection is stimulated by the application of shear.

In general, the numerical study has demonstrated that open magnetic ﬁeld boundary conditions and an arbitrary $a$ proﬁle can be incorporated self-consistently. The code can later be used with a measured alpha proﬁle if vector magnetograms become available for weak ﬁeld regions, and a more realistic global ﬁeld model could be used instead of the linear force-free ﬁeld model to ﬁx $B_z$ over the computational box.

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