WAVELET ANALYSIS OF SPATIAL COHERENT STRUCTURES IN THE PHOTOSPHERE


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ABSTRACT

In this work we present some applications of wavelet analysis to the dynamics of the photosphere as obtained from a good serie of high spatial resolution white light images taken at the Vacuum Tower Telescope (VTT) at the NSO/SPO in October 1996. The use of wavelets, in particular the space-scale localization property, permits us the recognition of coherent structures at different scales; by using a related method known as MultiResolution Analysis (MRA) we compute the wavelet spectrum for each image in our data set and with this informations we define a characteristic scale; for space and time distributions of wavelet spectra we compute the flatness which is a good marker for the presence of space and time intermittency. We verify the existence of a characteristic scale and we study its temporal variation; we find a spatial flatness near gaussian value for all considered scales, while the flatness computed on the derivative of wavelet coefficients temporal distribution shows a strong dependence of its maximum with the scale.

Key words: Atmospheric Dynamics.

1. INTRODUCTION

In recent years a great effort of solar physicists has been focused on the statistical approach to solar granulation. The appearance and the character of underlying phenomena governing this fundamental aspect in the life of our star are the natural place for a statistical description; moreover the experimental achievements in observations both in large and narrow bandwidths and the theoretical evolutions in the description of complex dynamical systems have pushed the solar community to find new methods of analysis and description for the solar granulation; as a matter of fact all recent improvements to our picture of the collective behaviour of granules has mainly been done thanks to the improvements in the analyzing techniques. Starting from the 90’s much effort has been done in the study of granules activity as seen as a fluid, with the aim of using the solar photosphere as a good laboratory for the test of turbulence models; the main method of investigation was the Fourier analysis (Espagne et al. 1993), but this approach has been strongly criticized (Nordlund et al. 1997). Lately the research has been directed to the study of morphological properties of individual granules; many techniques have been developed in order to recognize granules and then to study their collective properties; recently Roudier et al. 1999 pointed out that some results about velocities fields (in particular horizontal fields) are strongly dependent on the analysing technique.

In aid to this situation in the same period several related mathematical tools have been developed in the framework of wavelet analysis. These techniques are all based on a common principle: the use of a set of functions able to give a good description of a phenomenon in the coupled domain of space (or time) and scale simultaneously. By using these features many authors have characterized the scale properties of solar magnetic fields (Komm 1995), of chromospheric intensity fields (Berrilli et al. 1999), of velocity, intensity, and magnetic fields on the photosphere (Lawrence et al. 1999). In particular Lawrence et al. 1999 found many important results about the turbulent regime of the photosphere: through the analysis of flatness spectra they found that intensity and velocity fields are Gaussian distributed, then stochastic in nature, at least at the observable scales, while the magnetic field was intermittent. In addition they confirmed some previous results on the characteristic scales of granulation and detected a weaker presence of mesogranulation.

In this work we present a method based on wavelet analysis in order to study the global behaviour of granules from a statistical point of view; by means of Multiresolution Analysis we investigate at smaller scales than previous works the distribution of photospheric intensity fields and their evolution with time.


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In §2 we describe the data used which consist of a very good sequence of data taken at the NSO/KPNO; in §3 we will present the method of analysis and its peculiarities; in §4 we introduce the global spectrum measure and show that it is possible to define some useful scalars in order to describe the dynamical properties of the photosphere and we show that exist a coherent global behaviour of granules with time; in §5 we use the flatness of wavelet coefficients in the space and time domain in order to characterize different regions with different dynamical regimes.

2. OBSERVATIONS

The data set used in this work has been described elsewhere (Cauzzi et al. 1998); here we would like to remind some important features: data has been acquired at the VTT/NSO (Sacramento Peak), on October 16, 1996, from 14:30 UT to about 16:00 UT; seeing conditions were excellent; images are taken at the disk centre through an interference filter at 550 nm, with 10 nm FWHM with a 8-bit camera, 1920x1360 pixels, and with an exposure time of 8 ms; a sub-portion of 1024x1024 pixels made up the final images; the image scale was set at 0.123 arcsec/pixel, for a total field of view of about 2x2 arcminutes. All the 512 images have been corrected for the dark current and flat-field; a destretching algorithm has been applied to remove the atmospheric distortions; the 5-minute oscillations have been removed by using the standard technique as depicted by Title et al. 1989.

3. METHOD OF ANALYSIS

3.1. Multiresolution analysis

As well known the wavelet transforms are formally an extension of Fourier transforms; the wavelet coefficient is defined as

$$Wf(a, b) = \int f(x)\psi\left(\frac{x - b}{a}\right) dx,$$

where $f(x)$ is a generic function to be analyzed and $\psi_{ab}(x)$ is a primitive function shifted by $b$ and rescaled by $a$. $Wf(a, b)$ gives us informations about the spectral content of $f(x)$ in a space-scale domain, depending on the choice of $(a, b)$ parameters; when we are dealing with sets of experimental sampled data we can use a discrete set of $(a, b)$ values, producing a Discrete Wavelet Transform (DWT); within this context it can be possible to prove (Mallat 1989) that an optimal choice is a set of scales $a$ and dilations $b$ as powers of two and the introduction of a second function, $\phi(x)$, the scaling function. This computational scheme is known as Multi Resolution Analysis (MRA) and is one of the most powerful spectral techniques adopted is digital signal processing; it has the advantage of using a restricted set of coefficients, a fast iterative algorithm based on two strictly related functions: the wavelet function $\psi(x)$ acting as a high pass filter, and the scaling function $\phi(x)$ which acts as a low pass filter.

3.2. 2D Multiresolution Analysis

This algorithm has been extended to bidimensional functions: if $\psi(x)$ and $\phi(x)$ are the functions adopted for the unidimensional MRA, we can define a bidimensional scaling function as
\[ \Psi(x, y) = \phi(x) \phi(y) \]

and a set of three bidimensional wavelet functions as

\[ \Psi^h(x, y) = \phi(x) \psi(y), \]
\[ \Psi^v(x, y) = \psi(x) \phi(y), \]
\[ \Psi^d(x, y) = \psi(x) \psi(y), \]

directly related to the three possible elementary directions of application (h, v, and d stand for horizontal, vertical, and diagonal directions).

To reach out our purpose we applied the bidimensional MRA by making use of Haar functions as analysing scaling and wavelet functions; the result can be expressed as an equivalent image where pixels are representative of wavelet coefficients at different scales and positions for a particular combination of monodimensional wavelets (Daubechies 1992); the result can be represented as an equivalent image where the wavelet coefficients are hierarchically organized in descending scales for each direction. Unlike an integral transform here we have all coefficients at all the scales available simultaneously; the lack is of course in information details, but this is not an important aspect in a statistical approach. In this work we considered linear scales from 2 pixels \((\sim 0.17\text{Mm})\) to \(2^8\) pixels \((\sim 22\text{Mm})\); a typical result is shown in Figure 1: on the left there is the original image, while on the right is reported the image in the MRA space-scale wavelet domain.

4. GLOBAL SPECTRUM

Once wavelet spectra for each image in the sequence has been computed we can define the energy associated to each scale and direction as

\[ E^{h,v,d}(j) = \sum_{i=1}^{N_{\text{coeff}}} |j_{ij}^{h,v,d}|^2 \]

where \(j_{ij}^{h,v,d}\) are the wavelet coefficients at a given position \(i\) and scale \(j\) for the three directions. Figure 2 shows the global energy for the image of Figure 1 and the horizontal direction with superimposed the reconstructed spline; it is evident the presence of a maximum near 1140 Km which clearly define a characteristic scale for the image; this value represents also the mean value over the whole serie for the horizontal direction; values of 1110 Km and 1610 Km have been obtained for the vertical and diagonal directions respectively. The difference for the diagonal direction lies in the very simply functional form of Haar’s functions which act as square box functions and consequently detect the diagonal in a geometrical sense.

An interesting aspect offered by these spectra is their time evolution; in Figure 3 we show the plot of the eight spectral points as function of time; is clearly present a regular variations of energies at all scales, a phase relation among group of scales and a remnant of 5 minute oscillations at smaller scales surprisingly present despite the application of a sub-sonic filter.
5. FLATNESS

The previous section has been devoted to the morphological features detectable with wavelet transforms; here we show how is possible to investigate with wavelets the turbulent nature of photospheric intensity fields. As well known (Farge 1992) wavelet coefficient distributions can reveal the turbulent behaviour of a fluid through a typical marker, the intermittency (for a more detailed treatment see Bianchini et al. 1999 elsewhere in these Proceedings); its presence tends to enhance the wings in gaussian distributions and can be revealed by using the fourth moment of the distribution of flatness

\[ \Phi(j) = \frac{\langle f_{ij}^4 \rangle}{\langle f_{ij}^2 \rangle^2} \]

As indicated by Lawrence et al. 1999 we expect a stochastic behaviour of velocity and temperature (related to intensity) fields and hence a value of 3 for the flatness at all scales; with our data we test this assumption up to scales of \( \sim 170 \text{Km} \) and we confirm the results of Lawrence et al. 1999.

In addition with our data set, which has a good spatial and temporal extension, we investigate the fluid evolution; to do that we compute the flatness of the differences of wavelet coefficients taking into account their spatial positions; the results are bidimensional maps, one for each considered scale and direction. Here we show the results for the horizontal direction; in particular in Figure 4 (left) we show the flatness map relative to a scale of 713 Km. Although the mean value is representative of a gaussian distribution, we find a relationship between the maximum of flatness and the scales used; this result is shown in Figure 4 (right). The abrupt change of flatness maximum at the smallest scale can be interpreted as a clear evidence of the limit of the experimental resolution which is near to 200 Km in the best observing conditions; under this value the intensity fields probably is masked by a random noise field and the result is a lowering of maximum flatness.

6. CONCLUSIONS

In this work we use the Multiresolution Analysis method in order to investigate morphological and statistical properties of intensity solar fields by using an excellent data set of white light images taken at the VTT/NSO Sacramento Peak Observatory. Our analysis confirms some previous results about characteristic scales of coherent structures on the solar photosphere: we found characteristic scales of 1.1 Mm comparable to those found by Lawrence et al. 1999 and Berrilli et al. 1997; a temporal investigation of wavelet spectra reveals the presence of residual 5 minute oscillations near granulation scales. A statistical analysis of spatial flatness confirms the preceding results of Lawrence et al. 1999; the flatness computed on the derivatives of time distribution of each wavelet coefficient reveals a dependence of maximum values with the scale. The dependence of flatness with scales seems to be in accordance with the general opinion that intermittency resides in smaller scales. In a future work we intend to investigate about the behaviour of these singular cases.

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