GENERATION OF OSCILLATIONS IN THE SOLAR CONVECTION ZONE: LINEAR MECHANISM OF MODE CONVERSION IN SHEAR FLOWS

G. D. Chagelishvili\(^1\), A. G. Tevzadze\(^1,3\), M. Goossens\(^3\)

\(^1\)Abastumani Astrophysical Observatory, 2\(^{nd}\) Ave. Kazbegi 380060 Tbilisi, Georgia
\(^2\)Space Research Institute, 84/32 Str. Profsoyuznaya, 117810 Moscow, Russia
\(^3\)Center for Plasma Astrophysics, K.U.Leuven, Celestijnenlaan 200B, 3001, Heverlee, Belgium

ABSTRACT

The linear conversion of convection by shear flow into acoustic waves is identified as a new excitation mechanism for Solar oscillations. This excitation mechanism has been discovered during our study of the linear perturbations in an unstably stratified compressible three dimensional constant shear flow by means of the nonmodal analysis. Being extensively used recently, the nonmodal approach led to the discovery of the linear mode coupling in shear flows, including nonresonant phenomenon of vortex-wave mode conversion [Chagelishvili et al. 1997a]. Here we report on a similar phenomenon that occurs for convectively unstable, exponentially growing modes which provide a source for the acoustic wave modes due to the mean flow velocity shear. This phenomenon seems to be the most efficient dynamical source of waves in the Solar convection zone. This nonresonant phenomenon of mode conversion results in a qualitative change of the temporal variation scales of the perturbations and hence presents a new significant contribution into the channel of energy transfer from the dynamically active interior to the atmosphere of the Sun.

Key words: hydrodynamics; convection; Sun: oscillations;

1. INTRODUCTION

The study of the generation and propagation of waves in the solar convection zone is important for understanding the internal structure and dynamics, as well as the energy transport from the interior to the atmosphere of the Sun. The suggestion that the solar acoustic oscillations are excited by the turbulent convection in the convection zone may be found in Stein 1968, Goldreich & Keeley 1977, Blamforth 1992, Goldreich & Kumar 1990 and Goldreich, Murray & Kumar 1994.

The theory of acoustic wave generation by free turbulence in a homogeneous medium was developed by Lighthill (1952). Stein (1967) has generalised this theory for stratified fluids. From a physical point of view, Lighthill’s theory of aerodynamic sound generation is based on the concept of the stochastic excitation of oscillations (waves). In this approach, the turbulent fluid dynamics is described by an inhomogeneous wave equation, with the nonresonant form of the oscillatory part and the nonlinear inhomogeneous term standing for the source function. The principal source terms in a stratified medium are usually classified by the multipole order, according to their physical origin, as well. Notwithstanding the essential difference of the dynamics of convective and free turbulence, Goldreich and Kumar (1990) have shown that the main source of sound remains the nonlinear quadrupole term – double divergence of the Reynolds stress. The amplifying effect of the large mean flow shear on the fluctuating Reynolds stress and thus on the emission of the acoustic waves has been noted by Lighthill (1954). However, this idea has not received further development within the concept of stochastic excitation.

Significant advances in the investigation of the dynamics of flows with velocity shear have been achieved together with the disclosure of specific features of the shear flow phenomena (see Reddy et al. 1993, Trefethen et al. 1993). Operators arising in the mathematical formalism of the canonical modal analysis in the study of the linear dynamics of shear flows are not self-adjoint. Consequently eigenmode interference introduces principal complications in the problem. The nonmodal approach has proved to be an alternative successful approach. This approach employs the study of temporal evolution of the spatial Fourier harmonics of perturbations. Impressive progress has been made by use of the nonmodal analysis (see Goldreich & Linden-Bell 1965, Batler & Farrell 1992, Balbus & Hawley 1992, Chagelishvili et al. 1994, 1996a,b, 1997a,b,c, Lubow & Spruit 1995, Rogava et al. 1997, 1998, Poedts et al. 1998, 1999, Zaqarashvili 1999). The same approach has led to the disclosure of the coupling and nonresonant conversion of different modes in shear flows. Conversion of vortices into acoustic waves has been described in Chagelishvili et al. 1997a. The same mechanism is found to operate for magnetosonic (Tevzadze 1998) as well as plasma Langmuir oscillations (Rogava et al. 1998).

In this report we introduce a new dynamical source of acoustic waves in unstably stratified shear flows.

Proc. 9th European Meeting on Solar Physics, 'Magnetic Fields and Solar Processes', Florence, Italy,
12-18 September 1999 (ESA SP-448, December 1999)

© European Space Agency • Provided by the NASA Astrophysics Data System
Namely, we present the linear phenomenon of the nonresonant mode conversion in shear flows: conversion of convective into acoustic wave mode. Convectively unstable exponentially growing buoyancy perturbations should be able to generate acoustic wave oscillations in presence of a mean flow velocity shear. We identify the linear conversion of modes in shear flows with a new excitation mechanism of the solar oscillations and waves. It differs in principle from the stochastic excitation mechanism and should significantly contribute to the process of acoustic wave generation in the solar convection zone.

The paper is organised as follows. The physical model and mathematical formulation of the problem are outlined in Sec. 2. Acoustic wave sources are identified in Sec. 3. Exact numerical solutions are used to demonstrate mean flow induced wave emission by convection in Sec. 4. Summarising discussion and conclusions are given in Sec. 5.

2. BASIC EQUATIONS

The equations governing the dynamics of a compressible stratified flow are the following:

\[ \partial_t (\mathbf{V}) + \rho \mathbf{V} \cdot \nabla \mathbf{V} = 0, \]  
\[ \partial_t (\rho \mathbf{V}) = -\nabla p + \mathbf{g}, \]  
\[ \partial_t (\rho \mathbf{V}) + \nabla [\rho (\mathbf{V} \cdot \nabla) \rho] = \frac{\gamma p}{\rho} \partial_t (\mathbf{V} \cdot \nabla) \rho. \]  

The background is a horizontal shear flow \( \mathbf{V}_0 = (A y, 0, 0) \) in a constant vertical gravity field \( \mathbf{g} = (0, 0, -g) \). Thus we assume that \( A = \text{const} \) and \( g = \text{const} \). This yields the stratified equilibrium state:

\[ \frac{p_0(z)}{\rho_0(z)} = \frac{p_0(0)}{\rho_0(0)} = \exp(-z k_H), \]

where \( k_H \equiv \gamma g / c_s^2 \) and \( c_s^2 \equiv \gamma p_0 / \rho_0 \). As much as we are interested in the convectively unstable medium, described state is considered as an unstable equilibrium. Particularly, this equilibrium state allows for exponentially unstable mode of the perturbation spectra.

We neglect the perturbations of the gravitational field and introduce the linear perturbations in the following way:

\[ \mathbf{V} = \mathbf{V}_0 + \frac{p_0(0)}{\rho_0(z)} \mathbf{V}', \quad \rho = \rho_0 + \rho', \]

Here velocity perturbations are already normalised to exclude the exponential height dependence due to the vertical stratification of the background flow. Following the standard method of nonmodal analysis (see Criminale & Drazin 1990 for a rigorous mathematical interpretation) we introduce the spatial Fourier harmonics (SFHs) of the perturbations with particularly time dependent phases:

\[ \Psi(r, t) = \psi(k(t), t) \exp \left( i z k_x + i y k_y(t) + i \hat{k}_z \right), \]

\[ k_y(t) = k_y(0) - A k_x t, \]  
where \( \hat{k}_z \equiv k_z + i k_H / 2 \). For compactness of notation we introduce the generalised vector of perturbations \( \Psi \) and its SFH \( \psi \) as:

\[ \Psi \equiv (V', p', \rho'), \quad \psi \equiv (u, p, \rho). \]

To exclude complex coefficients in the dynamical equations, we construct the normalised entropy and vertical velocity perturbation in the following way:

\[ s \equiv \frac{c_s^2 \hat{k}_z^* + ig}{\gamma - 1} \left( \frac{p - c_s^2 \rho}{\gamma - 1} \right), \]

\[ v \equiv \frac{c_s^2 \hat{k}_z^* + ig}{u_z}, \]

where \( \hat{k}_z^* = k_z - ik_H / 2 \). Following Eqs. (1-5) we reach the set of differential equations that describe the linear dynamics of the perturbation SFHS in the stratified shear flows:

\[ \dot{c}_s^2(t) = c_s^2(k_x u_x + k_y(t) u_y) + v, \]

\[ \dot{u}_x(t) = - A u_y - k_x p, \]

\[ \dot{u}_y(t) = - k_y(t) p, \]

\[ \dot{v}(t) = (N_B^2 - c_s^2 k_z^2) p - N_B^2 s, \]

\[ \dot{s}(t) = v, \]

where \( N_B^2 \) is frequency of the the Brunt-Väisälä:

\[ N_B^2 \equiv g k_H \gamma - 1 / \gamma \]

and \( \hat{k}_z^2 = |\hat{k}_z|^2 = k_z^2 + k_H^2 / 4 \). In an unstable stratified flows negative buoyancy (\( N_B^2 < 0 \)) requires that adiabatic index \( \gamma < 1 \). Such an effective value may be assigned to this parameter under a certain thermodynamic approach (see Ryu & Goodman 1992). However, in Eqs. (6.a-e) we retain only \( N_B^2 \) and argue that these equations are somewhat more general than the underlying \( \gamma \) prescription.

Further we note the time invariant of Eqs (6.a-e) that stands for the vorticity conservation law in the wave-number space:

\[ I = k_x u_y - k_y(t) u_x - \frac{A}{c_s^2} (p - s). \]

For acoustic and buoyancy perturbations \( I = 0 \), hence, \( I \) is the vortex mode measure in the perturbation spectrum.

The spectral energy of the perturbations may be defined in the following way:

\[ E = \frac{\rho_0^2}{2 c_s^2} (E_K + E_P + E_T), \]

\[ E_K = c_s^2 (u_x^2 + u_y^2) + \frac{1}{(c_s^2 k_z^2 - N_B^2)^2}, \]

\[ E_P = p^2, \]

\[ E_T = \frac{N_B^2}{(c_s^2 k_z^2 - N_B^2)^2} s^2. \]
where $E_K$, $E_p$ and $E_T$ correspond to the kinetic, elastic and thermobaric energies of the perturbations, respectively. Formally the perturbation energy is conserved in the shearless limit: $E = A c_s^2 u_s u_y$. The instability of the convective eddies is revealed by the negative sign of the thermobaric energy.

The different modes may be classified using the dispersion equation, that is obtained from Eqs. (6.a-e) in the shearless limit ($A = 0$):

$$\omega^2 = \frac{1}{2} c_s^2 k_2^2 \left\{ \frac{1 \pm \sqrt{1 - 4 N_B^2 k_2^2}}{c_s^2 k_2^2} \right\},$$

(11.b)

where the subscripts $v$, $s$, $c$ define the frequencies of the vortex, acoustic and convective modes, respectively. In an unstratified flow, i.e., when $N_B^2 < 0$, $\omega_c$ defines the growth rate of the buoyancy perturbations.

From a physical point of view, it is worth to introduce the local approximation and consider small scale perturbations: $k_2 \gg k_p$. This approximation strongly simplifies the mathematical formulation and is strongly justified for the following two reasons. Firstly, our analysis needs constant vertical gravity, an assumption that may be adopted for perturbations with vertical height scales less than the stratification scale. Secondly, this approximation is necessary for consideration of the constant linear shear of the flow velocity, especially in the turbulent flows. Using Eq. (7) this approximation may be represented by the following condition:

$$c_s^2 k_2^2 \gg N_B^2.$$  

(12)

Hence, the mode frequencies may be well approximated by the following expressions:

$$\Omega_1^2 = c_s^2 k_2^2, \quad \Omega_2^2 = N_B^2 \frac{k_2^2}{k_s^2}. \quad (13.a,b)$$

The local approximation is used in our further analysis.

3. SOURCE TERMS

To analyse the dynamics of acoustic oscillations in the considered flow, we rewrite Eqs. (6.a-e) in the form of coupled second order differential equations:

$$\ddot{y}(t) + f(t) \dot{y}(t) + \Omega_1^2(t)p(t) = \lambda_1(t) \dot{y}(t) + \lambda_2(t) y(t), \quad (14.a)$$

$$\ddot{y}(t) + \Omega_2^2(t)y(t) = 0, \quad (14.b)$$

where the convection variable $y(t)$ is introduced in the following form:

$$y(t) \equiv \frac{k_2}{k_s} \left( s(t) - \frac{k_s^2}{k_2^2} p(t) \right) \quad (15)$$

and

$$\Omega_1^2(t) = c_s^2 k_2^2(t) + 2 A^2 \frac{k_2^4}{k_s^2(t)} - 4 A^2 \frac{k_2^2 k_s^2(t) k_2^2}{k_s^2(t) k_s^2(t)} \quad (16.a)$$

$$\Omega_2^2(t) = \frac{N_B^2 k_s^2(t)}{k_2^2(t)} + 2 A^2 \frac{k_2^2 k_s^2(t)}{k_s^2(t) k_s^2(t)} \times \left[ 3 k_s^2(t) k_2^2(t) - 4 k_s^2(t) k_2^2(t) - k_s^2(t) k_s^2(t) \right] \quad (16.b)$$

$$f(t) = 2 A \frac{k_2 k_s(t)}{k_s^2(t)} \quad (16.c)$$

$$\lambda_1(t) = -2 A \frac{k_s k_s(t)}{k_s(t) k_s(t)} \quad (16.d)$$

$$\lambda_2(t) = -2 A^2 \frac{k_2 k_s^2}{k_s^2(t)} \quad (16.e)$$

In deriving Eqs. (14.a-b) two simplifications have been made. First, we have retained only the terms describing the effect of the buoyancy perturbations on the acoustic waves, and we have neglected the effect of the acoustic pressure perturbations on the evolution (exponential amplification) of the buoyancy perturbations in the right hand side (rhs) of Eq. (14.b). Second, we have neglected the source terms in the rhs of both dynamical equations that describe convective induced coupling between the vortex and acoustic wave modes (in Eq. 14.a) and vortex and buoyancy modes (in Eq. 14.b). In fact, the coupling of the vortex and acoustic wave modes is the process that has been studied to reveal the mean flow shear induced nonresonant mode conversion phenomenon in Chagelishvili et al. 1997. However, in the present case, the source terms of the acoustic waves that are proportional to the vortex mode measure, conserved quantity $I$ (see Eq. 8), are dominated by the source terms, associated with the exponentially amplifying convective modes: $y(t)$ and $\dot{y}(t)$. It should be emphasized that the present approach is justified only for the convectively unstable medium, where $N_B^2 < 0$ and buoyancy modes undergo exponential amplification.

The dynamics of the acoustic waves in the absence of the buoyancy perturbations is described by the homogeneous part of Eq. (14.a). The acoustic wave frequency and amplitude variations are described by the parameters $\Omega_1(t)$ and $\lambda_1(t)$ (see Chagelishvili et al. 1997b for a detailed study). The dynamics of the convective mode perturbations is described by Eq. (14.b). Eq. (16.b) shows the transient stabilization effect of the mean flow shear in an unstably stratified medium. The stabilization occurs at times, when $|k_s(t)/k_2| < 1$ and reaches its maximum at $t \approx t^*$. when $k_s(t^*) = 0$ (see Eq. 16.b).

The terms $\lambda_1(t) y(t)$ and $\lambda_2(t) y(t)$ in the rhs of Eq. (14.a) describe the coupling between the convection and acoustic waves. Shear flow origin of these source terms is obvious from Eqs. (16.d,e). Hence, Eqs. (14.a,b) describe the mean flow shear induced buoyancy - acoustic wave mode conversion phenomenon in a convectively unstable medium. Some specific features of the described phenomenon are due to its linear nature: SFH of the exponentially growing buoyancy perturbations are able to generate SFH of the
acoustic waves with the same wave-numbers. The amplitude of the excited wave mode depends on the values of the source terms $\lambda_1(t)$ and $\lambda_2(t)$. So, convective modes with $k_x = 0$ are not able to generate acoustic waves at all ($\lambda_1 = \lambda_2 = 0$). While maximal efficiency of the mode conversion phenomenon should occur at $k_x = 0$, or in a realistic physical approximation (see Eq. 12) at $k_x^2 \geq N_B^2/c_s^2$. Naturally, acoustic wave emission from convection should generally increase when the mean flow shear parameter $A$ increases. For a more detailed study of the described wave generation phenomena we use numerical analysis.

4. NUMERICAL ANALYSIS

For the purpose of the presentation of the numerical results we introduce nondimensional notations. We normalise physical the quantities on the mean flow pressure at the $z = 0$ height:

$$U_{x,y}(\tau) \equiv \frac{c_s u_{x,y}(t)}{P_0(0)}, \quad V(\tau) \equiv \frac{\nu(t)}{c_s k_H P_0(0)},$$
$$P(\tau) \equiv \frac{p(t)}{P_0(0)}, \quad S(\tau) \equiv \frac{s(t)}{P_0(0)}.$$

(17)

With the use of the nondimensional time and shear scales:

$$\tau \equiv \frac{c_s k_H t}{R}, \quad \tau_0 \equiv \frac{A}{c_s k_H},$$

(18)

we normalise the perturbation wave-numbers in the following way:

$$K_x \equiv \frac{k_x}{k_H}, \quad K_y(\tau) \equiv \frac{k_y(t)}{k_H}, \quad K_z \equiv \frac{k_z}{k_H},$$

(19)

Hereafter, the local approach (small scale of perturbations $k_x \gg k_y$) is reduced to the condition $K_x \gg 1$. Finally, we introduce the normalised buoyancy parameter:

$$\sigma \equiv \frac{N_B^2 k_H^2}{c_s^2} = \frac{\gamma - 1}{\gamma},$$

(20)

describing the stratification state of the flow. A neutral fluid is described by $\sigma = 0$, while positive and negative values of this parameter correspond to a stably and unstably stratified medium, respectively.

Bearing in mind Eqs. (14.a,b) we introduce the nondimensional convection variable:

$$Y(\tau) \equiv \sqrt{K_x^2 + K_y^2(\tau)} S(\tau) - \frac{K_x^2}{K_x^2} P(\tau),$$

(21)

where $K_x^2(\tau) = K_x^2 + K_y^2(\tau) + K_z^2$. Nondimensional total energy $E(\tau)$ (see Eq. 9) and compression $\Sigma(\tau)$ take the following form:

$$E(\tau) = \frac{U_x^2(\tau) + U_y^2(\tau)}{K_x^2 - \sigma} + \frac{V^2(\tau)}{K_z^2 - \sigma} + \frac{P^2(\tau)}{K_z^2 - \sigma} + \frac{\sigma}{K_z^2 - \sigma} S(\tau),$$
$$\Sigma(\tau) \equiv \frac{K_x U_x(\tau) + K_y(\tau) U_y(\tau) + V(\tau)}{\sqrt{K_x^2 + K_y^2(\tau) + K_z^2}}.$$

(22)

(23)

To illustrate the acoustic wave generation by convection we present exact numerical solutions of Eqs. (6.a-e) without any additional assumptions used in Sec. 3. Fig. 1 presents the dynamics of the buoyancy modes in the convectively unstable medium at low mean flow shear. The initial perturbations are filtered from the vortex and acoustic modes. Thus, the numerical results illustrate the dynamics of the purely buoyancy perturbations. The dynamics of the total energy, as well as convection variable reveals the transient stabilisation effect of the shear flow on the convective instability in the vicinity of $|K_y(\tau)/K_x| \ll 1$. Pressure and compression perturbations are influenced by the minor variations of the convective mode in the same time interval due to the shear flow.

The dynamics of the buoyancy perturbations at high values of the shear parameter is shown in Fig. 2. The evolution of pressure and compression perturbations clearly demonstrate wave generation phenomenon. The excitation of oscillations occurs at time $t = t^*$, when the wave-number along the shear axis is zero: $K_y(t^*) = 0$.

Numerical analysis has shown that the efficiency of mode conversion generally increases with the flow shear parameter. Also, it proceeds quite differently at different wave-numbers: wave emission from the buoyancy perturbations with mainly vertical wave-numbers ($K_z \gg K_x$) is nearly suppressed even at high shear parameters.

Numerical solutions of Eqs. (6.a-e) well support the analytical results obtained in Sec. 3.

5. DISCUSSION AND CONCLUSIONS

We have presented a study of the linear dynamics of perturbations in an unstably stratified compressible flow by means of the nonmodal analysis. The linear character of the processes considered here enables to identify the perturbation modes and describe their dynamics individually. We find a mode conversion phenomenon that is originated from the mean flow shear: exponentially growing unstable buoyancy perturbations are able to generate acoustic waves in the linear stage of the evolution. The process discussed here offers a novel approach to the problem of the acoustic wave generation.

The presented linear mode conversion phenomenon differs in nature from the essentially nonlinear process of stochastic wave excitation. Advantages of the present analysis should help us better understand the acoustic wave generation in unstably stratified flows. A particularly important application is the solar convection zone and the acoustic oscillations of the sun. First indications of shear flow induced contribution to the solar acoustic oscillations come from the principal difference of these two mechanisms. In the stochastic excitation mechanism turbulent eddies of a certain temporal variation scale provide the sources of acoustic waves with similar frequencies. While, in the mode conversion phenomenon acoustic wave sources are eddies with similar wave-numbers. The frequency dependence of the generated acoustic wave energy comes from the wave dispersion equa-
Figure 1. Evolution of a buoyancy perturbations in the convectively unstable medium: $\sigma = -2$ ($\gamma = 0.5$). Here $K_x = K_z = 100$, $K_y(0) = 2000$ and $R = 2$. Convection $Y(\tau)$, pressure $P(\tau)$, normalised total energy $E(\tau)/E(0)$ and compression perturbations $\Sigma(\tau)$ are shown on a, b, c and d graphs, respectively.

The nonresonant phenomenon presented here results in a qualitative change of the temporal variation scales of the perturbations. It seems to be a most efficient dynamical source of oscillations in the solar convection zone. Therefore, the induced mode conversion phenomenon presents a new significant contribution into the channel of energy transfer from the dynamically active interior to the atmosphere of the sun.

Finally, we note that the general nature of the presented mechanism suggests that a similar phenomenon should be realised in the magnetised flows as well, where convection should be able to generate magnetohydrodynamic waves.

ACKNOWLEDGMENTS

A. G. Tevzadze would like to acknowledge the financial support as "bursaal" of the "FWO Vlaanderen", project G.0335.98. This work was supported in part by the INTAS grant GE97-0504.

REFERENCES


© European Space Agency • Provided by the NASA Astrophysics Data System
Figure 2. Evolution of perturbations in the convectively unstable medium: $\sigma = -2$ ($\gamma = 0.5$). Here $K_1 = K_2 = 100$, $K_3(0) = 10000$ and $R = 25$. Convection $Y(t)$, pressure $P(t)$, normalised total energy $E(t)/E(0)$ and compression perturbations $\Sigma(t)$ are shown on a, b, c, and d graphs, respectively. Purely convective mode buoyancy perturbations are excited initially. Evolution of the pressure and compression perturbations reveal the acoustic wave generation phenomenon. Generation of oscillations occurs at time $\tau = 4$, when wave-number along the flow shear is zero $K_3(4) = 0$. High frequency pattern of the acoustic oscillations is visually damaged due to the limited numerical temporal resolution.

Goldreich, P. & Linden-Bell, D. 1965, MNRAS 130, 125
---------, 1999, Space Science Reviews 87, 295

© European Space Agency • Provided by the NASA Astrophysics Data System