Balmer Line Formation in Convective Stellar Atmospheres

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Abstract. We give a calibration of the mixing-length parameter $\alpha_{\mathrm{MLT}}$ based on different criteria. While the continuum colors indicate $1 < \alpha_2 < 2$, fitting of the Balmer lines typically requires $\alpha_3 \approx 0.5$.

1. Introduction

Convection affects the temperature structure of a stellar atmosphere in a twofold way: it influences the mean vertical stratification and introduces horizontal inhomogeneities. This poses several questions: (i) What is the "right" $\alpha_{\mathrm{MLT}}$ to produce the correct mean temperature stratification in the framework of mixing-length theory? (ii) What errors are introduced by representing the horizontally averaged spectrum of an inhomogeneous atmosphere by the spectrum of a plane-parallel mean stratification? (iii) Is there something like a spectroscopically equivalent mean stratification of an inhomogeneous atmosphere?

In order to address these questions, we have computed synthetic spectra of the first three Balmer lines from a small sample of 2D solar granulation models. Through a differential comparison between the inhomogeneous atmospheres and different 1D mean models, we try to find a calibration of the mixing-length parameter $\alpha_{\mathrm{MLT}}$ based on three different criteria: $\alpha_1$, reproducing the mean $T(\tau_{\mathrm{Ross}})$ relation around $\tau_{\mathrm{Ross}} \approx 1$; $\alpha_2$, giving the best fit to the visual spectral energy distribution (continuum fluxes and colors); and $\alpha_3$, producing the optimum overall fit of the profiles of $\mathrm{H}\alpha$, $\mathrm{H}\beta$, and $\mathrm{H}\gamma$.

2. Model Atmospheres and Synthetic Spectra

Hydrodynamical Models of Solar Granulation. This investigation is based on time-dependent radiation hydrodynamics (RHD) simulations of the convective surface layers of the Sun. The models include the highly superadiabatic boundary layers at the base of the photosphere where the continuum radiation and the

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Balmer lines originate. Multi-dimensional, non-local, frequency-dependent radiative transfer with realistic opacities as well as a detailed equation of state are taken into account (Freytag et al. 1996, Ludwig et al. 1994). A total of 13 representative snapshots from three different numerical simulations were selected, each having an instantaneous radiative surface flux that deviates by less than 1% from the nominal solar flux. For each of these inhomogeneous hydrodynamical models, we have constructed an associated plane-parallel model atmosphere by averaging \( P \) and \( T^4 \) on (corrugated) surfaces of constant Rosseland optical depth \( \tau_{\text{Ross}} \) (1DT4TAU mean models; \( T^4 \) average virtually conserves total flux).

**Mixing-Length Models** For a differential comparison with mixing-length theory (MLT), we have computed a set of six standard mixing-length models with \( \alpha_{\text{MLT}} = 0.25, 0.50, 0.75, 1.0, 1.25, \) and 1.5 (mixing-length \( l = \alpha_{\text{MLT}} H_p \)). To make sure that the comparison of hydrodynamical and mixing-length models really measures the differences in the theory of convection, we have computed the MLT models with the same physics / numerics that is employed in the hydrodynamical simulations. In particular, we have used the same equation of state, the same radiative transfer scheme, and a similar spatial resolution.

**Temperature Structure of the Lower Photosphere** We have compared the temperature structure of the 1DT4TAU mean models with the \( T(\tau_{\text{Ross}}) \) relations obtained from MLT with different values of \( \alpha_{\text{MLT}} \). Considering the crucial layers around \( \tau_{\text{Ross}} = 1 \), it is obvious that the mean temperature structure of the RHD models differs qualitatively from those of the MLT models: Depending on the optical depth considered, the temperature of the RHD models is matched by very different values of the mixing-length parameter, ranging from \( \alpha_{\text{MLT}} < 0.5 \) to \( \alpha_{\text{MLT}} > 1.5 \). For this reason, it is very difficult to assign an \( \alpha_{\text{MLT}} \) value to the RHD models from this kind of comparison: \( \alpha_1 \) is not uniquely defined. In the following, we therefore try to calibrate \( \alpha_{\text{MLT}} \) through a comparison of different observable quantities, namely the continuum fluxes and the Balmer line profiles.

**Continuum Fluxes and Colors** The hydrodynamical granulation models have been used to compute continuum fluxes at \( \lambda \) 4340, 4860, and 6560 Å (1.5D approximation). The spatial variation of the continuum flux is considerable: Typically, the rms intensity contrast is about 20% at \( \lambda \) 4340 Å.

The horizontally averaged 1.5D fluxes were used to calculate the positions of the RHD-models in a special "color-magnitude-diagram" (Fig. 1) where the **color** is defined as the ratio of the continuum fluxes at \( \lambda \) 4860 Å and \( \lambda \) 6560 Å, and the **magnitude** is defined as the sum of the continuum fluxes at \( \lambda \) 4340, 4860, and 6560 Å (normalized to zero for a black body at \( T = 5780 \) K). Remarkably, the RHD models as well as the associated 1DT4TAU mean models fall onto the same line as the sequence of MLT models with different values of \( \alpha_{\text{MLT}} \).

We interpret this line in the "color-magnitude-diagram" as a sequence of atmospheres characterized by constant effective temperature but different temperature gradients in the continuum forming layers. The average temperature gradients, in turn, depend on the efficiency of the convective energy transport, which is determined by \( \alpha_{\text{MLT}} \) in the case of the MLT models, and by the number of downdrafts per unit surface area in the case of the RHD models. A higher surface density of downdrafts implies a higher convective efficiency and a smaller
mean temperature gradient. The sample of snapshots used in this investigation covers a considerable range of convective efficiencies. Hence, $\alpha_2$ matching the continuum fluxes of the RHD models is not yet well defined. A much larger sample of RHD-models is needed to determine a statistically significant average of the continuum colors. Note that as a result of the averaging procedure employed in constructing the 1DT4TAU mean models, their colors are systematically redder than those of the respective 2D atmospheres, indicating horizontal temperature fluctuations in excess of $\pm 500$ K.

*Balmer Line Profiles* For each of the 13 sample snapshots, we have computed the spatially resolved profiles of H$_{\alpha}$, H$_{\beta}$, and H$_{\gamma}$. The spatial variation of the line strength is even larger than that of the continuum flux (variation by more than a factor of 10; strongest lines in the continuum-bright areas). The horizontally averaged 1.5D flux profiles were taken to be the “observations” which are to be matched by a “spectroscopically equivalent” plane-parallel atmosphere. Specifically, we compared the average line profiles from the RHD simulations with those from the associated 1DT4TAU mean models as well as from the MLT models with $\alpha_{\text{MLT}} = 0.25, 0.50, 0.75, 1.0, 1.25$, and 1.5.

For the sake of a quantitative analysis, we have computed the rms relative deviation of the 1D line profiles from the average 2D profile ($\langle \tilde{F}_{1D} - \tilde{F}_{2D} \rangle_{\text{rms}}$, $\tilde{F} \equiv F_{\text{line}}(\lambda)/F_{\text{cont}}(\lambda)$) as a measure of the quality of the fit, excluding the inner parts of the lines where $\tilde{F}_{2D} < 0.8$. It turns out that H$_{\alpha}$ is insensitive to the choice of $\alpha$; all 1D models provide a relatively good fit. H$_{\beta}$ requires $\alpha_{\text{MLT}} \approx 0.75$ for the optimum fit. The discrimination between different $\alpha_{\text{MLT}}$ is largest for H$_{\gamma}$, favoring $\alpha_{\text{MLT}} \approx 0.5$. The 1DT4TAU mean models give a similar fit quality as the MLT models with $\alpha_{\text{MLT}} \approx 1.25$.

For each of the RHD snapshots we have finally determined $\alpha_3$ giving the minimum rms deviation summed over the three Balmer lines. In Fig. 2 we
show the correlation between $\alpha_2 (F_{4860} / F_{6560})$ and $\alpha_3$ (diamonds) [and between $\alpha_2 (F_{4860} / F_{6560})$ and $\alpha_2 (F_{4340} / F_{4860})$ (crosses)]; $\alpha_2 (F_{4860} / F_{6560})$ was determined from the color-magnitude-diagram shown in Fig. 1 [$\alpha_2 (F_{4340} / F_{4860})$ was found in a similar procedure using the fluxes at $\lambda$ 4340 and 4860 Å for the definition of the color]. Clearly, the different $\alpha$ are correlated (linear regression lines in Fig. 2), but not identical: $\alpha_3 < \alpha_2 (F_{4340} / F_{4860}) < \alpha_2 (F_{4860} / F_{6560})$. Fig. 2 demonstrates that strictly speaking, there is nothing like a unique spectroscopically equivalent mean stratification for a given inhomogeneous atmosphere. Different mean stratifications are needed to represent different photometric / spectroscopic properties of an inhomogeneous stellar atmosphere.

3. Conclusions

(i) No single choice of $\alpha_{\text{MLT}}$ matches the average $T'(\tau)$ relation obtained from the hydrodynamical simulations over a sufficiently large interval of optical depth around $\tau_{\text{Ross}} \approx 1$. The right choice of $\alpha_{\text{MLT}}$ depends on the observational quantity considered.

(ii) A given visual continuum colors of an inhomogeneous stellar atmosphere can be reproduced by a MLT model of the correct effective temperature by adjusting the value of $\alpha_{\text{MLT}}$. However, $\alpha_{\text{MLT}}$ needed for this purpose depends on the considered wavelength range: $\alpha_2 (F_{4340} / F_{4860}) \approx 0.75 \alpha_2 (F_{4860} / F_{6560})$.

(iii) The optimum overall fit to the first three Balmer line profiles originating from an inhomogeneous stellar atmosphere requires an even smaller value of $\alpha_{\text{MLT}}$: $\alpha_3 \approx 0.4 \alpha_2 (F_{4860} / F_{6560})$. However, the small value of $\alpha_3$ cannot be taken as evidence for a low efficiency of solar-type convection!

References