RESONANT HEATING AND ACCELERATION OF PROTONS AND IONS IN CORONAL HOLES: TWO-PROTON CLOSURE

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Abstract. The ion-cyclotron mode does not extend above the proton cyclotron frequency. Thus only roughly half of the protons can be in resonance. We calculate the trajectories of individual protons in the electric, magnetic, and gravitational fields in a coronal hole, and we include the resonant heating and acceleration for an average resonant particle. To provide closure we consider two protons, which are proxies for the resonant and non-resonant halves of the distribution. For dispersive waves, $k_{\text{res}}$ becomes large. If the dissipation is determined by a turbulent cascade, $k_{\text{res}}$ controls the relative importance of resonant acceleration and resonant heating. Such models yield good agreement with what is known about the behavior of protons in coronal holes. We also consider, in a cruder model, the behavior of minor ions such as $O^{+5}$. We show qualitatively how such ions can be more than mass-proportionally heated, and that dispersion is important. We also show that the observed radial profile of $T_e$ for $O^{+5}$ requires a steep power spectrum. It is concluded that the cyclotron resonance can account for many observed features of protons and heavy minor ions in coronal holes. However, the source of the resonant waves remains uncertain.

1. Introduction. In situ observations of solar wind protons and ions have motivated a long series of studies which invoked the cyclotron resonance (Hollweg and Turner, 1978; Dusenberg and Hollweg, 1981; Isenberg and Hollweg, 1982, 1983; Marsch et al., 1982; McKenzie and Marsch, 1982; Isenberg, 1984; Gomberoff and Elgueta, 1991; Gomberoff et al., 1996; Gomberoff and Astudillo, 1999). These studies emphasized processes occurring in interplanetary space rather than in the corona.

The importance of the cyclotron resonance has been further supported by data from UVCS. Four lines of evidence point to the action of the cyclotron resonance in coronal holes (Kohl et al., 1998). First, the protons and heavy ions are heated primarily perpendicular to the magnetic field. Second, heavy ions, such as $O^{+5}$, are considerably hotter than the protons, by more than the mass ratios. Third, the heavy ions flow faster than the protons. Fourth, all positively charged particles are considerably hotter than the electrons.

Hollweg (1986), Hollweg and Johnson (1988), and Isenberg (1990) were the first to consider the possibility that the cyclotron resonance could heat coronal protons and drive the fast solar wind. These models have recently been re-examined by Li et al. (1999) and Hu and Habbal (1999). The results are generally in excellent agreement with the UVCS data, and other observations of the fast solar wind flows from polar coronal holes. These models postulated that the Sun launches long-period (hours or longer) waves, which cascade their energy to the cyclotron-resonant frequencies. This viewpoint is motivated by a wealth of in situ and remote sensing data. In an alternate viewpoint, McKenzie et al. (1995), Marsch and Tu (1997), Tu and Marsch (1997), and Ruzmaikin and Berger (1998) propose that solar reconnection events launch very high frequency, several kHz, waves into the corona. In this paper we again present models which assume that the wave dissipation rate is determined by a turbulent cascade.

Hollweg (1999a,b,c) discussed some basic physical principles governing the effects of the cyclotron resonance in coronal holes. In those papers we emphasized the kinetic physics, and illustrated certain kinetic effects by computing trajectories of individual particles (see also Cranmer et al., 1997, 1999). We demonstrated the importance of wave dispersion, especially in producing the more than mass-proportional heating of $O^{+5}$. One of the main points of this paper will be to further discuss the importance of wave dispersion, but for the protons. We will show that the ratio of work done to perpendicular heating is more important when there is dispersion. We will find that the resonant acceleration can then make an important contribution to the dynamics.

Our previous papers were flawed in their lack of closure, especially for the protons. The difficulty with the protons is that only half of the protons can resonate with ion-cyclotron waves. In this paper we present a primitive closure scheme for the protons. Our closure is different in spirit from the traditional closure which is achieved by assuming that the distribution function is bi-Maxwellian. Our approach has the disadvantage of applying only to regions which are essentially collisionless.

For completeness, in Section 5 we will recapitulate some previous results for heavy minor ions (Hollweg, 1999c).

2. Dispersion. In a cold electron-proton plasma, parallel-propagating ion-cyclotron waves with angular frequency $\omega$ and wavenumber $k$ obey the dispersion relation $\omega/k = \nu_A = (1 - \omega/\Omega_p)^{1/2}$ where $\nu_A$ is the Alfvén speed and $\Omega_p$ is the proton cyclotron frequency. Figure 1 displays this relation as the heavy curve. Also shown, with the heavy diagonal line, is the extension of the low-frequency MHD dispersion relation. The
The condition for proton resonance is \( \frac{\omega}{\Omega_p} = (k \| v_A / \Omega_p) (v_\| / v_A) + 1 \) where \( v_\| \) is the proton's parallel velocity. This is plotted as the thin straight lines for three values of \( v_\| \). The intersections of those straight lines with the dispersion relation give the resonant values of \( \omega \) and \( k_\| \). Since the ion-cyclotron branch does not extend above \( \omega / \Omega_p = 1 \), only protons with \( v_\| < 0 \) in the bulk proton frame can resonate. In a coronal hole, \( v_A \) can be as large as 5000-10000 km s\(^{-1}\). With parallel proton thermal speeds of the order of 100 km s\(^{-1}\), we expect \( v_\| / v_A \) to be of the order of 0.01. For purposes of illustration, we plot \( v_\| / v_A = - 0.1 \). Note that \( k_\| \) is larger for the dispersive waves. Marsh and Tu (1997) and Tu and Marsch (1997) took the protons to be in resonance with waves having \( \omega / \Omega_p = 0.1 \). This occurs for protons having \( v_\| = - 9 v_A \); in the corona there will be essentially no particles with these velocities. As an approximation which will be used later, for the dispersive case we have \( k_{\text{res}}^3 = - \Omega_p^2 / (v_\| v_A)^2 \) if \( v_\| \) is small.

![Graph showing the relationship between v_\| and k_\|](image)

**Fig. 1.** The curves indicated 'dispersive' and 'non-dispersive' represent the dispersion relations for parallel-propagating ion-cyclotron waves in a cold electron-proton plasma. The light lines represent the resonance condition for protons moving at the indicated values of \( v_\| \). The intersections of the light and heavy curves yield the values of \( \omega \) and \( k_\| \) for the resonant waves.

### 3. Two-Proton Closure

The quasilinear resonant interaction of particles with waves is proportional to \( P_B(k_{\text{res}}) \). \( P_B \) is the magnetic fluctuation power spectrum defined such that \( \langle B^2 \rangle = \int P_B(k_{\text{res}}) \, dk_{\text{res}} \), where \( B \) is the magnetic fluctuation, the angle brackets denote the variance, and the integral is between 0 and \( \infty \).

Consider velocity space \( (v_T, v_\perp) \), where \( v_T \) is radial velocity in a Sun-centered frame. We take the magnetic field to be radial, so \( v_T \) is along the magnetic field, and \( v_\perp \) is perpendicular to the magnetic field. We consider only waves propagating outward from the Sun. A distribution of particles having \( v_\| < 0 \) and small \( v_\perp \) will tend to resonantly diffuse to larger \( v_\perp \). At the same time the particles will be resonantly accelerated to larger values of \( v_T \).

For the average diffusing particle we write (Hollweg, 1999a)

\[
\frac{n}{2} \frac{d}{dt} \frac{v_\perp^2}{v_{\text{res}}^2} = \frac{q^2}{mc^2} \frac{\Omega}{k_{\text{res}}} \quad \text{(1)}
\]

where \( q \) and \( m \) are particle charge and mass, and \( c \) is the speed of light (cgs units are used throughout). The corresponding resonant acceleration is \( (dv_\perp / dt)_{\text{res}} = \left( k_{\text{res}}^2 / 2 \Omega \right) (dv_\perp^2 / dt)_{\text{res}} \). As \( v_\perp^2 \) increases, the mirror force provides the dominant acceleration, \( (dv_\perp / dt)_{\text{mag}} = -(v_\perp^2 / 2) d \log B / dr \). Magnetic moment conservation gives \( (dv_\perp^2 / dt)_{\text{mag}} = v_\perp^2 v_T d \log B / dr \).

![Graph showing the velocities of two protons](image)

**Fig. 2.** Schematic of the motions of the two protons in \( v_\perp - v_T \) space. Curves 'A' and 'D' represent the diffusive motion due to resonance with the waves. Curve 'C' represents the adiabatic motion of a particle in the Sun's magnetic field.

The resonant effects apply only to particles moving slower than the bulk solar wind speed, \( V_{SW} \). In this paper we obtain a self-consistent description by considering the trajectories of two protons, which are proxies for the slow and fast halves of the distribution function. The basic behavior of the model is shown very schematically in Figure 2, which shows the particle motions in velocity space. (In this discussion we ignore gravity and the electric field.) Particles 1 and 2 start at \( t = 0 \) with small values of \( v_\perp \) and different values of \( v_T \), representing the parallel spread of the distribution function. \( V_{SW} \) is the average \( v_T \). Initially
only particle 1 is in resonance. It moves up in $v_\perp$ and to larger $v_T$. This behavior is sketched as curve 'A'. Particle 2 meanwhile stays put in velocity space (ignoring for the moment the weak mirror force on that particle). This behavior persists until particle 1 reaches point 'B'. At that point it becomes the faster particle, and drops out of resonance. The mirror force still accelerates particle 1, but as it moves outward in the decreasing coronal magnetic field $v_\perp$ decreases. This behavior is sketched as curve 'C'. At this time particle 2 is the slower particle and it becomes resonant, moving up to and the right along curve 'D', which is analogous to curve 'A'. This behavior persists until particle 2 again becomes the faster particle, which occurs at the two points marked 'E'. The cycle then repeats, as indicated by the two arrows. The figure suggests that the separation in $v_T$ decreases; the numerical calculations confirm this expectation. If the waves are dispersive, $k_{res}$ will then become very large; see the last line of Section 2.

To model the resonance acting only on the slower particle, we multiply the right-hand side of (1) by the factor $(1/2)(1 + \tanh[(V_{SW} - v_T)/3])$ where $V_{SW}$ and $v_T$ are here in km/s. Thus if the particles are within about 6 km/s of each other we regard them as indistinguishable, and each shares half of the resonant heating and acceleration.

To complete the model, we calculate the evolving mass density. As the model advances $t_1(t)$ and $t_2(t)$ in time, along with the positions $r_1(t)$ and $r_2(t)$ (which are very nearly the same), each particle sees an electron concentration $n_e(t) = n_{eo}B(t)/[B(t_0)V_{SW}(t)]$, where the subscript 'o' indicates conditions at $t = 0$. (In the numerical calculations $n_{eo}$ is chosen to correspond to the observed value at $t_0$. Then $V_{SW}(t)$ is taken to give a flux ($n_e V_{SW}$) = $2.2 \times 10^8$ cm$^{-2}$ s$^{-1}$ at 1 AU.) The magnetic field $B$ is prescribed: $B = 1.5 (f_{max} - 1) R^{-3.5} + 1.5 R^{-2}$ Gauss where $R$ is r in solar radii and $f_{max} = 9$.

The radial electric field $E_r$ follows from the electron momentum equation with $m_e = 0$ and $T_e = constant = 10^6$K. With $n_e V_{SW}/B = constant$, we obtain the equation of motion for particle 1:

$$\frac{dv_1}{dt} = -\frac{k_{Te}}{m} \left[ 1 - \frac{v_T}{v_1(v_1 + v_2)} \right] (\frac{d\log B}{dr} + a)$$

where $k$ is Boltzmann's constant, $v$ d$V_{SW}$/dt, and 'a' represents the accelerations from waves, mirroring, and gravity. The equation for particle 2 is similar.

We consider a power-law power spectrum $P_B = L k^{-\gamma}$. Following Isenberg (1984), we introduce a quantity related to the wave action: $S = <\delta B^2>/v_A(1 + V_{SW}/v_A)^2/B$.

We have also $k_{res} = -\Omega_p(v_T - v_{ph} - V_{SW})$ where $v_{ph}$ is phase speed in the solar wind frame. For the non-dispersive case we take $v_{ph} = v_A$. For the dispersive case we use the last line in Section 2.

4. Kolmogorov Dissipation. We use an approach introduced by Hollweg (1986), Hollweg and Johnson (1988), and Isenberg (1990). Most power is at large spatial scales. A Kolmogorov cascade is postulated, transferring energy to the small resonant scales, where it is absorbed. The rate of energy transfer is taken to be $\delta = \rho <\delta V^2>_3/2L_{corr}$ where $\rho$ is mass density, and $L_{corr}$ is the correlation scale which scales as the distance between field lines: $L_{corr} \approx 1/\sqrt{r}$.

We evolve $S(t)$ according to the (Isenberg, 1984) $dS/dt = -4\pi V_{SW}D/B$. The energy $D$ goes into resonant heating, and into the work done by the resonant acceleration. The level $L$ of the power spectrum is adjusted so that the resonant terms always add up to $D$.

The ratio of work done to perpendicular heating is easily shown to be $k_{res} v_{ph} / \Omega_p$. Since $k_{res}$ is larger in the dispersive case, resonant acceleration will then be more important. As it stands, our model exaggerates this effect, since $k_{res} \to 0$ when $v_1 = v_2$. But the distribution function contains more than just our two particles, so there will always be some spread even when $v_1 = v_2$. As a crude attempt to remedy this, we replace $v_v$ in the last line of Section 2 with $v_v \to Max(30km/s, |v_T - V_{SW}|)$.

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Fig. 3. The transverse speeds $v_\perp(t)$ of the protons vs. heliocentric distance $r(t)$. In the dispersive case, relatively more energy goes into work and less goes into heat. The extended plateau for the dispersive case agrees with the UVCS data.

We consider first the dispersive case. Both protons start at $R = 1.5$, with $v_1 = 150$ km s$^{-1}$ and $v_2 = 250$ km s$^{-1}$ at $t = 0$. At $t = 0$, $n_e = 1.19 \times 10^6$ cm$^{-3}$. For both particles we take (m $v_\perp^2$)/2$k = 10^6$ K at $t = 0$. 

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The initial wave level is specified by taking \( \langle \delta V^2 \rangle^{1/2} \) = 75 km s\(^{-1}\). If this were extrapolated back to the Sun with no dissipation, the flux density at the Sun would be \( 3 \times 10^5 \) erg cm\(^{-2}\) s\(^{-1}\). \( L_{corr} \) = 7600 km at the Sun, and \( \gamma = 2 \).

The dark curves in Figure 3 display \( v_{\perp}(t) \) vs. \( r(t) \) for the dispersive case. The tendency of \( v_{\perp} \) to have an extended plateau near 250 km s\(^{-1}\) agrees with the proton data reported by Kohl et al. (1998). The dispersive curve in Figure 4 displays \( v_T(t) \) vs. \( r(t) \). The rapid acceleration close to the Sun is in fact in reasonable accord with the steep density decline. The mirror force is still dominant, but the resonant acceleration plays an essential role. The importance of the resonant acceleration is due in part to the large values of \( k_{res} \).

Look now at the non-dispersive case. Since \( k_{res} \) is smaller, the work done by the acceleration is less important. The gray curves in Figure 3 show that this case leads to a higher perpendicular energy than the dispersive case. The two cases yield nearly the same velocity profile; see Figure 4. In the non-dispersive case, the perpendicular heating is strong giving a large mirror force, while the resonant acceleration is weak. The dispersive case has less direct heating and a smaller mirror force, but the deficit is balanced by an enhanced resonant acceleration. The flow is faster in the dispersive case in spite of having a smaller mirror force.

Thus measurements of the coronal proton temperature do not tell the whole story. For example, the UVCS data (Kohl et al., 1998) suggest that the perpendicular proton temperature is fairly flat between 2 \( r_S \) and 4 \( r_S \), with \( T_{\perp} = 3.8 \times 10^6 \) K, corresponding to \( v_{\perp} = 250 \) km s\(^{-1}\) in Figure 3. By itself, this temperature is somewhat too low to produce the rapid acceleration of the wind close to the Sun. However, if the waves are dispersive, the resonant acceleration can compensate for the low temperature, and yield an acceleration which is consistent with the data.

5. Heavy Ions. In this section we will focus on \( O^{+5} \) ions, which are the best observed by UVCS. The principles apply to other heavy ions as well, with the probable exception of \( He^{++} \), which because of its abundance significantly affects the dispersion relation, as well as the momentum and energy balance. As discussed by Hollweg (1999a,b,c), \( O^{+5} \) has several advantages over the protons. Both forward- and backward-going \( O^{+5} \) particles can be in resonance, whereas only backward-going protons can resonate. For positive \( v_{\perp} \), \( O^{+5} \) can resonate simultaneously with two sets of waves. \( O^{+5} \) resonates with waves having lower-frequency than do the protons; the lower-frequency waves propagate faster and presumably have more power.

![Fig. 4.](image)

Fig. 4. The radial flow speeds \( v_T(t) \) vs. \( r(t) \). Even though the mirror force is smaller in the dispersive case, the flow is faster because the resonant acceleration plays a greater role.

![Fig. 5.](image)

Fig. 5. Velocity space in the solar wind frame. Particles diffuse on the indicated circles, which are centered on the phase speeds of the waves with which the particles are in resonance. The dark arcs indicate where the particles are able to satisfy the resonance condition. The ability of \( O^{+5} \) to reach larger values of \( v_{\perp} \) than the protons implies more than mass-proportional heating. If the protons were originally distributed in the boxes near the origin, they would be cooled in the direction parallel to \( B \).

If we ignore dispersion for the moment, a particle conserves its energy in the wave frame. In that frame, a distribution of particles will diffuse in pitch-angle. This is illustrated in Figure 5, which shows how particles move in velocity space measured in the proton
frame. If there is energy conservation in the wave frame, the particle motions will be restricted to circles centered on the phase speeds of the waves. The circles for $0^+5$ have larger radii than the circles for protons, because $0^+5$ resonates with faster waves (taking the protons to resonate with waves moving at 0.4vA) is illustrative only). A group of particles starting with small v\perp and the same values of v\parallel will diffuse along the circles as indicated. But they will only diffuse along the dark arcs; otherwise they will be out of resonance. The diffusion to larger values of v\perp implies heating perpendicular to B. The fact that $0^+5$ is able to reach larger values of v\perp than the protons implies more than mass-proportional perpendicular heating, as is in fact observed by UVCS. The particles also move to the right; thus the cyclotron resonance accelerates as well as heats.

Consider again a power-law power spectrum with a low-k\perp cutoff. We now assume that the total power obeys the relation for undamped WKB Alfvén waves: 
\[\frac{\Delta B^2}{\Delta B^2} \propto n_e^{1/2} (1 + v_{SW}/v_{A})^{-2}\]
We take v\perp and v\parallel (the phase speed of the waves with which the particles are in resonance) to be dominant velocities. (This fails for test protons as v\perp \rightarrow v_{SW}.) We then find
\[\frac{1}{m} \left(\frac{dT}{dt}\right)_{\text{res}} \propto (q/m)^{2-\gamma} v_{\parallel}^{\gamma+1} B^{1-2\gamma} n_e^{\gamma/2}\]  
(3)
where $T_\perp = m v_\perp^2/2k$ measures particle energy, not the temperature of a distribution. The factor $(q/m)^{2-\gamma}$ by itself means that more than mass-proportional heating requires $\gamma > 2$. This can be overridden by the fact that $v_{\parallel}$ is larger for heavy ions than for protons; if $\gamma = 2$, the dependence is $v_{\parallel}^3$, which is significant. Thus dispersion plays an important role in the mass and charge dependence of the heating rate.

For $0^+5$ and other heavy minor ions we have $v_{\parallel} \approx v_A$. The spatial dependence of the heating rate is then:
\[\frac{1}{m} \left(\frac{dT}{dt}\right)_{\text{res}} \propto B^{2-\gamma} n_e^{-1/2}\]  
(4)
Recall now the UVCS observation showing that for $0^+5$ $T_\perp$ still increases with r out to the limit of observation in spite of the presence of the adiabatic cooling term which itself increases in proportion to $T_\perp$. In our numerical computations, we found that this is a difficult observation to reproduce. The difficulty is in part due to the factor $B^{2-\gamma}$ in (4), which decreases with increasing r if $\gamma < 2$. In our particle trajectory computations, we have found that the $0^+5$ temperature will monotonically increase with r in the observed range only if $\gamma > 2$.

An example is shown in Figure 6. We follow trajectories of $0^+5$ and resonant protons in an undamped wave field with a power-law power spectrum. Here to facilitate comparison with the UVCS data we plot $v_\perp/L_e = (2kT_\perp/m)^{1/2}$. Both $0^+5$ and protons are heated extremely rapidly after they are released at t = 0. (The particles are started where we estimate that collisionless behavior begins.) The $0^+5$ attains more than mass-proportional $T_\perp$. But in the top panel, with $\gamma = 5/3$, $v_\perp/L_e$ for $0^+5$ flattens out for $R > 2$. This stands in contrast to the observations, indicated by the gray shading in the lower panel. If we take $\gamma = 2$, in the lower panel, we find that the $0^+5$ temperature rises within the range spanned by the data, in agreement with UVCS. At the same time the protons are hotter than the data indicate, by a factor of about 2. This may be a reflection of the fact that we are only considering a proton which is in resonance; as already emphasized, this may represent only about half of the protons.

For further details, see Hollweg (1999c).

6. Summary. The results from UVCS indicate that heating in coronal holes acts mainly on the transverse degrees of freedom of protons and heavy ions. This suggests that the heating (and acceleration) is occurring
via the ion-cyclotron resonance. We noted that only half the protons are able to resonate. Proton distribution functions are therefore not expected to be even close to bi-Maxwellians (Isenberg et al., 1999). In this paper we employed an alternate closure scheme. We considered two protons, each being a proxy for the resonant half or the non-resonant half of the distribution function. The two protons trade roles, alternately being resonant or non-resonant. The perpendicular energies increase due to the resonance, while at the same time the parallel velocity spread decreases.

Different results are obtained when the waves are dispersive or non-dispersive. \( \kappa_{\text{res}} \) becomes much larger in the dispersive case. In a specific model, we assumed that the overall wave dissipation was determined by a turbulent cascade. The resonant interaction determines how the energy is apportioned between perpendicular heating and work done by the resonant acceleration. This model was in good agreement with coronal observations of perpendicular temperature and solar wind flow speed. We also found that the larger values of \( \kappa_{\text{res}} \) in the dispersive case mean that less energy goes into work and less into heat, when compared with the non-dispersive case. Even though the perpendicular energy (corresponding to perpendicular temperature), and also the mirror force, were less in the dispersive case, the flow speed was faster. In fact, the dispersive model, being cooler but yet faster, is in better agreement with the observations than the non-dispersive model. It is not sufficient to simply look at the proton temperature.

A potentially serious issue is that it is not clear whether a turbulent cascade will proceed in this fashion. Based on MHD simulations of turbulence, the cascades seem to produce large \( k_L \) rather than the larger \( k_L \) necessary for the ion-cyclotron resonance as we have described it. This is a major issue which still needs to be resolved. At present it is fair to say that the source of high-frequency ion-cyclotron waves is unknown.

We also considered heavy minor ions such as O\(^+\). The model here was cruder: we did not allow for wave dissipation and we considered only the resonant protons (which required that \( V_{SW} \) be assumed at the outset). We found that wave dispersion is also important in giving the more than mass-proportional heating observed for O\(^+\). We also found that the positive slope of \( T_L \) for O\(^+\) out to \( R = 3.5 \) could only be reproduced by assuming a steep power spectrum (in agreement with Cranmer et al., 1999).

In future work it will be necessary to carefully include the effects of He\(^+\), both on the dispersion relation and on the overall energetics. It will also be necessary to consider the effects of the parallel proton temperature on the dispersion relation near \( \omega = \Omega_p \).

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References


