MHD Waves in Open Magnetic Structures

V. M. Nakariakov and B. Roberts

School of Mathematical and Computational Sciences, University of St Andrews, St Andrews, Fife KY16 9SS, Scotland

K. Murawski

WIBiS, Politechnic of Lublin, ul. Nadbystrzycka 40, 20-618 Lublin, Poland

Abstract. The dynamics of Alfvén and magnetoacoustic waves of finite amplitude in open magnetic structures is considered. A transversal inhomogeneity in the magnetic structure in the density and/or in a field-aligned steady flow gives rise to Alfvén wave phase mixing, which, in the almost collisionless and dissipationless plasma of the corona, nonlinearly generates obliquely propagating fast magnetoacoustic waves. The generated fast waves are more effectively damped than the Alfvén waves and so the process of heating by phase mixing is enhanced. This "indirect" heating leads to the spreading of the heated plasma across the magnetic field. We point out also that an inhomogeneous flow gives rise to the secular generation of longitudinal motions through Alfvén wave phase mixing, accelerating the plasma along the field.

1. Introduction

Magnetohydrodynamic (MHD) waves are believed to be primary candidates for heating of open magnetic structures, such as holes and streamers, in the solar corona, and for acceleration of the solar wind. The presence of MHD wave activity in the coronal holes is shown by: the recent discovery of upwardly propagating compressible waves in coronal plumes (DeForest & Gurman 1998), found in SOHO EIT observations of the lower corona (< 1.4R☉), which can most probably be interpreted as slow magnetoacoustic waves; the interpretation in terms of MHD waves of apparent non-thermal broadening of ion emission lines observed by SOHO UVCS at about 1.7R☉ (Ofman & Davila 1997); and by in situ observations of Alfvén waves in the solar wind (Balogh et al. 1995). Moreover, the observations show that MHD waves can have sufficiently strong amplitudes for nonlinear effects to come into play. Consequently, our understanding of heating and acceleration processes in coronal holes requires the construction of a theory of weakly nonlinear MHD waves.

The presence of bright plumes, readily seen in extreme ultraviolet images of the solar corona, demonstrates that coronal holes are highly inhomogeneous in the direction transversal to the magnetic field. Transversal structuring dramatically affects MHD wave dynamics, leading to magnetoacoustic wave guiding
accompanied by appearance of dispersion (e.g. Roberts 1991) and Alfvén wave phase mixing (Heyvaerts & Priest 1983). Phase mixing has been suggested as an effective mechanism for heating of inhomogeneous plasmas. However, even enhanced heating by phase mixing seems to be insufficient to heat the coronal hole plasma close to the Sun, because the coefficients of viscosity and resistivity are extremely small (Ruderman, Nakariakov & Roberts 1998), although this difficulty can be removed by assuming that the coefficients are strongly increased by micro-turbulence. Anyway, application of the mechanism does not answer several other questions, such as how do MHD waves accelerate the solar wind and how does heating spread across the magnetic field?

Our aim here is to introduce an alternative physical mechanism for the heating of open, transversally inhomogeneous magnetic structures, namely “indirect” heating by Alfvén wave phase mixing, based on generation of obliquely propagating fast magnetoacoustic waves by phase mixing.

2. The Model

Our model consists the following:

- The plasma is assumed to be cold (plasma-β = 0), this assumption is quite realistic for the corona, although it does not allow us to consider slow magnetoacoustic waves.

- Perturbations of the plasma are magnetohydrodynamical (ω ≪ ω_B, ω_p, where ω_B,ω_p are the ion gyrofrequency and Langmuir frequency, respectively), satisfying

\[
\rho \left[ \frac{\partial \mathbf{V}}{\partial t} + (\mathbf{V} \nabla) \mathbf{V} \right] = -\frac{1}{4\pi} \mathbf{B} \times \nabla \mathbf{B},
\quad \nabla \cdot \mathbf{B} = 0,
\]

\[
\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{V} \times \mathbf{B}),
\]

\[
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{V}) = 0,
\]

all notations are standard.

- The amplitudes of the perturbations are small but finite, allowing us to apply a weakly nonlinear approximation.

- The plasma is homogeneous in the y- and z-directions.

- The applied magnetic field B_0 \parallel z is straight and homogeneous.

- There are smooth inhomogeneities in the plasma density ρ_0(x) and the steady flow speed U_0(x) of the plasma. The steady flow speed is much less than the Alfvén speed C_A(x) and, consequently, the structure considered is stable with respect to the Kelvin–Helmholtz instability and to negative energy wave instabilities.

- 2.5D consideration; we do not consider variations of physical values in the y-direction, but perturbations V_y and B_y are non-zero.
3. Governing Equations

According to Nakariakov, Roberts & Murawski (1997, 1998), the quadratic nonlinear dynamics of Alfvén and fast waves in our model is described by the pair of wave equations,

$$
\left( \frac{\partial}{\partial t} + U_0(x) \frac{\partial}{\partial z} \right)^2 V_y - C_A^2(x) \frac{\partial^2}{\partial z^2} V_y = \mathcal{N}_{AX}[V_y B_z],
$$

$$
\left( \frac{\partial}{\partial t} + U_0(x) \frac{\partial}{\partial z} \right)^2 B_x - C_A^2(x) \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} \right) B_x = \mathcal{N}_{F, AX}[B_x^2] + \mathcal{N}_{AA}[V_y^2],
$$

where $\mathcal{N}_{AX, F, AX, AA}$ are linear differential operators applied to quadratical forms.

3.1. Linear Limit

In the linear limit, when the righthand sides of equations (2) and (3) are zero, (2) describes linearly polarised Alfvén waves (which perturb variables $V_y$ and $B_y$) and (3) describes fast waves (perturbing variables $V_x$, $B_x$, $B_z$, $\rho$ and $V_z$).

When $\mathcal{N}_{AX} = 0$, equation (2) possesses the solution

$$
V_y = \Psi_L(x) f \left[ z - (U_0(x) - C_A(x)) t \right] + \Psi_R(x) g \left[ z - (U_0(x) + C_A(x)) t \right],
$$

where $\Psi_L$, $\Psi_R$, $f$ and $g$ are arbitrary functions. In the absence of a steady flow, the terms on the right hand side of (4) correspond to waves propagating in the negative and positive directions of $z$, respectively. In particular, solution (4) describes Alfvén wave phase mixing, the secular generation of transversal gradients in the Alfvén waves. During the linear development of phase mixing, the longitudinal wavelength of the Alfvén waves remains the same while the transversal wavelength tends to zero. When viscous effects come into play, phase mixing leads to enhanced dissipation of the wave, as $\sim \exp(-t^3)$.

The linear form of equation (3) shows refraction of fast waves and thus the possibility for their trapping by the inhomogeneities (see Nakariakov et al. 1998).

3.2. Quadratic Nonlinear Limit

According to the structure of the nonlinear terms (2) and (3), if Alfvén waves are initially absent from the system ($V_y(t=0) = 0$), they are not generated by fast waves. In this case, weakly nonlinear dynamics of fast waves is described by the self-consistent weakly-nonlinear wave equation (3) with $\mathcal{N}_{AA} = 0$. In particular, self-interaction of fast waves and coupling of various modes of the inhomogeneity occurs (Nakariakov & Oraevsky 1995; Nakariakov, Roberts & Petrukhin 1997).

Here we consider the opposite limiting case, when fast waves are initially absent from the system ($B_x(t=0) = 0$). Then the righthand side of (2) is still zero, and Alfvén waves evolve according to the linear solution (4). The development of phase mixing generates transversal gradients in the Alfvén waves which act as sources for fast waves (through the righthand side of (3)). The righthand side of (3) has the form

$$
RHS(3) = tA^2 \frac{d}{dx} [U_0(x) + C_A(x)] \frac{d}{dz} \left( g \frac{dg}{dz} \right),
$$

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Figure 1. The kinetic energy of the fast wave as a function of time. The lowest curve corresponds to the initial Alfvén pulse with the amplitude $0.1 \, C_A$. The upper curves correspond to $0.2 \, C_A$, $0.3 \, C_A$, and $0.4 \, C_A$, respectively. The kinetic energy is expressed in units $\rho_0 C_A^2 a^2/2$ and the time in the Alfvén transit time, $a/C_A$. (From Nakariakov et al. 1998)

where $g(z)$ describes the initial shape of the Alfvén wave. Consequently, the source term is secularly growing, leading to efficient generation of fast magnetoacoustic waves. Results of 2.5D full MHD numerical simulations shows the growth of the generated fast wave energy (Figure 1). From the point of view of nonlinear wave theory, this effect is the nonlinear generation of the second harmonics, dramatically enhanced by phase mixing. The generated fast waves propagate obliquely with respect to the magnetic field and are subject to much more efficient dissipation (e.g., in coronal conditions, due to Landau damping (Wentzel 1989)). The effect takes place whenever there is a transversal inhomogeneity in either the Alfvén speed $C_A(x)$ or the steady flow $U_0(x)$, or both. Secular growth occurs for perturbations of $V_x$, $\rho$, $B_z$ and $B_x$. Moreover, in the presence of a flow inhomogeneity, phase mixing generates secularly growing longitudinal motions,

$$ V_x = \left( \frac{\partial}{\partial t} + U_0(x) \frac{\partial}{\partial z} \right)^{-1} \frac{dU_0}{dx} V_x, \quad (6) $$

which are especially interesting for the acceleration of the solar wind. This subject will be developed elsewhere.
4. Conclusion

We summarise our results as follows:

1. Transversal structuring of open coronal structures in the solar atmosphere (e.g. the presence of plumes in coronal holes) plays a crucial role in MHD wave dynamics.

2. We propose the mechanism of "indirect" heating of the plasma by Alfvén wave phase mixing as a promising mechanism for the heating of coronal holes. This mechanism is based on the nonlinear generation of oblique propagating fast magnetosonic waves, which under coronal conditions are subject to more effective dissipation than Alfvén waves.

3. Indirect heating results in the spreading of heating of the plasma across the magnetic field. The extent of the heated region is defined by the dissipation length of the fast wave.

4. In the presence of an inhomogeneous steady flow, Alfvén wave phase mixing generates secularly growing longitudinal motions of the plasma, providing a potential mechanism for solar wind acceleration.

5. Even very slow inhomogeneous flows, with speeds well below the thresholds of the Kelvin–Helmholtz instability and negative energy wave instabilities, may lead to secular (instability-like) excitation of magnetosonic waves. This is important for the construction of a theory of magnetoacoustic turbulence in magnetic structures with inhomogeneous flows.

References