THE INFLUENCE OF A STEADY STATE ON p- AND f-MODES

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ABSTRACT

The influence of a steady equilibrium state on p- and f-modes is evaluated theoretically for a simple model of the solar plasma, consisting of a polytrope and steady solar interior, above which is an isothermal magnetic atmosphere. The atmosphere is permeated by a horizontal magnetic field, and the introduced steady flow is parallel with the atmospheric magnetic field in the interior. This model may serve as a first approximation of the observed highly dynamical solar interior (e.g., meridional flows, convective motion, etc.) especially for high degree l. Frequency changes and Doppler shifts in phase factors due to the presence of a steady interior are calculated analytically in the long-wavelength approximation using hypergeometric functions. The obtained dispersion relation is solved numerically for other cases.

Key words: helioseismology; p- and f-modes; steady flow.

1. INTRODUCTION

The solar p- and f-modes are acoustic waves trapped in the solar interior within a cavity, formed at its lower level by the increasing sound speed in the convective zone and below, and at the upper level by the reflective properties of the photosphere and chromosphere (see Leibacher et al. 1985). The power spectrum of oscillations shows a concentrated peak at 3.3 mHz (5 minute). The observed power spectrum lies on approximately parabolic ridges designating the overtones (see Deubner and Gough 1984; Christensen-Dalsgaard et al. 1985). p/f-modes are clearly a reflection of the structure of the solar interior providing important information about the Sun’s internal subsurface regions. Large scale motions in the subsurface region (meridional flow, convective motion) influence the p/f-modes resulting in observable frequency shifts (see, e.g., Braun & Fan 1998). The influence of a chromospheric magnetic field on p/f-modes is studied in, e.g., Cambell & Roberts (1989), Jain & Roberts (1993, 1994a, 1994b).

In the present paper we evaluate theoretically the influence of a steady equilibrium state on p- and f-modes for a simple model of the solar plasma, consisting of a polytrope and steady solar interior, above which is an isothermal magnetic atmosphere. The atmosphere is permeated by a horizontal magnetic field, and the introduced steady flow is parallel with the atmospheric magnetic field in the interior. This model may serve as a first approximation of the observed highly dynamical solar interior (e.g., meridional flows, convective motion, etc.) especially for high degree l. Frequency changes and Doppler shifts in phase factors due to the presence of a steady interior are calculated analytically in the long-wavelength approximation using hypergeometric functions (see Campbell & Roberts). The obtained dispersion relation is solved numerically for other cases. Relevance of the steady state in the solar interior for high-resolution helioseismic data is pointed out, for p- and f-modes in particular.

2. BASIC EQUATIONS AND MAIN ASSUMPTIONS

2.1. The Convection Zone

In the convection zone (z > 0) we suppose the magnetic field to be absent and to have a constant steady flow along the z-axis uzu = (V, 0, 0). The governing equations of motion can be written as

\[
(\omega_D^2 - \gamma^2 k^2) u_z = -\omega_D^2 c_s^2 \frac{d\Delta}{dz} - g(\gamma \omega_D^2 - k^2 c_s^2) \Delta, \tag{1}
\]

\[
\frac{du_z}{dz} = -\frac{g k^2}{\omega_D^2} u_z - \left( \frac{k^2 c_s^2}{\omega_D^2} - 1 \right) \Delta. \tag{2}
\]

where \( \omega_D = \omega - kV \) is the Doppler shifted frequency and \( \Delta = -iku_z + \frac{du_z}{dz} \). Elimination of \( u_z \) yields a second-order ordinary differential equation

\[
\frac{d^2 \Delta}{dz^2} + \left( \frac{c_s^2}{c_i^2} + \gamma g \right) \frac{d\Delta}{dz} + \left[ \frac{\omega_D^2 - k^2 c_s^2}{c_i^2} - \frac{g k^2}{\omega_D^2} \left( \frac{c_s^2}{c_i^2} - (\gamma - 1) g \right) \right] \Delta = 0, \tag{3}
\]
where the prime denotes the derivative with respect to $z$. In the special case when the sound speed is given by
\[ c_s'(z) = c_s^0 + c_s^2 z, \]
equation (3) has the solution
\[ \Delta(z) = e^{-k(z+z_0)} \left[ C M(-a, m+2, 2kz + 2kz_0) + D U(-a, m+2, 2kz + 2kz_0) \right], \]
(4)
where $z_0 = c_s^0/c_s^2$, $M$ and $U$ are confluent hypergeometric functions (Abramowitz & Stegun 1965) and $C$ and $D$ are arbitrary constants. The parameter $\alpha$ is given by
\[ 2\alpha = \frac{m}{\gamma} \left( \frac{\omega^2}{gk} \right) + \frac{gk}{\omega_D} \left[ (\gamma - 1) \frac{g}{c_s^2} - 1 \right] = (m+2), \]
and
\[ m = \frac{\gamma g}{c_s^2} - 1 \]
is the polytropic index. In the case of an adiabatically stratified atmosphere, to be considered here,
\[ c_s^2 = (\gamma - 1)g, \]
and so
\[ 2\alpha = m \left( \frac{\omega^2}{gk} \right) - (m+2), \quad \gamma = 1 + \frac{1}{m}, \]
\[ z_0 = (1+m)H_0, \]
(5)
where $H_0 = c_s^2/(\gamma g)$ is the pressure scale height. We require the kinetic energy density $1/2\rho_0 u^2$ to be finite as $z \to \infty$. This implies that $C = 0$ in (4). Thus the solution (4) takes the form
\[ \Delta(z) = D e^{-k(z+z_0)} U(-a, m+2, 2kz + 2kz_0), \quad z > 0. \]
(6)

2.2. The Chromosphere

In the chromosphere ($z < 0$) the steady flow is absent and a horizontal magnetic field $B_0 = (B_0(z), 0, 0)$ is present. The magnetohydrostatic equation is given by
\[ \frac{d}{dz} \left( \rho_0(z) + \frac{B_0^2(z)}{2\mu} \right) = \rho_0(z)g. \]
(7)
The equations governing the motion in the chromosphere are
\[ \frac{du_z}{dz} = -\frac{gk^2}{\omega^2} u_z - \left( \frac{k^2 c_s^2}{\omega^2} - 1 \right) \Delta, \]
(8)
\[ \frac{d}{dz} \left[ \frac{\rho_0(c_s^2 + v_A^2)(\omega^2 - k^2 c_s^2)}{(\omega^2 - k^2 c_A^2)} \frac{du_z}{dz} \right] = \frac{\rho_0 g^2 k^2}{\omega^2 - k^2 c_s^2} \frac{d^2 u_z}{dz^2} - \rho_0(\omega^2 - k^2 v_A^2) - \frac{gk^2}{\omega^2} \left( \frac{\rho_0 v_A^2}{\omega^2 - k^2 c_A^2} \right) u_z, \]
(9)
where $v_A(z) = B_0(z)/(\mu_0 \rho_0)^{1/2}$ is the local Alfvén speed, and $c_T = c_s v_A/(c_s^2 + v_A^2)^{1/2}$ is the MHD cusp speed.

We assume the chromosphere to be isothermal with a temperature equal to that at the top of the field-free medium, i.e., in $z < 0$ we take $c_s(z) = c_s^0$. Also we suppose that the Alfvén speed is constant. With these assumptions the plasma $\beta = c_s^2/v_A^2$ parameter is constant and equation (9) reduces to
\[ \frac{d^2 u_z}{dz^2} + \frac{1}{H_B} \frac{du_z}{dz} + Au_z = 0, \]
(10)
where
\[ A = \frac{(\Gamma - 1)k^2 g^2 + (\omega^2 - k^2 v_A^2)(\omega^2 - k^2 c_s^2)}{(c_s^0 + v_A^2)(\omega^2 - k^2 c_s^2)}, \]
and
\[ \Gamma = \frac{2\beta g}{2\beta + \gamma}, \quad H_B^{-1} = \frac{\rho_0}{\rho_0} = \frac{g}{c_s^2} \]
are the magnetically modified adiabatic exponent and pressure scale height, respectively (in the absence of a magnetic field, $\Gamma = \gamma$ and $H_B = H_0$). Equation (10) possesses solutions of the form
\[ \sim \exp \frac{z}{2H_B} \left[ -1 \pm (1 - 4AH_B^2)^{1/2} \right]. \]

We suppose that $4AH_B^2 < 1$ and choose the plus sign in (10). This corresponds to evanescent disturbances in the chromosphere. So we have
\[ u_z = e^{\lambda z}, \quad \lambda = \frac{1}{2H_B} \left[ -1 + (1 - 4AH_B^2)^{1/2} \right], \quad z < 0. \]
(11)

3. BOUNDARY CONDITIONS AND THE DISPERSION RELATION

From continuity of the normal component of the Lagrangian displacement ($-iu_{\omega}/\omega$ in $z < 0$ and $-iu_{\omega}/\omega_D$ in $z > 0$) across $z = 0$, using the equation (1), we have
\[ \frac{u_z}{\omega} \bigg|_{z=0} = -\frac{1}{\omega_D(\omega^2_D - g^2 k^2)} \times \left[ \frac{2(\Delta_z c_s^2 d\Delta_z/dx + g(\gamma^2 - k^2 c_s^2))}{\omega^2 c_s^2} \right] \bigg|_{z=0}. \]
(12)
Substituting $u_z$ from (11) and $\Delta$ from (6) in equation (12), we obtain
\[ \frac{\omega^2}{\omega_D} \left( \omega^2_D - g^2 k^2 \right) = D e^{-kz_0} U \times \left[ \frac{\omega^2 c_s^2 k - 2k\omega_D c_s^2}{\omega^2_D - g(\gamma^2 - k^2 c_s^2)} \right] U', \]
(13)
where \( U = U(-a, m + 2, 2kz_0) \).

From the continuity of the equilibrium total pressure across \( z = 0 \) we have the relation
\[
\left[ p_0 + \frac{B^2}{2\mu} \right]_{x=0} = p_0(0_+),
\]
and
\[
\rho(0_+) = 1 + \frac{\gamma}{2} \left( \frac{v_A^2}{c_0^2} \right) = 1 + \frac{\gamma}{2\beta}.
\]

We have
\[
B_x = i\frac{u_x}{\omega} \frac{dB_0}{dz} + i\frac{B_0}{\omega} \frac{du_x}{dz},
\]
\[
p = i\frac{u_x}{\omega} \frac{dp_0}{dz} + i\frac{c^2}{\omega} \frac{dp_0}{dz},
\]
and
\[
p = i\frac{u_x}{\omega} \frac{dp_0}{dz} + \frac{i\gamma}{\omega} \frac{c^2}{\omega} \frac{dp_0}{dz} \Delta, \quad \text{when} \ z < 0.
\]

Using (16) and (17) we can write
\[
P = p + \frac{B_0 B_x}{\mu} = \frac{i}{\omega} \left[ c^2 \rho_0 \Delta + \frac{B_0^2}{\mu} \frac{du_x}{dz} \right] - \rho_0 g \xi_z, \quad \text{when} \ z < 0,
\]

and
\[
p = i\frac{c^2}{\omega} \rho_0 \Delta - \rho_0 g \xi_z, \quad \text{when} \ z > 0,
\]
where \( \xi_z \) is the normal component of the Lagrangian displacement. Substituting (18) and (19) in the boundary condition
\[
[P + \rho_0 g \xi_z]_{x=0} = [p + \rho_0 g \xi_z]_{x=0+},
\]
we obtain
\[
\frac{1}{\omega} \left[ c^2 \rho_0 \Delta + \frac{B_0^2}{\mu} \frac{du_x}{dz} \right]_{x=0} = \frac{1}{\omega} \left[ c^2 \rho_0 \Delta \right]_{x=0+},
\]
or, taking into account the equation (8)
\[
\frac{1}{\omega} \left[ \rho_0 \left( c^2 + v_A^2 \right) (\omega^2 - k^2 c_0^2) \frac{du_x}{dz} \right]_{x=0} = \frac{1}{\omega} \left[ c^2 \rho_0 \Delta \right]_{x=0+}.
\]

Substituting \( u_x \) from (11) and \( \Delta \) from (6) in (15) and taking into account (20), we obtain
\[
\frac{\omega}{\omega} D e^{-kz_0} U(-a, m + 2, 2kz_0) = \frac{g k^2 c_0^2 + (c_0^2 + v_A^2)(\omega^2 - k^2 c_0^2)}{(\omega^2 - k_0^2 c_0^2)(c_0^2 + \frac{1}{2} \gamma v_A^2)},
\]

From (13) and (21) we obtain the dispersion relation
\[
2k^2 c_0^2 \frac{U'}{U}(-a, m + 2, 2kz_0) + \gamma g \omega D - k^2 c_0^2 = \frac{(\omega^2 - k^2 c_0^2)(g^2 k^2 - \omega_0^2)(c_0^2 + \frac{1}{2} \gamma v_A^2)}{g k^2 c_0^2 + (c_0^2 + v_A^2)(\omega^2 - k^2 c_0^2)}.
\]

4. THE LONG WAVELENGTH LIMIT

Introducing \( \Omega^2 = \omega^2/(gk) \) and \( \Omega_D^2 = \omega_D^2/(gk) \) the dispersion relation (22) takes the form
\[
2 \frac{\Omega_D^2}{\Omega} + (m + 1) \frac{\Omega_D^2}{kz_0} - (1 + \Omega_D^2) = \frac{(m + 1)(1 - \Omega_D^2)(m \Omega^2 - kz_0)}{\Omega X},
\]
where
\[
X = m k z_0 + \lambda z_0 \left( 1 + \frac{1}{\beta} \right) \left( m \Omega^2 - \frac{k z_0}{1 + \beta} \right).
\]

From (Abramowitz & Stegun 1965)
\[
U'(-a, m + 2, 2kz_0) = aU(1 - a, m + 3, 2kz_0)
\]
and
\[
U(A, B, x) = \frac{\pi}{\sin \pi x} \left[ \frac{M(A, B, x)}{\Gamma(1 + A - B) \Gamma(B)} \right] - \frac{M(1 - m) M(-m)}{\Gamma(1 - m) \Gamma(m + 2)},
\]
we have
\[
\frac{2 \Omega_U'}{\Omega} = \frac{(1 + m)}{kz_0} \times \left[ M_1 - \frac{(2kz_0)^{m+1}}{\Gamma(1 - a - m) \Gamma(m + 3)} \right] \times \left[ M_3 - \frac{(2kz_0)^{m+1}}{\Gamma(1 - a - m) \Gamma(m + 2)} \right]^{-1}.
\]

where
\[
M_1 = M(-1 - a - m, -1 - m, 2kz_0),
\]
\[
M_2 = M(1 - a, m + 3, 2kz_0),
\]
\[
M_3 = M(-1 - a - m, -m, 2kz_0),
\]
\[
M_4 = M(-a, m + 2, 2kz_0).
\]

Further it is convenient to treat \( p_\text{r} \) and \( f_\text{r} \)-modes separately.

4.1. \( p_\text{r} \)-Modes

Write the dispersion relation (23) in the form
\[
\frac{2 \Omega_U'}{\Omega} = \frac{(1 + m + 1 + \Omega_D^2)}{\Omega_D^2 \Gamma X} + \frac{(m + 1)(1 - \Omega_D^2)(m \Omega^2 - kz_0)}{\Omega_D^2 \Gamma X},
\]
where
\[
\Omega = \Omega_D + \frac{V}{c_0 \sqrt{m} (kz_0)^{1/2}},
\]
and expand the right-hand side in a series in \((kz_0)^{\frac{1}{2}}\) to yield
\[
2 \frac{U'}{U} = -\frac{2(m+1)(\Omega_D^4 + 1)V}{c_0\sqrt{m}\Omega_D(\Omega_D^4 - 1)}(kz_0)^{-\frac{1}{2}} + \frac{(m+1)(1 + 12\Omega_D^2 + 3\Omega_D^4)V^2}{\Omega_D^6(\Omega_D^4 - 1)^2c_0^2m} + 1 - \frac{a_0\Omega_D^2 + a_4\Omega_D^4 + a_0}{\Omega_D^6(\Omega_D^4 - 1)} + O(\sqrt{kz_0}).
\]
(30)

Here
\[
a_8 = \frac{\gamma}{4\beta(\beta + 1)} \left( 1 + \frac{2\beta}{\gamma} \right)^2,
\]
(31)
\[
a_4 = -1 - \frac{\gamma}{\beta} + 2(\Gamma - 1)a_8,
\]
(32)
\[
a_0 = 1 + \frac{\beta - 3\beta\Gamma - \Gamma^2}{(\beta + 1)\Gamma^2}.
\]
(33)

Observe that the expansion of the right-hand-side of equation (28) breaks down if \(\Omega_D^2\) is close to zero or unity. The case \(\Omega_D^2 = 1\) corresponds to the f-mode and is considered below. We see from (30) that \(2(kz_0)^{\frac{1}{2}}U'/U\) remains finite as \((kz_0)^{\frac{1}{2}} \to 0\). For this to be compatible with equation (27), it is necessary that \(\Gamma(-a)\) tends to infinity in such a way that \((kz_0)^{m+\frac{1}{2}}\Gamma(-a)\) remains finite. In particular, then we require that
\[
a \to n - 1, \quad n = 1, 2, 3, ... \quad (34)
\]
From the definition of \(a\) under adiabatic conditions (see eq. 5), we see that
\[
\Omega_D^2 \to \Omega_n^2 = 1 + \frac{2n}{m}.
\]
(35)

To obtain a correction to this result, set
\[
\Omega_D^2 = \Omega_n^2 + p_{V_1}(2kz_0)^{m} + p_{V_2} + \delta_0(2kz_0)^{m+\frac{1}{2}},
\]
(36)

with \(p_{V_1}, p_{V_2}, \delta_0\) and \(s\) to be determined, where \(\delta_0\) is a correction to the eigenfrequency due to a magnetic atmosphere (see, e.g., Campbell & Roberts 1989) and \(p_{V_1}, p_{V_2}\) are terms due to the effect of a steady flow in the convective part below \(z = 0\). After some algebra we obtain
\[
p_{V_1} = -\frac{2\sqrt{2}\Gamma(1 + m + n)(\Omega_n^4 + 1)V}{\Gamma(m + 1)\Gamma(m + 2)\Gamma(n)c_0\sqrt{m}\Omega_n(\Omega_n^4 - 1)},
\]
(37)
\[
p_{V_2} = -\frac{\Gamma(1 + m + n)(3 - 4\Omega_n^4 + \Omega_n^8)V^2}{\Gamma(m + 1)\Gamma(m + 2)\Gamma(n)c_0^2m^2\Omega_n^2(\Omega_n^4 - 1)^2},
\]
(38)
\[
\delta_0 = -\frac{\Gamma(1 + m + n)}{m(m + 1)\Gamma(m + 1)\Gamma(m + 2)\Gamma(n)c_0^2m^2\Omega_n^2(\Omega_n^4 - 1)^2} \times \left[ a_8\Omega_n^8 + a_4\Omega_n^4 + a_0 \right] + \frac{\Omega_n^2}{\Omega_n^4 - 1} - \frac{m\Omega_n^2}{m + 2}
\]
(39)
and \(s = m + 3/2\). So, we can write
\[
\Omega_D = \Omega_n + \frac{p_{V_1}}{2\Omega_n}(2kz_0)^{m} + \frac{p_{V_2} + \delta_0}{2\Omega_n}(2kz_0)^{m+\frac{1}{2}},
\]
(40)
or
\[
\Omega = \Omega_n + \frac{V}{c_0\sqrt{2m}(2kz_0)^{\frac{1}{2}}} + \frac{p_{V_1}}{2\Omega_n}(2kz_0)^{m} + \frac{p_{V_2} + \delta_0}{2\Omega_n}(2kz_0)^{m+\frac{1}{2}},
\]
(41)
with \(p_{V_1}, p_{V_2}\) and \(\delta_0\) defined above. In the case when there is no flow we obtain back the results by Campbell & Roberts (1989).

4.2. f-Mode

Carying out a similar analysis we can write the solutions to the dispersion relation for the f-mode in the form
\[
\Omega_D^2 = 1 + f_{V_1}(kz_0)^{m} + (f_{V_2} + f_0)(kz_0)^{m+\frac{1}{2}},
\]
(42)

with \(f_{V_1}, f_{V_2}, f_0\) and \(s\) to be determined. Here, \(f_0\) is a correction due to a non-zero magnetic chromosphere, while \(f_{V_1}\) and \(f_{V_2}\) arise from a steady state in the internal part of the Sun. After some algebra we obtain
\[
f_{V_1} = -\frac{2m+2V}{c_0\sqrt{m}\Gamma(m + 2)},
\]
(43)
\[
f_{V_2} = \frac{2m+1V^2}{c_0^2m\Gamma(m + 2)},
\]
(44)
\[
f_0 = \frac{(1 + 2\beta)2m}{\beta(\beta + 1)m\Gamma(m + 2)},
\]
(45)
and
\[
\Omega_D = 1 - \frac{2m+1V(kz_0)^{m+\frac{1}{2}}}{c_0\sqrt{m}\Gamma(m + 2)} + \frac{1}{\Gamma(m + 2)} \left[ \frac{2mV^2}{c_0^2m^2} + \frac{(1 + 2\beta)2m-1}{\beta(\beta + 1)m} \right](kz_0)^{m+\frac{1}{2}},
\]
(46)
or
\[
\Omega = 1 + \frac{V}{c_0\sqrt{m}(kz_0)^{\frac{1}{2}}} - \frac{2m+1V(kz_0)^{m+\frac{1}{2}}}{c_0\sqrt{m}\Gamma(m + 2)} + \frac{1}{\Gamma(m + 2)} \left[ \frac{2mV^2}{c_0^2m^2} + \frac{(1 + 2\beta)2m-1}{\beta(\beta + 1)m} \right](kz_0)^{m+\frac{1}{2}}.
\]
(47)

5. NUMERICAL RESULTS

Numerical solutions to the dispersion relation (22) are shown in Figs. 1 and 3. Figures 2 and 4 show the frequency shifts obtained by solving the dispersion relation (22). In these numerical results we have taken...
Figure 1. Cyclic frequency \( \nu = \omega / (2\pi) \) in mHz as a function of flow in km/s and without magnetic field for \( f^- \) (with \( k = 2.7 \) Mm\(^{-1}\)) and \( p \)-modes with \( n = 1 \) (\( k = 1.3 \) Mm\(^{-1}\)), \( n = 2 \) (\( k = 0.9 \) Mm\(^{-1}\)).

Figure 2. Frequency difference \( \Delta \nu = \nu(V) - \nu(0) \) in \( \mu \)Hz as a function of the wavenumber \( k \) (Mm\(^{-1}\)) with \( V = 1 \) km/s and without magnetic field for \( f^- \) and \( p \)-modes (\( n = 1, 2 \)).

Figure 3. Cyclic frequency \( \nu = \omega / (2\pi) \) in mHz as a function of flow in km/s with \( B = 100 \) G for \( f^- \) (with \( k = 2.7 \) Mm\(^{-1}\)) and \( p \)-modes with \( n = 1 \) (\( k = 1.3 \) Mm\(^{-1}\)), \( n = 2 \) (\( k = 0.9 \) Mm\(^{-1}\)).
Figure 4. Frequency difference $\Delta \nu \equiv \nu(V) - \nu(0)$ in $\mu$Hz as a function of the wavenumber $k$ ($\text{Mm}^{-1}$) with $V = 1$ km/s and $B = 100$ G for $f$- and $p$-modes $n = 1, 2$.

Figure 5. Ratio $R = C_B/C_V$ of the correction due to magnetic field $C_B = \omega(k, B, V) - \omega(k, 0, V)$ over the correction due to flow $C_V = \omega(k, B, V) - \omega(k, B, 0)$ for $f$- and $p$-modes $n = 1, 2$ as a function of $V$ (km/s) and $B$ (G) with $k = 0.1$ Mm$^{-1}$.

the sound speed in the isothermal atmosphere to correspond to that at the temperature minimum, yielding $c_0 = 6.76$ km/s for an adiabatic index $\gamma = 5/3$. The scale height $H_0$ is then 100 km and the polytropic index $m$ is 3/2. For conditions typical at the temperature minimum, we may take

$$\beta = \frac{180}{B_0}^2,$$

where $B_0$ is the field at the base of the magnetic atmosphere, measured in gauss (see also Campbell & Roberts 1989). Cut-offs in these figures are denoted by dotted lines.

In order to assess the importance of a steady state over a chromospheric magnetic field (or vice-versa!) Fig. 5 is obtained using the long wavelength limit formulae (41) and (47) for $p$- and $f$-modes. Here the ratio of the correction due to a magnetic field over the correction due to a steady state is plotted.

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