MAGNETOACOUSTIC-GRAVITY SURFACE WAVES IN STEADY PLASMAS

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ABSTRACT

We consider the linear theory for the parallel propagation of magnetoacoustic-gravity (MAG) surface waves for an interface of a plasma embedded in horizontal magnetic field above a field-free steady plasma medium. This steady state can be interpreted as an approximation for, e.g., the convection motion below the solar surface, or large scale steady motions like the meridional flows right below at the solar surface. The plasma either side of the interface is taken to be isothermal.

The dispersion relation is derived and is studied in details for the case of a constant Alfvén speed. In the absence of the equilibrium steady flow the interface may support MAG surface modes (Miles and Roberts, Solar Physics, 141, 205, 1992). The effect of steady state on these modes is examined and dispersion diagrams are shown. New modes, called flow- or v-modes are found as a consequence of the steady state. Relevance for solar applications is shown, in particular.

Key words: MHD (surface) waves; steady flow; Sun: convective zone, chromosphere.

1. INTRODUCTION

The solar atmosphere is strongly magnetically structured (see, e.g., Acton et al. 1992, Brekke et al. 1997, Fludra et al. 1997, Kjeldseth-Moe and Brekke 1998, Schrijver et al. 1997). The presence of magnetic structuring drastically changes the character of MHD wave propagation in plasmas. For instance, MAG surface waves can exist in magnetically structured plasmas. Such waves can propagate wherever there is a sharp change of plasma parameters across a surface called ‘magnetic interface’. Surface MHD waves on magnetic interfaces have been intensively studied (see, e.g., Roberts 1981; Cambell & Roberts 1989, Miles & Roberts 1992; Miles et al. 1992). In the solar atmosphere the surface MHD waves can propagate, e.g., along the boundaries of the photosphere and convection zone, along the boundaries of sunspots, coronal holes, coronal loops, and in the canopy regions in the chromosphere. MAG surface waves are of natural interest as they also appear in coronal heating studies, oscillations in sunspot penumbras, or in helioseismology (see, e.g., Christensen-Dalsgaard et al. 1985, Deubner et al. 1984, Jain & Roberts 1993, 1994ab, and Erdélyi & Taroyan 1999 in this Volume). The solar atmosphere is highly inhomogeneous even on largest scales. This inhomogeneity can be approximated by regions separated by tangential surface discontinuities. When a magnetic interface is a true discontinuity the surface waves are eigenmodes of the ideal linear MHD equations (Nye & Thomas 1976, Rae & Roberts 1981). Dissipation in the solar atmosphere (e.g. viscosity, thermal conductivity, and electrical resistivity) causes surface wave damping. The surface wave damping in the solar corona was considered by, e.g., Gordon & Hollweg 1983 and Ruderman 1991. It was found that for typical coronal conditions dissipation in the solar coronal plasma is not enough to cause substantial damping of surface waves in the inner part of the solar corona unless wave periods are very short (of the order of ten seconds or less).

The Sun is also highly dynamical showing, e.g., large scale equilibrium motions both in the interior (e.g., convective motion) or in the atmosphere (e.g., meridional flows). Large scale motions in the surface and subsurface region influence the MAG surface modes resulting in observable frequency shifts (see, e.g., Braun & Fan 1998, Erdélyi & Taroyan 1999).

Surface discontinuities may support MAG surface waves which are influenced by the inhomogeneity (e.g., either in density or in magnetic field) or by steady state. In the present paper we evaluate theoretically the influence of a steady equilibrium state on MAG surface waves for a simple model of the solar plasma, consisting of a polytrope and steady solar interior, above which is an isothermal magnetic at-
atmosphere.

The paper is organized as follows. In the next section we describe the main assumptions, the equilibrium state and basic equations. In Sect. 3 we obtain the dispersion equation for surface waves. In Sect. 4 this dispersion equation is solved numerically. We also draw here our conclusions.

2. GOVERNING EQUATIONS

2.1. EQUILIBRIUM STATE

The equilibrium model is a single magnetic interface in a stratified atmosphere with a parallel constant steady flow, \( u_s \), below the discontinuity. The temperatures \( T_0 \) and \( T_e \) either side of the interface are taken to be isothermal. The magnetic interface is located at \( z = 0 \) (see Fig. 1).

\[
\frac{d}{dz} \left( p(z) + \frac{B^2(z)}{2\mu} \right) = -g \rho(z). \tag{1}
\]

In both regions the assumption of a constant Alfvén speed results in

\[
\rho(z) = \begin{cases} 
\rho_0 \exp(-z/H_B), & z > 0 \\
\rho_e \exp(-z/H_e), & z < 0,
\end{cases}
\tag{2}
\]

where \( H_B = \frac{c_{A0}^2}{g} \) is the density scale-height in Region I, and \( H_e = \frac{c_{As}^2}{g} \) is the density scale-height in Region II. Here \( \Gamma \) denotes the magnetically-modified adiabatic exponent. The two regions are connected by the pressure balance at \( z = 0 \), e.g.,

\[
\rho_0(0^+) + \frac{B_0^2(0^+)}{2\mu} = \rho_e(0^-), \tag{3}
\]

where Eq. (3) also serves as a boundary condition.

2.2. THE GOVERNING DIFFERENTIAL EQUATIONS

In compressible ideal MHD perturbations are governed by

\[
\frac{D\rho}{Dt} = -\rho \nabla \cdot \mathbf{u}, \tag{4}
\]

\[
\rho \frac{D\mathbf{u}}{Dt} = -\nabla p + \mathbf{j} \times \mathbf{B} + \rho \mathbf{g}, \tag{5}
\]

\[
\nabla \cdot \mathbf{B} = 0, \tag{6}
\]

\[
\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B}), \tag{7}
\]

\[
\frac{\partial e}{\partial t} = -\nabla \cdot (e \mathbf{u}), -p \nabla \cdot \mathbf{u}, \tag{8}
\]

where \( D/Dt \equiv \partial/\partial t + \mathbf{u} \cdot \nabla \), and \( \rho \) is the mass density, \( p \) the kinetic gas pressure, \( \mathbf{g} \) the gravitational acceleration, \( \mathbf{u} \) the velocity, \( \mathbf{B} \) the magnetic field \( \mathbf{j} \) the electric current density and \( e \) \( \rho \) the internal energy. In addition \( \mathbf{j} \) and \( p \) are connected by

\[
\mathbf{j} = \frac{1}{\mu} \nabla \times \mathbf{B} \text{ and } p = \frac{k_B}{m} \rho T. \tag{9}
\]

Two-dimensional, linear, isentropic perturbations are considered in the form

\[
\mathbf{u}(x, z, t) = (u_x(z), 0, u_z(z)) \exp i(\omega t - k_z z). \tag{10}
\]
Introducing $\Delta = \nabla \cdot \mathbf{v}$ and the Doppler shifted wave frequency $\Omega = \omega - k \cdot \mathbf{u}$, and substituting the perturbations of Eq. (10) in the governing equations (4)-(8) will result in and ODE of the form

$$
\frac{d}{dz} \left[ \rho \left( v_A^2 + c_s^2 \right) \Omega^2 \right] = \frac{d}{dz} \left( \frac{k^2 c_s^2 - \Omega^2}{k^2 c_s^2} \right) u_z \quad - \left[ \rho \left( \Omega^2 - k^2 c_s^2 \right) + \frac{g^2 k^2 \rho}{k^2 c_s^2} \right] + g k^2 \frac{d}{dz} \left( \frac{\rho c_s^2}{k^2 c_s^2 - \Omega^2} \right) \quad u_z = 0
$$

Here we use the usual notation, e.g.,

$$
c_s^2 = \frac{\gamma \rho(z)}{\rho(z)} \quad v_A^2 = \frac{B^2(z)}{\mu \rho(z)} \quad \gamma = \frac{5}{3}.
$$

3. DISPERSION RELATION

Applying the boundary conditions at $z = 0$, e.g., the total pressure balance is continuous

$$
[p_T - \rho g \xi]_{z=0} = 0,
$$

and the Lagrangian displacement is continuous

$$
[x]_{z=0} = 0
$$

will result in the dispersion relation for MAG surface waves in the form

$$
\frac{\omega^2}{k^2} = \frac{\rho_0 v_A^2}{\rho_0 + \frac{\Omega^2}{\omega^2} \frac{m_0^2 (M_e - \Omega^2)}{m_{e0}^2 (M_e - \Omega^2)}} - \frac{g \rho_0 c_s^2}{k^2 c_s^2 - \Omega^2} + g \frac{\rho_0 c_s^2}{k^2 c_s^2 - \Omega^2},
$$

where

$$
m_0^2 = \left( \frac{k^2 c_s^2 - \omega^2}{c_s^2 + v_A^2} \right) \left( k^2 c_s^2 - \omega^2 \right)
$$

and

$$
m_e^2 = \frac{k^2 c_s^2 - \Omega^2}{c_s^e}.
$$

The dispersion relation Eq. (16) describes the parallel propagation of surface waves at a single magnetic interface in a gravitationally stratified atmosphere under the assumption of a constant Alfvén speed in the magnetic region and a constant homogeneous flow in the nonmagnetic region.

4. NUMERICAL RESULTS

For a given set of parameters the dispersion relation Eq. (14) does not have surface wave solutions for any arbitrary wave number resulting in cutoff curves $R_n, i = 1, \ldots, 6$. Permitted regions for surface wave propagation are shaded in grey in Figs. 2-4. Note, these regions are NOT necessarily stable.

Figs. 2-4 show the solutions of the dispersion relation, Eq. (14). The dimensionless phase-speed, $\omega/k c_s$, is plotted as a function of the dimensionless horizontal wave number $k x H_e$ (solid line). Figs. 2-3 show cases when the upper region, Region I is cooler while Fig. 4 corresponds to cases when Region I is hotter. $R_i, i = 1, \ldots, 6$ are the cutoff curves determining the propagation windows. The dash-dot line divides the propagation windows into stable (shaded light grey) and unstable (shaded dark-grey) regions. An increasing flow introduces, generally speaking, stable propagation windows.

In Fig. 2 solutions are shown to the dispersion relation for a nonmagnetic case ($v_A/c_s = 0$) and $c_{e0}/c_s = 0.9$ for different flow values ($u_s/c_s = 0.01, 0.2$ and $0.3$). In the last case (Fig. 2d) a stable propagation window appears as a result of the applied equilibrium flow. This window will always be present for a steady equilibrium flow.

Fig. 3 is similar to Fig. 2, however there is now a magnetic field present in Region I ($v_A/c_s = 0.5$). The equilibrium flow ($u_s/c_s$) introduces a new mode, the so-called $v$-modes which are purely as a result of the bulk motion. Notice the bifurcation of the two $v$-modes at around $u_s/c_s = 655$.

Finally, Fig. 4 shows the numerical solutions to the dispersion relation in the case when the upper region, Region I, is hotter as Region II (i.e., $c_{e0}/c_s = 1.4$). Region I is embedded in a magnetic field characterised by $v_A/c_s = 0.75$. We plot solutions when the equilibrium flow is given by $u_s/c_s = 0.3, 0.6$, and 0.8, respectively. Due to the presence of a magnetic field in this case the acoustic cut-off frequency splits up the stable propagating window. Once again, the flow introduces a stable propagating window where the $v$-mode can propagate.

Further improvements of the results presented here are in progress. These include more realistic equilibrium models especially as regards the equilibrium flow. Another important step is to consider effects due to the inhomogeneous nature of the solar atmosphere.

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Figure 2. Solutions to the dispersion relation for a nonmagnetic case ($v_\|c_\perp = 0$) and $c_{\perp 0}/c_\perp = 0.9$ for $a$, $u_\perp/c_\perp = 0$; $b$, $u_\perp/c_\perp = 0.1$; $c$, $u_\perp/c_\perp = 0.2$; and $d$, $u_\perp/c_\perp = 0.3$.

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Figure 3. Solutions to the DR when there is a magnetic field in Region I, $v_A/c_{se} = 0.5$ for $a$, $u_e/c_{se} = 0$; $b$, $u_e/c_{se} = 0.2$; $c$, $u_e/c_{se} = 0.6$; and $d$, $u_e/c_{se} = 0.7$. Notice, the steady state introduces the so-called $v$-modes purely as a result of the bulk motion.
Figure 4. Solutions to the DR when the upper region, Region I, is hotter than Region II where the flow takes place. There is a magnetic field in Region I, $v_A/c_{te} = 0.75$, which splits up the stable propagating window due to the presence of the acoustic cut-off frequency. Once again, the flow introduces a stable propagating window where the $v$-mode can propagate.