Convection and the Eclipsing Binary AI Phoenicis: How Well Can We Constrain the Mixing-Length Parameter from Stellar Modeling?

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Abstract. For comparison with theoretical predictions we determined the mixing-length parameters for the components of the eclipsing binary AI Phe by fitting standard evolutionary models to them. We provide new estimates of their effective temperatures and helium content. Our fitting procedure takes advantage of constraints stemming from the light curve analysis of the system.

1. Introduction

Recent theoretical results indicate that the mixing-length parameter \( \alpha \) is not a constant but shows slight variations with stellar parameters (see Freytag et al., this volume). This investigation aims at a test of these findings. We determined \( \alpha \) empirically by fitting standard stellar models to an eclipsing binary system with accurately known parameters. Central questions were: (i) Are the \( \alpha \)-values of the binary components different from the solar one? (ii) Are they consistent with the theoretical predictions?

Three aspects led us to do the fitting by an automatic least-squares procedure. In first place the number of involved variables was so large that a fitting “by hand” appeared unpractical. Second, the quantitative nature of the questions demanded for reliable estimates of the uncertainties of the derived parameters. And finally the automatic procedure allowed an easy inclusion of all information about the system, in particular the knowledge about correlations among measurements which stemmed from the light curve analysis. The methodical development and a new determination of the effective temperatures of the binary components are the novel features of this analysis.

AI Phoenicis (HD 6980, HIP 5438, spectral types FV5 + K0IV) appeared particularly well suited for this investigation since: (i) the components of the system differ significantly in their atmospheric parameters making a detection of a change of \( \alpha \) easier, (ii) the more massive component is a subgiant where the radius is sensitively dependent on \( \alpha \), and (iii) its physical parameters and metallicity are known with high accuracy.
2. Effective temperatures

The effective temperatures of the components strongly influence the determination of $\alpha$. For this reason we reconsidered the published temperatures and tried to include as many constraints as possible in their determination. Basically, the effective temperatures of the components were derived from the Strömgren colors $b - y$, $c_1$ and the observed metallicity. Since the systems undergoes total eclipses the colors of the larger A-component can be measured directly while the colors of the B-component are a result of the light curve analysis. We have used the empirical scale of Alonso et al. (1996) for converting colors into temperatures. The typical uncertainty of the temperature scale is $\pm 110$ K.

The filled squares in Fig. 1 show our results. We obtain the "uncorrected" temperatures when we apply the colors as observed. The temperature scale of Alonso et al. was derived for dwarfs. In order to utilize it for the evolved stars here we applied a correction to the observed colors derived from theo-
Retical model atmospheres (ATLAS9, Buser & Kurucz 1992). We calculated color differences at fixed effective temperatures between the main sequence with $\log g = 4.4$ and the observed $\log g$. Applying these corrections to the observed colors leads to the temperatures indicated by "cool". The reddening towards AI Phe is uncertain but due to its rather small distance and high galactic latitude it should be small. We adopted an educated guess of Hrivnak & Milone (1984) $E(b - y) = 0.015$ and $E(c_1) = 0.003$ leading to the temperatures labeled "hot". The effect of reddening on the temperatures is also indicated by an arrow.

The observed flux ratios in the various bands (Strömgren $ubvy$ and Johnson $UBV$) define relations between the effective temperatures of the components shown in Fig. 1. They were again calculated from ATLAS9 model atmospheres for the observed $\log g$ and metallicity of the components. Note, that only the relative model fluxes enter this procedure. It is encouraging that the "cool" as well as "hot" solution falls onto the relation defined by the flux ratios. The width of the strip spanned by the various colors is consistent with the photometric errors.

Taking into account the flux ratios we finally adopt the 1σ error ellipse drawn around the "hot" temperatures as our best estimate. For comparison previous determinations are shown labeled "HM" (Hrivnak & Milone 1984), "MSK" (Milone, Stagg, & Kurucz 1992), and "ACGNV" (Andersen et al. 1988). Note the significant absolute change and reduction of error space in comparison to "ACGNV". From the effective temperatures and observed radii together with the apparent magnitude of the system one can derive distance estimates depending on the amount of interstellar extinction. The thin black lines correspond to no extinction, the thick ones to the extinction estimated by Hrivnak & Milone. Note, that the distance is not sensitively dependent on the assumed extinction.

3. The fitting procedure

The observations provide the masses, radii, effective temperatures, and the metallicity of the components as well as relations among them with a certain precision. Our state-of-the-art standard evolutionary models (for a description see Salaris et al. 1997) provide the temperature $T$ and radius $R$ of a star as a function of mass $M$, helium abundance $Y$, metallicity $Z$, mixing-length parameter $\alpha$, and age $t$: $T = T(M, Y, Z, \alpha, t)$, and $R = R(M, Y, Z, \alpha, t)$. While $M$ and $Z$ are known, $Y$, $\alpha$ and $t$ of the stars have to be determined by the fit to the observed temperatures and radii. This is possible since one makes the plausible assumption that both stars have the same age and the same chemical composition: $Y_A = Y_B$ and $t_A = t_B$. Note, that we consider metallicity and helium abundance as independent and that we make no assumption about the value of the mixing-length parameter at this stage.

For the fitting procedure we use the well-known $\chi^2$-technique. Defining the vector of variables

$$\vec{x} \equiv (R_A, T_A, M_A, \alpha_A, R_B, T_B, M_B, \alpha_B, Y, Z, t)^T$$

one is searching for an improved vector $\vec{x} + \delta\vec{x}$ minimizing the expression

$$\chi^2 = \frac{1}{2} \delta\vec{x}^T \Sigma^{-1} \delta\vec{x}$$

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under the constraints stemming from the evolutionary models. Σ is the covariance matrix of x. Note, that in this formulation the distinction between observations (M, T, R, Z) and to be fitted model parameters (Y, α, t) is rather arbitrary. They are treated on an equal footing. This has the advantage that one can easily experiment with assumptions about the unknown parameters (e.g. fixing Y to the solar value etc.).

R and T are provided in the form of evolutionary tracks for given Y, Z, and α with the age as parameter. For the fitting we linearized these functions for each component of the system around a point in parameter space close to the observed one. During the course of the investigation it turned out that this might be a too coarse an approximation which we plan to improve the future.

4. Results and conclusions

The table below summarizes the results obtained for the "hot" temperature of the system under various assumptions about the mixing-length parameters of the components. We found: (i) The fits were not always stable. I.e. the outcome was dependent on the way the evolutionary tracks were linearized. That indicates the need for a better analytical representation of the evolutionary tracks. (ii) Nevertheless it was never possible to obtain α to a precision better than about 0.1 making a critical test of the theoretical calibration of α difficult. (iii) We found the tendency for an α greater than the solar value for both components. This conclusion depends sensitively on the temperatures of the components. (iv) We found the tendency for a subsolar helium abundance. However, formally the 1σ uncertainties are consistent with a solar Y for AI Phe.

<table>
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<tr>
<th>Assumption</th>
<th>αA</th>
<th>αB</th>
<th>Y</th>
<th>log t/Gyr</th>
<th>χ²</th>
<th>deg. of freedom</th>
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<td>solar α-values</td>
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<td>0.029</td>
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<td>αA=αB</td>
<td>1.86</td>
<td>1.86</td>
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<td>0.12</td>
<td>0.026</td>
<td>0.029</td>
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<td>α free</td>
<td>1.82</td>
<td>1.75</td>
<td>0.258</td>
<td>9.658</td>
<td>0.0</td>
<td>0</td>
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**References**

Böhm-Vitense E., 1958, ZAp, 46, 108