INFLUENCE OF ACHROMOSPHERIC MAGNETIC FIELD ON SOLAR ACOUSTIC MODES

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ABSTRACT

This paper studies the effect of a magnetic atmosphere on the global solar acoustic oscillations in a simple cartesian model. First, the influence of the ratio of the coronal and the photospheric temperature $\tau$ and the strength of the magnetic field at the base of the corona $B_z$ on the oscillation modes is studied for a convection zone-corona model with a true discontinuity. The ratio $\tau$ seems to be an important parameter. Subsequently, the discontinuity is replaced by an intermediate chromospheric layer of thickness $L$ and the effect of the thickness on the frequencies of the acoustic waves is studied. In addition, nonuniformity in the magnetic field, plasma density and temperature in the transition layer gives rise to continuous Alfvén and slow spectra. Modes with characteristic frequencies lying within the range of the continuum may resonantly couple to Alfvén and/or slow waves.

Key words: MHD; Sun: oscillations.

1. INTRODUCTION

Solar acoustic oscillations are the window to the interior of the Sun. Observations of their frequencies are the starting point of helioseismology. They have been determined with high precision, up to the order of $\mu Hz$ (Cox, Gustik, and Kidman, 1989; Elsworth et al., 1990; Libbrecht and Woodard, 1990; Woodard and Libbrecht, 1991). Over the last decade, special efforts have been made to measure frequency shifts over the solar cycle (Ronan et al., 1994).

Meanwhile, many theoretical solar models were investigated to reproduce the observed frequencies. But the discrepancy between theory and observations seems to be an ever lasting problem: errors up to $20\mu Hz$ were found; this is still more than observational accuracy.

The acoustic five-min oscillations reside below the Sun's surface where the dominant restoring force is fluid pressure. To ignore magnetic and other effects from layers above, is a logical starting point for theoretical investigations of the modes (Christensen-Dalsgaard and Däppen, 1992). These closed models lead indeed to the parabolic ridges found by Deubner (1975).

The layers above the Sun's surface open the door to a further refinement of the basic model. In these layers, magnetic fields and temperature gradients play a non negligible role in the determination of $p$ mode frequencies. The discussion about this part of the investigations has started at the end of the eighties. Campbell and Roberts (1989), Evans and Roberts (1990), Wright and Thompson (1992) studied parallel propagation. Non-parallel propagation was the subject of study by Jain and Roberts (1994).

In the theoretical papers mentioned above, the authors neglected the possibility of coupling between $p$ modes and local Alfvén and/or cusp waves. This coupling will show up in the theoretical analysis when the presence of an intermediate layer where the magnetic field is nonuniform, is taken into account. In this way, an Alfvén and cusp continuum is introduced. Global acoustic oscillation modes with characteristic frequencies lying within the range of the continuous spectra, may resonantly couple to local Alfvén and/or cusp modes. This phenomenon was discussed in Tirry and Goossens (1996), Tirry et al. (1998) and Pintér and Goossens (1998). In Vanlommel and Čadež (1998), it was shown how solar acoustic oscillations are affected by variations of a temperature distribution only.

The present paper studies the combined effect of a temperature gradient and of a magnetic field in the atmosphere and in the transition layer where the Alfvén speed is nonuniform. We pay special attention to the case where the transition layer is replaced by a sharp boundary. The important parameters are the temperature ratio $T_0/T_p$ with $T_p$ the photospheric temperature and the value of the magnetic field $B_z$ at the base of the corona. The results are obtained for various $\theta$, the angle between the propagation vector and the horizontal magnetic field in the corona. In a second part, we replace the discontinuity by a smooth transition layer. We study the influence of the thickness $L$ of the layer on the frequencies and on eventual damping of the modes.
2. THE BASIC EQUATIONS

We describe the Sun’s interior, the chromosphere and the overlying corona by a model in cartesian coordinates. Five-min oscillations observed at the surface of the Sun are standing oscillations of sound waves (pressure waves, or p modes). They are global oscillations of the Sun which are commonly described in terms of spherical harmonics \( Y^m_l(\theta, \phi) \) of co-latitude \( \theta \) and longitude \( \phi \). The degree \( l \) measures the total horizontal wave number \( k_h \) on the surface by \( k_h = \sqrt{l(l+1)}/R_0 \), where \( R_0 = 696 \) Mm is the solar radius. The azimuthal order \( m \) measures the number of nodes along the equator.

We define a cartesian coordinate system with the z-axis pointing downwards to the centre of the sun. We assume a static equilibrium configuration with a horizontal magnetic field, \( \vec{B} = (B_0(z), 0, 0) \) and superimpose linear isentropic harmonic motions on the equilibrium state. They are assumed to be of the form

\[
\xi(z, y, z; t) = (\xi_x(z), \xi_y(z), \xi_z(z)) \exp \left[ i(\omega t - \vec{k} \cdot \vec{r}) \right]
\]

for angular frequency \( \omega \) and horizontal wavenumber \( \vec{k} = k_x \hat{\xi}_x + k_y \hat{\xi}_y \). We define \( \theta \) as the angle between \( \vec{k} \) and the equilibrium magnetic field, so that \( k_x = k \cos \theta \) and \( k_y = k \sin \theta \). The linearised ideal MHD equations can be combined to give two linear first order differential equations for \( \xi_x \), \( \xi_y \) and \( P_T \), the perturbed total pressure which need to be solved in the corona, the convection zone and the chromosphere, where resistivity can be ignored. The two differential equations have two regular singular points, the Alfvén resonance point and the cusp resonance point at the zeroes of the coefficient function \( D \). They are mobile singularities and define the continuous Alfvén and slow spectrum: \( \omega_1^2 = \omega_A^2(z) \) and \( \omega_2^2 = \omega_S^2(z) \). The orientation of \( \vec{k} \) with respect to the equilibrium magnetic field is crucial for the resonant coupling of the acoustic eigenoscillations to local Alfvén modes. Namely, for \( k_y = 0 \), there is no coupling.

3. THE EQUILIBRIUM MODEL

The model we consider consists of three regions: a semi unbounded corona with uniform temperature, the adiabatic chromosphere and finally the polytropic convective zone. The corona occupies the half space \( z \leq -L \), the chromosphere is located in the region \(-L \leq z \leq 0 \), while the convective interior lies at \( z \geq 0 \).

The Alfvén speed is uniform in the corona, i.e. \( v_A^2(z) = v_A^2 \equiv \text{constant} \). It decreases linearly in the transition layer to zero at the top of the convective layer. In the convection zone, the magnetic field and so the Alfvén speed is zero. In the analysis that follows, all quantities are scaled to their values at \( z = 0 \).

The ratio of the coronal temperature to the photospheric temperature is denoted as \( \tau \equiv T_c/T_P \). The square of the Alfvén speed can be expressed in terms of the plasma \( \beta_p \) at \( z = -L \) and \( \tau \).

![Figure 1. The profile for sound speed and Alfvén speed in the corona, the chromosphere and the convection zone. In the case when \( L = 0 \), we deal with a sharp boundary between the corona and the convection zone.](image)

The analytic expression for the temperature profile can now be written as:

\[
T_0(z) = T_p \begin{cases} 
\tau, & z \leq -L, \\
1 - (\tau - 1) \frac{z}{L}, & -L \leq z \leq 0, \\
1 + \frac{z}{L}, & z \geq 0,
\end{cases}
\]

with \( z_0 = (1 + m_p)H_0 \).

The square of the Alfvén speed is given by:

\[
v_A^2(z) = v_{A_c}^2 \begin{cases} 
1, & z \leq -L, \\
-\frac{z}{L}, & -L \leq z \leq 0, \\
0, & z \geq 0.
\end{cases}
\]

Note that \( v_{A_c}^2 = \frac{2P_T}{\rho_0 v_A^2} \). The squares of the sound speed and of the Alfvén speed are shown schematically on Fig 1.

4. SOLUTIONS

For each of the three regions, the differential equations for \( \xi_x \) and \( P_T \) have to be solved. In the corona, the solutions have to be evanescent. This condition gives rise to cutoff frequencies. In the solar interior, confluent hypergeometric functions define the solutions. In the chromosphere, the differential equations have to be integrated numerically. A global mode can couple to localized Alfvén and/or cusp waves when the real part of the eigenfrequency equals the local Alfvén and/or cusp frequency: \( \omega = \omega_A(z_A) \) and \( \omega = \omega_C(z_C) \). This happens in the transition layer where the Alfvén and cusp frequency change continuously from their coronal values at \( z = -L \) to zero at \( z = 0 \). The dissipative terms in the MHD equations become important in a narrow layer around the resonance position. Outside this narrow layer, the eigenmode is accurately described by the equations of ideal MHD. Tirry and Goossens (1996) and Tirry et al. (1996) derived the jump condition for the eigenvalue problem. The frequencies of the global eigenmodes are complex, \( \omega = \omega_r + i\omega_i \) when they couple to local Alfvén and/or cusp modes; the mode is exponentially decaying in time. The imaginary part is much smaller than the real part.
5. EIGENVALUE SOLUTION

We use the same procedure to calculate the eigenvalue solution as described in Vanlommel and Cadede (1998), except for the resonance. We start the integration at \( z = -L \) with the appropriate boundary values for \( \xi \) and \( P_T \). If a resonance is encountered during the integration, then the dissipative solutions (see Tiry and Goossens, 1996) are used. These equations give the jump conditions. They specify the variation of the vertical displacement and the total pressure over the dissipative layer. After passing the dissipative layer, the integration of the ideal MHD equations proceeds till the point \( z = 0 \) is reached. At \( z = 0 \), continuity of the vertical displacement and total pressure is demanded.

In the special case of a sharp boundary \( (L = 0) \), where temperature and density are discontinuous, we do not need any integration. Resonances cannot occur in that case because the continuous spectra are absent. We only need to apply the continuity conditions for \( \xi \) and \( P_T + \mu g \xi \) at \( z = 0 \). By putting the determinant of the system equal to zero, we obtain the dispersion relation (Jain and Roberts, 1994).

6. RESULTS AND DISCUSSION

Our numerical results, are always for \( T_p = 4170 \)K.

6.1. Sharp boundary: \( L = 0 \)

Both temperature and Alfvén speed are discontinuous at \( z = 0 \). The eigenfrequencies are calculated for different values of \( B_c, \tau, \theta \) and \( l \). Coupling in the case of an intermediate layer, is possible if the frequency of the mode is smaller than the cusp and/or Alfvén frequency at the base of the corona. The dispersion curves and the position of the Alfvén and cusp frequencies in the case of a true discontinuity give in this sense a good indication for possible resonances if we deal with a smooth transition region \( (L \neq 0) \). We therefore shall refer in this section to the case \( L \neq 0 \) where coupling can exist.

In Figures 2 and 3 we have plotted the eigenfrequencies for different modes as a function of the order \( l \) together with the magnetothermoacoustic cutoff frequencies and the coronal values of the Alfvén \( v_{Ac} \) and cusp frequency \( v_{Cc} \). The magnetic field \( B_c \) is taken to be 100G, \( \beta_c \) equals 3.9 in this case.

In Figure 2(a), \( \theta \) and \( \tau \) are taken to be 0° and 1, respectively. The propagation vector \( \vec{k} \) lies along the horizontal magnetic field; the Alfvén resonance is excluded if we let \( L \) be different from zero. There are three cutoff frequencies, namely \( \nu_{f1}, \nu_{f2} \) and \( \nu_{Cc} \). As can be seen, neither the \( f \) nor the \( p \) modes will be damped in the case of \( L \neq 0 \), because their characteristic frequencies are much higher than the cusp frequency. The mode just below the cusp frequency is recognised as a surface mode \( a \) (Hindman and Zweibel, 1994). Above the upper magnetothermoacoustic cutoff \( \nu_{f1} \), modes propagate in the corona and therefore leak energy into that region from below. When

Figure 2. Dispersion curves for different values of the the temperature ratio \( \tau \): a) 1 and b) 20 with \( L = 0 \). In both figures, the angle \( \theta \) equals 0° and \( B_c = 100G \). The eigenmodes are the surface mode \( a \), the \( f \) mode \( (f) \) and the spectrum of acoustic \( p \) modes \( (p_n) \).

Figure 3. Dispersion curves for \( \theta = 45^\circ \) with \( L = 0 \). The temperature ratio \( \tau \) equals 200 and \( B_c = 100G \).
modes reach the cutoff line $\nu_l$, the dispersion curves are simply terminated. This can be seen in both Figures 2(a) and 2(b). This behaviour is also present for $\theta = 45^\circ$ and $\tau = 200$ (Figure 3), but occurs for values of $l$ that are not plotted.

Figure 2(b), $\tau = 20$, shows an avoided crossing of the $a$ and the $f$ mode. The $p_n$ modes disappear when their frequencies reach the cutoff and reappear at higher values of $l$ (see Vanlommel and Čadež, 1998). The dispersion curves stop at $\nu_{II}$ and reappear at $\nu_{C_C}$ because modes with a frequency lying between $\nu_{II}$ and the cusp frequency $\nu_{C_C}$ are not trapped.

The figure shows that resonances will be possible if $L \neq 0$ for the $f$ mode for values of $l$ larger than $l_1(n = 0, \theta, \tau, B_c)$ with $n$ the nodal number. $n = 0$ indicates the $f$ mode.

For non parallel propagation ($k_x \neq 0$, $k_y \neq 0$), the character of the modes can change at five instead of three frequencies. An important phenomenon to notice in Figure 3, for $\tau = 200$ and $\theta = 45^\circ$, is the fact that dispersion curves do not simply stop at $\nu_{II}$ to continue at $\nu_C$, but they run along $\nu_{II}$ and disappear before bumping into the mode of the next higher radial order. The curve reappears at $\nu_{C_C}$. The magnetic field clearly influences the modes; the dispersion curves stop at $\nu_{II}$ and turn aside along $\nu_{III}$; this was not seen in the fieldfree case. In the case $L \neq 0$, damping at the Alfvén and/or cusp resonances is possible for many modes as long as their characteristic frequencies are below $\nu_{A_C}$.

Figure 4 shows how the eigenfrequencies change with $\tau$ for $l = 300$ and $\theta = 45^\circ$. $B_c$ is again taken to be 100G. The number of modes decreases rapidly with $\tau$ until all of them disappear in broad intervals of $\tau$. As $\tau$ is further increased, modes start gradually to reappear again. Only the two lowest modes exist at all values of small $\tau$. The same phenomenon occurs as in Figure 3: modes turn off and run along $\nu_{II}$, the curves disappear when they reach the curve above.

The frequencies change only slightly (except for frequencies close to the cutoffs $\nu_l$ and $\nu_{II}$), with the understanding that the frequency $\nu_{II}$ of the $p_n$ mode is practically equal to the frequency of the $p_{n-1}$ mode at higher $\tau$. As in the fieldfree case (see Vanlommel and Čadež, 1998), the frequencies of the modes decrease with $\tau$ at small values of $\tau$ until the cutoff is reached. There are two exceptions on this rule. The frequency of the first mode, i.e., the $f$ mode increases rapidly, while the frequency of the second mode, i.e., the $p_1$ mode remains constant. After reappearing at $\nu_l$ and $\nu_{C_C}$, the frequencies increase and tend to a constant value.

For $\tau = 1$ and $\theta = 0^\circ$, we can compare our results with the constant Alfvén speed model of Campbell and Roberts (1989). We calculated for this purpose $\Delta \nu = \nu(B_c) - \nu(B_c = 1G)$ as function of $B_c$ in Figure 5. It shows that the $f$ mode frequency increases while the $p_1$ and $p_2$ mode frequencies decrease in the interval $B_c \in [1, 200]$G (see Figure 5(a)). This decrease is caused by the presence of the upper cutoff. These findings are in agreement with Campbell and Roberts (1989); the $f$ mode and the $p$ modes are systematically split further apart from one another by the presence of a magnetic field. Campbell and Roberts let $B_c$ vary from 0 to 4,000G. In our case, the frequency of the $p_2$ mode decreases, the mode disappears when $B_c$ reaches 100 Gauss. The $p_0$ mode would reappear with increasing frequency if we let the magnetic field tend to much higher values. This is what Campbell and Roberts called the 'windoweffect'. In solar conditions, 4,000G is rather strong. If the parameter $\tau$ equals 200, the influence of the upper cutoff is negligible for these modes and the frequencies increase with increasing magnetic field strength as seen on Figure 5(b). The curves in Figure 5(b) are interrupted because of the windoweffect. So, the temperature jump $\tau$ seems to be an important parameter to determine the behaviour of the cutoff and in this way the behaviour of the modes.

6.2. Transition layer: $L \neq 0$

We introduce a transition layer between the convective zone and the corona, namely the chromosphere. The density, temperature and Alfvén speed...
now change continuously. The parameter \( \tau \) is the ratio of temperatures at the two sides of the layer. Because we introduce in this way two continuous spectra, namely the Alfvén and cusp continuum, the modes can couple to local waves. As a consequence their frequencies become complex, the modes are damped. The real part \( \omega_r \) is slightly shifted. The imaginary part of the frequency is small compared to the real part. However, by introducing a transition layer, the frequency of a mode that is not damped is also shifted.

In figure 6, \( \theta = 0^\circ \). This choice exclude the Alfvén resonance, but leaves the possibility of the slow resonances depending on the value of \( \tau \). Figure 6 shows how the frequencies of modes that are not damped, are shifted when we vary the width of the chromosphere. We plot \( \Delta \nu = \nu(L, \tau, \theta, l, B_c) - \nu(0, \tau, \theta, l, B_c) \) as function of the thickness \( L \) of the transition layer. The plots are for four values of the modal degree \( l \): 300, 400, 500 and 600. For the \( f \) mode, the shift is negative. From a certain value of the base frequency, the shift is positive: the \( p_l \) mode for \( l = 500 \) and \( l = 600 \). The absolute value of the shift is larger for higher \( l \) and seems to tend to a constant value if the width is increased. We plotted additionally the frequency shifts and damping ratio for \( \theta = 45^\circ \) and \( \tau = 200 \) in Figures 7 and 8. The \( f \) mode couples to Alfvén and cusp waves. There is a well defined minimum in the curves in Figure 8 where the ratio \( \nu'/\nu \) for the \( f \) mode is plotted. This ratio is a rate for the relative damping (see Tirry et al., 1998). The largest damping rate is produced at a value of \( L = L_m \), depending on \( l \) and \( n \). The damping rate is one order higher than that found by Tirry et al. (1998). The width \( L_m \) where this minimum occurs can be recognized easily in Figures 7(a) and 7(b) where \( \Delta \nu(L) \) is plotted.

The imaginary part of the frequency determines the damping of the mode. Observed linewidths measure the lifetime of a mode, which is in turn related to the damping. The linewidth \( \Gamma \) (Libbrecht, 1988) can be expressed as

\[
\Gamma = 2 \nu_i.
\]
We have plotted $\Gamma$ as a function of $L$ for the first $p$ mode in Figure 9. The linewidths for solar $p$ modes are tabulated in Libbrecht (1988, Table 1): $l \approx 20$. We took 500 for the modal degree, because in our model for $r = 200$ the modes with $l = 20$ are not damped. As a consequence, we have to be careful with our conclusions. However, Libbrecht (1988) showed that the linewidth can be represented by a function of $\nu$ only, independent of $l$ and radial order $n$, at least for $l < 60$. The linewidths due to resonant absorption are of the order of a few $\mu$Hz and comparable to those observed by Libbrecht. This confirms the idea that resonant absorption should be taken into account in producing a definite solar model reproducing the observed frequencies.

7. Conclusions

First, for the true discontinuity, we recovered the results obtained by Campbell and Roberts (1989), Evans and Roberts (1990), but also showed that if we take other values for the parameter $\tau$, the shifts can be reversed as seen in Figure 5(b). It is our impression that the quantity $\tau$ plays a rather important role. The temperature jump determines the cutoff frequency and this on its turn, influences the frequency dependence on the different parameters. A good choice of $\tau$ and $L$ can lead to shifts comparable with those in observations. The figures with $L = 0$ give a good idea of what could happen in the case $L \neq 0$. It is possible to predict in this way coupling between modes and loss of energy of the fundamental and acoustic modes.

For the case with an intermediate layer, we showed that the frequencies are influenced by the value of $L$. If resonances occur, the influence of the width $L$ is even more pronounced. The linewidths found are also in agreement with observations done by Libbrecht (1988).

To make an appropriate model that would recover the observed frequencies of acoustic modes with required accuracy, it is therefore necessary to make an adequate estimate of the width $L$ over which the temperature changes and of the temperature ratio $\tau$. We can conclude that the parameter $\tau$ plays a crucial role for the modes being damped or not if we deal with a transition layer. Resonances influence the frequency shifts and determine the behaviour of the mode in the chromosphere.

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