Implementation of a Filter for the Restoration of Solar Granulation Images

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Abstract. Statistical analysis of brightness fluctuations in solar granulation at different positions on the solar disk is seen as a powerful technique to model the variation of the temperature with depth in the photosphere. Nevertheless, prior to any measurement, an unavoidable task is to correct the observed images for the degradation induced by the terrestrial atmosphere and the telescope. For this purpose we describe here a restoration filter based on estimates of the PSF from the lunar limb in solar eclipse observations. Independently of the procedure for estimating the PSF, the central point of our presentation is the description of a novel method for constructing a noise filter for images of solar granulation.

1. Introduction

Several diagnostic tools are routinely used to model the variation with the optical depth, τ, or the geometrical height, z, of the physical variables related to photospheric structures. Thus, the analysis of the center-to-limb variation of different spectral lines allows ΔT(z) and ν(z) to be determined. The study of the dependence of the continuum intensity fluctuations, ΔI, with wavelength is another way to infer the temperature as a function of z. ΔT(z) can also be derived from measurements of ΔI at different heliocentric positions μ = cosθ. We are concerned here with the latter technique; namely, determinations of power spectra and normalized contrast, ΔIrms, of the intensity fluctuations in the quiet granulation as a function of μ.

Granulation contrast is severely attenuated by the degradation caused by atmospheric turbulence and telescope instrumental profile and aberrations. Several procedures have been developed to compensate for these effects (see Bonet, 1998 for a recent review). Partial solar eclipses offer an opportunity to get an estimate of the point spread function (PSF) representing the degradation produced by the telescope and the atmosphere. In this paper we will concentrate on the implementation of a filter based on an estimate of the PSF from partial solar eclipse observations, to restore images of granulation at different positions on the solar disk. Special emphasis will be put on the procedure for the construction of the noise filter to be applied prior to the restoration itself. The discussion on
the determinations of power spectra and contrasts, as well as other statistical considerations is postponed to a forthcoming paper.

2. Data Material

The material analyzed here was obtained at the Swedish Vacuum Solar Telescope (SVST, Observatorio del Roque de los Muchachos, La Palma; see Scharmer et al. (1985) during the partial solar eclipse of 1994 May 10.

A set of CCD images showing quiet granulation fields at different \( \mu \)-positions on the solar disk, partially covered by the Moon’s limb, was recorded at \( \lambda 6708 \pm 20 \text{Å} \) with an exposure time of 13 ms and an image scale of \( 0.1124/\text{pixel} \). From this set only the best images were selected for pre-treatment (flat-fielding and dark current subtraction) and further processing.

3. Image Reconstruction

3.1. Presentation of the problem

The image of an incoherent object \( i_o(x, y) \), formed by an optical system, will be recorded as

\[
i(x, y) = i_o(x, y) * s(x, y) + n(x, y),
\]

where \( * \) represents spatial convolution, \( n(x, y) \) the noise assumed uncorrelated with the signal, and \( s(x, y) \) the PSF characterizing the optical system composed in the present case by the atmosphere and the telescope.

The recovery of the object (i.e. the “true” intensity distribution in solar granulation) from the observed image, can be easier performed in the Fourier transform domain since there convolutions turn into products. Thus the suitable formula to do that is

\[
\hat{i_o}(x, y) = \mathcal{F}^{-1}\left[ \frac{I(u_x, u_y) \cdot \Phi(u_x, u_y)}{S(u_x, u_y)} \right],
\]

where \( \mathcal{F}^{-1} \) stands for the inverse Fourier transform; \( I(u_x, u_y) \) and \( S(u_x, u_y) \) are the Fourier transforms of \( i(x, y) \) and \( s(x, y) \), respectively; \( S(u_x, u_y) \) is also called optical transfer function (OTF) of the imaging system. \( \hat{i_o}(x, y) \) is an estimate of the real object. \( \Phi(u_x, u_y) \) in (2) require that, prior to performing the image restoration, i.e. the quotient \( I(u_x, u_y)/S(u_x, u_y) \), noise filtering in the recorded image is essential, otherwise the noise contribution would be amplified and have a disastrous effect, mainly on the high frequency range where the signal-to-noise ratio is poorer.

The lunar limb crossing the solar disk has been taken as a reference object to obtain an estimate of the OTF assuming isotropic degradation (see e.g. Bonet et al. 1995). We shall denote this function by \( M(\nu) \), where \( \nu = u_x^2 + u_y^2 \). For this purpose we have measured the mean photometric profile of the transition Moon–solar photosphere all along the lunar limb included in each particular frame and compared the result with the “known true photometric profile”, i.e.
the Heaviside step function. The average of the above-mentioned profiles implies that isoplanatism is assumed.

In short, in our image treatment the following approximations are implicit: a) uncorrelated signal and noise, b) isoplanatism, and c) isotropic degradation. The first assumption is based on the fact that we expect the noise to be predominantly photon noise and that the solar granulation is a low-contrast structure. Isoplanatism could be partially justified by the uniform quality observed in our selected frames. Finally, we are compelled to assume isotropic degradation, i.e. azimuthal symmetry in $s(x, y)$ because our reference object (the lunar limb) is substantially a 1D structure. We are aware of the weight of the last two assumptions. They are essentially equivalent to estimating a sort of average degradation on each frame. Given the excellent quality of our images, this would mean, to our understanding, that the restoration process should not introduce substantial bias in the statistical analysis that we intend to do taking advantage of a singular phenomenon offered by nature (viz., the eclipse) and of the low computational cost of the method as compared with other more recent reconstruction techniques.

3.2. Noise filtering

Noise filtering is performed by weighting the spectral components of the image at various frequencies in some optimum way. Thus, our filter $\Phi(u_x, u_y)$ used in (2) is based on the so-called optimum filter described by Brault & White (1971), and is derived from the observed images themselves. This filter is constructed by imposing the reasonable condition that, in the measuring domain, the deviation of the image smeared by the PSF and not affected by noise (i.e. $i_0 * s$), from the image which results from the filtering, is a minimum in the root-mean-square error sense.

Assuming the noise to be uncorrelated with the signal, this optimum filter can be formulated as

$$\Phi(u_x, u_y) = \frac{|I(u_x, u_y)|^2 - |N(u_x, u_y)|^2}{|I(u_x, u_y)|^2}, \quad (3)$$

where $|I(u_x, u_y)|^2$ stands for the power spectrum of the observed image, and $|N(u_x, u_y)|^2$ for that of the noise. In the practical realization of $\Phi(u_x, u_y)$ it is usually adequate to replace both the signal and the noise power spectra by smooth fitted models.

In the present study the noise has been modeled as a constant power ("white") noise. White noise is typical for the statistical fluctuations in photon flux, which are the most prominent noise source in solar images taken with a good CCD camera. However the construction of a smooth model that fits the observed power spectrum well is much trickier. An accurate fit is increasingly important as the power level of the signal approaches the power level of noise in the high spatial frequency range.

In the paper by Brault & White (1971) are discussed simple functional forms to be fitted to the observed power spectra of 1D signals (spectral line profiles), depending of their peculiarities. In image restoration, the problem is much more troublesome. To the difficulty of dealing with 2D data is added the fact that the shape of the power spectra significantly changes depending on the nature of
the structures in the considered image, and consequently the choice of suitable functional forms to be fitted in each particular case is a very difficult task.

What saves us is the nature of the solar granulation. Brightness fluctuations due to granulation at the center of the solar disk can be considered as a statistically isotropic phenomenon and hence, the corresponding power spectrum is expected to be azimuthally symmetric. Away from the disk center, granular structures become progressively narrower in the direction perpendicular to the solar limb as one moves toward the limb. This effect produces power spectra closely approximating a bell with elliptical horizontal sections, which simplifies the fitting process. Since the shape (the ellipticity of the sections) of the power spectrum changes with the position in the solar disk, the noise filter has to be specific for each $\mu$-position and this compelled us to deal with small boxes for constructing the filter as well as for performing the restoration process. The size of the working boxes has been fixed to $100 \times 100$ pixels ($12''4 \times 12''4$).

Several attempts to fit 2D splines and Gaussian elliptical bells to the power spectra have failed, on the one hand due to the oscillations obtained in the spline fit (very dependent on the specific node positions), and on the other hand because the power bells have elliptical sections with variable ellipticity and orientation at different power levels so that they do not match Gaussian bells well. Consequently, we have chosen the fitting procedure described below.

To obtain the power spectrum of the observed signal at a given $\mu$-position on the solar disk, an average of power spectra is performed—the resulting mean power spectrum will hereafter be denoted by $\text{PS}$. To this end, several boxes placed within a range $\mu \pm \delta \mu$ ($\delta \mu = 0.05$ for $\mu \geq 0.75$ and $\delta \mu = 0.1$ for $\mu < 0.75$), are taken from the best-quality images that contain such a range of positions. However, for a given $\mu$, images obtained at different solar latitude and longitude are treated separately to prevent the possibility of the properties of the granulation differing as a function of the coordinates on the solar disk. Following Cooley et al. (1970), the boxes are overlapped by 50%. In that way, the number of power spectra included in the average is substantially increased thus reducing the "noise" in $\text{PS}$ stemming from the statistical differences between single boxes.

To fit a smooth model to $\text{PS}$, isocontours at various power levels are obtained and ellipses are fitted to them. The power levels have been chosen to have an isocontour representation as dense as possible, only being limited by the overlapping of the ellipse sampled points due to the small box size. No a priori assumptions about ellipticity or orientation of the fitted ellipses are made. The fitting method consist in taking many sets (typically 100000 for the outer iso-contours, and 700 for the inner ones) of three well separated points in an isocontour. Each set of three points defines an ellipse centered on the origin of frequencies. The parameters describing each of these ellipses are represented in the parameter space $A$-$B$-$\Theta$, where $A$ $\equiv$ major axis, $B$ $\equiv$ minor axis and $\Theta$ $\equiv$ ellipse orientation. The coordinates of the centroid $(A_c, B_c, \Theta_c)$ of the resulting cloud of points, defines the ellipse that is chosen as the best fit to the isocontour. This process has been applied to every isocontour obtaining a series of fitted ellipses at different power levels, all of them centered at the origin of spatial frequencies. Representation of these ellipses in the $12''4 \times 12''4$ box—i.e. sampling their analytical expressions—gives the ellipse data points describing $\text{PS}$ at those power levels where isocontouring has been performed.
The overall slope of the power spectrum flattens in the highfrequency range and the peaks due to the remaining statistical uncertainties cause the isocontours in this region to be disconnected so that an elliptical shape cannot be recognized. Thus, an information gap concerning the PS’s shape is found between the lowest isocontour elliptically shaped and the noise level, where the horizontal section of PS is a circumference of radius equal to the telescope cut-off spatial frequency. We assume an exponential decay of PS in this power range, described in every radial direction by the expression:

\[ p = a \cdot e^{-b \nu} + c, \]  

(4)

where \( \nu \) is the spatial frequency, \( a \) and \( b \) are determined by forcing the decay to pass through the lowest contour and the cut-off, and \( c \) is the free parameter that rules the slope of the decay. The ensemble of data points compounded by the sampling of the ellipses and of the exponential decay, form an irregular grid describing the entire PS. To interpolate these data values a Delunay triangulation of the set of data points has been performed to obtain a triangular grid structure (cf. Dahlquist & Björck, 1974). Then, linear interpolation has been applied to obtain a regular grid of interpolated surface values. Interpolated values are only computed from nearby points since the circumcircle of any triangle of a Delunay triangulation has the property of containing no other vertices in its interior. A logarithmic scale has been used to perform interpolation of the surface values in order to obtain a much realistic steeper decay between isocontours.

Fig. 1a shows typical contours of PS at \( \mu = 0.5 \) and their fitted ellipses. The corresponding global smooth model fitting PS is presented in Fig. 1b.

The constant power level of the white noise is computed by averaging the values of PS at frequencies higher than the telescope cutoff. Substituting \( |I(u_x, u_y)|^2 \) in (3) by the fit to PS described above and \( |N(u_x, u_y)|^2 \) by the fit to the noise, the optimum noise filter is obtained. Fig.1c shows a typical example of a noise filter corresponding to the case also represented in Figs.1a,b.

### 3.3. Image restoration

According to expression (2) the quotient of \( \Phi(u_x, u_y) \) and the OTF—\( M(\nu) \) in our case—rules the restoration process and will be termed the “restoration filter”. Hence, the construction of the restoration filter has been accomplished by dividing the noise filter, calculated in the preceding section, by the specific \( M(\nu) \) derived from each particular image to be restored. Note that the optimum noise filter is constructed by averaging power spectra from different frames, affected by different \( M(\nu) \)s, whereas in the restoration filter the specific \( M(\nu) \) for each frame is used. From numerical simulations of observed spectra, based on real measurements of \( M(\nu) \), we conclude that the resulting restoration filters are mainly sensitive to changes of the PS’s shape in the high-frequency range (\( \nu > 2 \) arcsec\(^{-1} \)). However, this sensitivity is only noticeable when the \( M(\nu) \) values are very close to zero (poor image quality) or when they differ significantly (ample spread of image qualities) in this frequency range. Accordingly, we have selected for restoration only those images with good and similar quality in this frequency range by analyzing and comparing the corresponding \( M(\nu) \) functions. The averaging of power spectra favors the decrease of the above-mentioned information gap in the power range close to the noise level. The more power spectra are
averaged the larger is the statistical stability in PS and consequently the closer are the isocontours that can be determined to the noise level.

A compromise between image quality and the minimum number of boxes to be considered has been reached to obtain PS. The boxsize determines the number of boxes included in a given $\mu$-range and consequently, this parameter has also to be considered to define the strategy for calculating PS. Based on the stability of parameters such as PS shape, centroid of the 1D azimuthally integrated spectrum and total integrated power, the final decision for the boxsize is 100×100 pixels (12''.4×12''.4).

It is important to note that this filter is based on the information contained in the PS of granulation and that the only free parameter is $c$, which determines the exponential decay from the lowest isocontour elliptically shaped to the noise level. This affects only the highest frequency part of the filter, beyond the maximum of restoration, so most of the restoration filter is completely data-based. The normalized rms contrast of the restored field (i.e. the integrated power) is only slightly sensitive to changes in $c$ because the largest contribution to the power comes from the region of large and medium spatial structures.

As can be seen in Fig.1d, the restoration filters are non-symmetric at positions other than the disk center due to the asymmetry in PS.

Once the restoration filter is ready for a given $\mu$-range and a particular frame, individual boxes of 12''.4×12''.4 included in this range and frame are restored following expression (2). In this formula $I(u_x, u_y)$ represents the Fourier transform of a particular box. A cosine apodization has been applied to the extreme 10% pixels to avoid oscillatory behavior at the borders of the restored boxes.

The efficiency of the restoration filter that we use is evidenced in Fig. 2 where pairs of non-restored and restored granular fields for different $\mu$-positions on the solar disk are shown for comparison.

4. CONCLUSIONS

Partial solar eclipses provide an opportunity to perform an estimate of the PSF representing the degradation originating in the terrestrial atmosphere and the telescope. Here we describe the sequential steps for constructing a restoration filter. Apart from the procedure followed for the PSF determination, we put special emphasis on the description of a novel method for the implementation of a noise filter based on the observed data themselves. This method is presently being applied to a statistical analysis of the center-to-limb variation of solar granulation that will be published elsewhere (Sánchez Cuberes et al., 1999a,b).

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Figure 1. a) PS isocontours and the corresponding fitted ellipses; b) smooth model of PS; c) noise filter, and d) restoration filter. All cases correspond to PS at $\mu = 0.5$. The abscissa values are expressed in pixels.
Figure 2. Pairs of reconstructed (top panel) and original images (bottom panel) at four \( \mu \)-positions on the solar disk. The boxes cover a field of 9\".92\times9\".92

References