On the Location of the Instability Strip

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1. Introduction

We investigate the stability properties of Cepheid models with masses $M$ satisfying $3M_\odot \leq M \leq 5M_\odot$, and we study the effect of metallicity and of the assumed mass-luminosity relation upon the location of the fundamental and first-overtone instability strip. The turbulent fluxes (heat and momentum) and their Lagrangian perturbations are obtained from a nonlocal, time-dependent, mixing-length formulation, and the radiative flux is treated in the Eddington approximation in both the equilibrium and the pulsation calculations. We further compare observed period ratios in double-mode Cepheids (DMC in the following) with model computations. Fairly accurate measurements of period ratios of fundamental mode/first overtone (F/1OT in the following) DMCs and of first overtone/second overtone (1OT/2OT) pulsators in the Large Magellanic Cloud (LMC) are provided by the MACHO collaboration (Alcock et al. 1995, 1998). The perhaps most consistent computations of period ratios in DMCs were reported by Baraffe et al. (1998), in which the stellar evolution and pulsation calculations were coupled. They neglected, however, in the equilibrium model the turbulent pressure and the pulsationally perturbed heat and momentum flux in the stability computations. Here we study models in which the turbulent pressure was consistently included in the equilibrium structure of envelope computations, and with the inclusion of the Lagrangian perturbation of the turbulent fluxes in the stability analyses, assuming, however, two mass-luminosity relations.

2. Computational details

The basic model calculations were essentially as described by Balmforth (1992), but with the addition of a representation of the acoustic emission from the turbulent eddies in the computation of the mean stratification. Models that are
located approximately in the middle of the instability strip exhibit very thin but efficient surface convection zones. As reported before by Baker & Gough (1979), this may lead to unbelievably large gradients in the turbulent pressure within the framework of a local formulation of convection. In the nonlocal mixing-length model used in this paper (see Balmforth 1992), the convective velocities and thus turbulent pressure are predicted to be smaller relative to a local formulation. However, the turbulent Mach number in these models still remains large, possibly as a consequence of the assumed Boussinesq approximation in the mixing-length formulation. This approximation neglects the acoustical radiation by turbulence, which drains energy from the turbulent velocity field. Thus its neglect may result in unrealistically large convective velocities, and consequently unrealistically large turbulent pressure gradients. The inclusion of a model of the acoustic emission by turbulence, in the manner of Houdek & Gough (1998), led to a reduction in the turbulent Mach number. Only for the most luminous stars did we still find very large gradients in the turbulent pressure, resulting in numerical stability problems for these models.

3. Results and discussion

We have computed model series with two mass-luminosity relations according to Chiosi (1990). They are defined as: \( \log(L/L_\odot) = A \log(M/M_\odot) + B \), with \( L \) denoting the model luminosity. In both mass-luminosity relations (ML1, ML2) we chose \( A = 3.52 \) and define \( B \) as:

ML1: \( B=0.7 \) ... for models with moderate overshooting,
ML2: \( B=0.5 \) ... standard models.

The stability coefficient, \( \eta = -\omega_i/\omega_r \) (\( \omega_i \) is the imaginary and \( \omega_r \) the real part of the complex eigenfrequency), as functions of the effective temperature \( T_{\text{eff}} \) across the instability strip, is shown in Fig. 1 for the two \( M - L \) relations ML1 (panel a) and ML2 (panel b). The stability coefficient for the three model masses \( M = 3, 4, 5 M_\odot \) is illustrated for the fundamental (\( \eta_1 \)) and first overtone (\( \eta_2 \)) modes computed for the metallicities \( Z = 0.01 \) and 0.005. The computations predict well-defined red edges for all model series. Decreasing \( T_{\text{eff}} \) even further leads to less stable modes, a feature which was also found by Baker & Gough (1979) and by Xiong et al. (1998). The whole strip of the first overtone mode (\( \eta_2 \)) is predicted to be located at higher temperatures relative to the fundamental mode (\( \eta_1 \)), again, a result which was already found by the aforementioned authors. The whole strip for both \( \eta_1 \) and \( \eta_2 \) is also shifted to higher temperatures with decreasing metallicities similar to the stability results in \( \delta \) Scti stars reported by Houdek (1997). Reducing the luminosity at constant model mass and metallicity results in smaller instability regions located at higher temperatures. A similar result was also found by Xiong et al. (1998) for Mira stars.

From evaluation of the work integral one may obtain the separate contributions to the stability coefficient \( \eta \) arising from the gas pressure fluctuations, \( \eta_g \), and from the perturbation of the momentum flux, \( \eta_t \), with \( \eta = \eta_g + \eta_t \) (e.g. Balmforth 1992). In Fig. 1c these contributions are illustrated for the \( 4 M_\odot \) Cepheid computed with the mass-luminosity relation ML2 and metallicity \( Z = 0.01 \), for the fundamental mode (\( n=1 \), top panel) and for the first-overtone mode (\( n=2 \), bottom panel). Interestingly, these results indicate that the effects
of turbulence remain significant well beyond the hotter boundary (blue edge) of the instability strip (despite the inefficient convection in these stars), somewhat extending it; even so, it is evident that the transition to stability at the blue edge is dominated by $\eta_g$. The return to stability at the red edge, however, is determined by both nonadiabatic effects $\eta_g$ and by dynamical effects, $\eta_k$.

In Fig. 2 we compare $F/1OT$ ($\Pi_1/\Pi_0$) and $1OT/2OT$ ($\Pi_2/\Pi_1$) period ratios for DMCs observed in the LMC with computed period ratios. The theoretical $F/1OT$ period ratios are displayed for models for which the fundamental and first-overtone modes are predicted to be unstable, and the $1OT/2OT$ period ratios are illustrated for models with unstable first- and second-overtone modes. The observations are from Alcock et al. (1995, 1998) and models are computed for $Z=0.01$ and 0.005. The two $M-L$ relations ML1 and ML2 and the model masses $M = 3, 4, 5 M_\odot$ are illustrated. It might be noted that the period ratios show a sharp decrease towards the red edge of the instability strip; this is not reflected in the observed period ratios, since the DMCs tend to be near the centre of the strip (see also Kolláth et al. 1998).

The dependence of the $F/1OT$ period ratios upon the $M-L$ relation increases with decreasing period $\Pi_0$. For models with $\Pi_0 \gtrsim 2.5$ d, the period ratios become almost independent of the assumed $M-L$ relation. The computed $1OT/2OT$ period ratios are in reasonable agreement with the observed values for models with $\Pi \gtrsim 0.9$ d. For smaller periods, however, the ratios predicted by the theory are too small. A possible reason for the discrepancy may be found in the inconsistent treatment between the evolutionary model parameters used in the equilibrium envelope model in terms of assumed $M-L$ relations and the stability analyses, as was discussed by Baraffe et al. (1998). The effect of the $M-L$ relation on the computed $1OT/2OT$ period ratios increases with period $\Pi_1$, as previously found by Christensen-Dalsgaard & Petersen (1995).
Figure 2. Comparison of period ratios of DMC in the LMC with theoretical models. Symbols mark data from Alcock et al. (1995, 1998).

We have also compared period ratios obtained from adiabatic and nonadiabatic stability analyses. For the F/1OT ratios the differences between adiabatically and nonadiabatically computed period ratios are found to be negligible. For the 1OT/2OT period ratios the differences are approximately in the same order of magnitude as the effect of changing metallicity. In general, adiabatically computed frequencies predict larger period ratios for the F/1OT modes and smaller ratios for the 1OT/2OT modes.

Support by the Particle Physics and Astronomy Research Council, and by the Danish National Research Foundation through its establishment of the Theoretical Astrophysics Center, is gratefully acknowledged.

References