SPHERICAL AND ASPHERICAL STRUCTURE OF THE SUN:
FIRST YEAR OF SOHO/MDI OBSERVATIONS

A.G. KOSOVICHEV, R.NIGAM, P.H. SCHERRER AND J. SCHOU
W.W. Hansen Experimental Physics Lab., Stanford University, U.S.A.

J. CHRISTENSEN-DALSGAARD
Teoretisk Astrofysik Center, Danmarks Grundforskningsfond, and
Institut for Fysik og Astronomi, Aarhus Universitet, Denmark

W.A. DZIEMBOWSKI
Copernicus Astronomical Center, Poland

P.H. GOODE
Big Bear Solar Observatory, New Jersey Institute of Technology,
U.S.A

D.O. GOUGH
Institute of Astronomy and Department of Applied Mathematics
and Theoretical Physics, University of Cambridge, U.K.

J. REITER
Technische Universität München, Germany

AND

E.J. RHODES, JR.
University of Southern California, U.S.A.

Abstract. We report the initial results of one year of continuous observations
of the Sun’s internal structure from the Michelson Doppler Imager (MDI)
on board SOHO. The results have been obtained by inverting frequencies
of p and f modes determined with two different methods of averaging over
split multiplets. Small systematic differences between the two frequency sets
depend primarily on mode frequencies, and, thus, did not significantly affect
the inversions. A preliminary study of the systematic effects resulting from
asymmetry of oscillation power peaks has also shown no significant influence
on the inversion results. The inferred sound-speed profile is in general agreement
with the previous data from MDI and ground-based networks. In the energy-
generating core, the resolution is substantially improved, and the inversion
results indicate a sharp negative perturbation of the sound speed in the core,
tending to a positive value near the center. High-precision measurements of
the f-mode frequencies have been used to determine the seismic radius of the Sun. The global asphericity estimated from frequency variation across the split mode multiplets has been found to be small, and is consistent with the asphericity during the previous activity minimum. Variations of the solar frequencies during the first year of MDI observations have also been detected.

1. Observations

The Medium-\(\ell\) Program of the Michelson Doppler Imager instrument on board SOHO provides continuous observations of oscillation modes of angular degree, \(\ell\), from 0 to \(\sim 300\) (Scherrer et al., 1995). The data for the program are partly processed on board because only about 3% of MDI observations can be transmitted continuously to the ground. The on-board data processing, the main component of which is Gaussian-weighted binning, has been optimized to reduce the negative influence of spatial aliasing of high-degree oscillation modes (Kosovichev et al., 1997). The first 360-day observing run, between May 1, 1996 and April 25, 1997, was nearly continuous with a duty cycle of 95%. Figure 1 shows the \(m\)-averaged oscillation power spectrum. The mode ridges (the lowest and weakest of which corresponds to the f mode) cover most of the \(l-v\)-plane extending to the upper limits of the observing program, \(l = 300\) and \(v = 8.333\) mHz.

![MDI 360-day medium-\(l\) spectrum](image)

*Figure 1.* Oscillation power spectrum obtained from 360 days of the medium-\(\ell\) data.

The noise in the medium-\(\ell\) oscillation power spectrum from MDI is substantially lower than in ground-based measurements. This enables us to detect lower-amplitude modes and, thus, to extend the range of measured mode frequencies, which is important for inferring the Sun's internal structure and rotation.

2. Frequency Estimations

Two different methods have been used to estimate the frequencies of the solar normal modes from the oscillation power spectra (Rhodes et al., 1997). In the first, so-called
“mean-multiplet” method (Schou, 1992), the power spectral peaks are assumed to have a symmetric Lorentzian shape, and a maximum likelihood method is employed to determine the parameters of Lorentzian profiles. The peaks are fit simultaneously in all of the $2l + 1$ individual power spectra for each multiplet so that the effects of overlapping peaks can be included in the fits. These $2l + 1$ frequencies are effectively averaged to yield a single mean frequency, $\nu_{nl}$, for that multiplet. In addition, an associated set of frequency splitting coefficients, $a_{k,nl}$, for which $k$ runs from 1 to 36, is obtained for the same multiplet.

The second frequency estimation technique (Korzennik, 1990; Reiter and Rhodes, 1997) is becoming known as the “averaged-spectrum” method because it employs the $m$-averaged power spectra rather than the $2l + 1$ individual power spectra which the mean-multiplet method employs. The advantage of this method is in greater stability of the peak fits. However, it relies on the multiplet splitting coefficients obtained by the first method and used for averaging the individual spectra. The results of the frequency measurements are shown in Fig. 2. The comparison between the two

![Figure 2](image)

Figure 2. Frequencies of solar modes determined from the 360-day medium-$\ell$ spectra using (a) the mean-multiplet method (Schou, 1992); (b) the averaged-spectrum method (Rhodes & Reiter, 1997). The error bars (shown in grey) correspond to 1000 standard deviations of the measurements.

sets of mode frequencies (Fig. 3a) reveals the systematic difference of $\approx 0.05 \mu$Hz at frequencies around 2000 $\mu$Hz, and $\approx 0.1 \mu$Hz above 3500$\mu$Hz. The systematic difference has been noticed from the 144-day MDI series (Rhodes et al., 1997). The 360-day data indicate that the systematic effect is essentially a function of frequency, and, therefore, most likely related to near-surface perturbations of solar oscillations.

3. Effects of Asymmetric Power Peaks

The medium-$\ell$ spectra from MDI have revealed asymmetry of the line profiles, which has the opposite sense in Doppler velocity and intensity spectra (Fig.4). The asymmetry has been noticed in the ground-based data (Duvall et al., 1993). Several authors (e.g. Gabriel, 1992; Roxburgh & Vorontsov, 1995; Abrams & Kumar, 1996) have studied this problem theoretically and have found that there is an inherent asymmetry whenever the waves are excited by a localized source. The reversal of the asymmetry has been recently explained (Nigam, et al., 1998) by a higher level of the solar
noise correlated to the excitation sources in the intensity spectra. The asymmetry effect leads to systematic errors in the determination of frequencies if the mode peaks are fitted with a Lorentzian profile (Hill et al., 1996; Abrams & Kumar, 1996). To estimate the effects of the asymmetry on the frequency estimates, we modified the numerical algorithm used in the average-spectrum frequency estimation program to fit asymmetrical profiles to all of the observed peaks in our 144-day spectra:

$$A(\nu) = p_1 \frac{1 - (\nu - p_4)^2/p_3^2}{1 + (\nu - p_2)^2/p_3^2} + p_5 + p_6 \nu,$$  \hspace{1cm} (1)

where parameters $p_1 - p_6$ represent the amplitude, the eigenfrequency, the linewidth, the asymmetry, and the background noise respectively. The asymmetry is measured via deviation of $p_4$ from $p_2$. This formula is an approximation to the theoretical power spectra.
Preliminary estimates of the asymmetry effect are shown in Fig. 3b. The similarity with the frequency dependence of the systematic errors in Fig. 3a is intriguing, and suggests that the systematic frequency shift between the two frequency estimates presented in the previous section may result partly from line asymmetry.

4. Radial Stratification

The spherically symmetric structure of the Sun was determined by using a version of the optimally localized averaging method (e.g. Gough & Kosovichev, 1988). Figure 5 shows the relative difference between the square of the sound speed in the Sun and a reference solar model. The reference model is model S of Christensen-Dalsgaard et al. (1996).

*Figure 5.* Sound-speed difference between the Sun and solar model S inferred from the averaged spectrum frequencies (points with crosses) and the mean-multiplet frequencies (solid curve) using the optimal averaging inversion method. The vertical error bars show the formal error estimates of the localized averages of the sound speed, and the horizontal bars show the characteristic width of the localized averages.

The results of the inversion of the two frequency sets described in Sec. 2 agree with each other within the estimated errors. The results confirmed the previous conclusions about a sharp peak at the base of the convection zone \((r \approx 0.7R)\), which is, probably, related to some kind of mixing (e.g. Gough et al., 1996), and rapid variation near the surface, which can be partly explained by correcting the solar radius (Schou et al., 1997). In addition, the new data provide substantially better resolution in the central region where the sound-speed perturbation profile is consistent with partial mixing as suggested from the initial inversion results by Gough & Kosovichev (1988). There is evidence from the averaged-spectrum data set of a sharp negative peak of the sound-speed perturbation between 0.15 and 0.2 \(R\), which may indicate the boundary of such mixing. Of course, at this stage, other possible explanations, e.g. variations of opacity and nuclear reaction rates, are not ruled out. Also, our inversion results contradict the recent results from GOLF (Turck-Chièze et al., 1997). Obviously, it will take longer time series to understand the structure of the core.
5. Seismic radius

In Figure 6, we compare the observed f-mode frequencies with the theoretical frequencies of solar model S. The main component of the relative frequency difference is a constant offset. This shift suggests that the theoretical frequencies are incorrectly scaled. The f-mode frequencies depend primarily on the gravitational acceleration at the solar surface and the horizontal wavenumber, and, therefore, are proportional to $R^{-3/2}$. This leads to the idea that the value of the solar photospheric radius, $R$, used to calibrate the model is somewhat different from the actual radius. A detailed analysis of the f-mode frequencies

![Plot showing relative differences between seismic model and model S for f-mode frequencies](image)

*Figure 6.* Relative differences between the f-mode frequencies of $l = 88 - 250$ computed for solar model S of Christensen-Dalsgaard et al. (1996) and the observed frequencies. The ‘seismic model’ frequencies are obtained by scaling the frequencies of model S with the factor 1.00067 which corresponds to reducing the model radius by the factor $(1.00067)^{2/3} \approx 1.00044$. The error bars are $3\sigma$ error estimates of the observed frequencies.

(Schou et al., 1997) has shown that the value of the solar photospheric radius used to calibrate the standard solar model has to be reduced by approximately 300 km in order to match the model frequencies with the observed frequencies. The f-mode frequencies provide a strict constraint on the density profile 4 - 10 Mm beneath the surface, but the precise correction to the calibration radius of a solar model depends on the description of the superadiabatic layer in the model. For the model S of Christensen-Dalsgaard et al. (1996), originally calibrated to 695.99 Mm, the new calibration radius is approximately 695.68 Mm. The uncertainty due to the statistical errors in the frequency measurements is only 0.008 Mm. However, the systematic error estimated from the deviation of the points of the seismic model in Fig.6 from the zero line could be about 0.03 Mm.
6. Asphericity

For these inversions, we represent the even-$a$ coefficients by
\[ a_{2k,\text{nl}} = a_{2k,\text{nl,rot}} + (-1)^{\frac{k}{2}} \frac{(2k - 1)!!}{(2k)!!} \frac{\gamma_k(\nu_{\text{nl}})}{\sqrt{l(l + 1)} I_{\text{nl}}}, \]  \hspace{1cm} (2)

where $I_{\text{nl}}$ is the mode inertia, $a_{2k,\text{nl,rot}}$ describe the contribution from the second-order effect of rotation which we have calculated using the rotation law following from the odd-$a$ MDI data, and $\gamma_k(\nu_{\text{nl}})$ is a characteristic of asphericity. The functions $\gamma_k(\nu_{\text{nl}})$ determined by inversion separately for the p and f modes are shown in Fig. 7. For the p modes, we have found that $\gamma_2$

![Graph](image1)

**Figure 7.** The asphericity coefficients, $\gamma_1, \gamma_2$ and $\gamma_3$ (see Eq 2) from the SOHO/MDI data and from two years of the BBSO data, 1986 and 1989, corresponding to the periods of low and high solar activity during the previous solar cycle.

dominates the results from the MDI data. This parameter corresponds to an aspherical structure described by the Legendre polynomial $P_4(\theta)$. This type of asymmetry is consistent with the results from the last activity minimum (compare with BBSO 86 data in Fig. 7). However, this $P_4$-asphericity is smaller than that from the LOWL data which cover a period earlier in the current activity minimum. During the rising phase of the last activity cycle, this term changed sign, and by 1989 was strongly negative (see Fig. 7). For the f modes, we have found that the $P_4$ asymmetry also dominates. It will be important to track the behavior of the asphericity, especially for the f modes, during the rising phase of the coming cycle.
7. Temporal variations

To detect temporal variations in the radial solar structure, the mode frequencies have been estimated for five consecutive 72-day intervals using the mean-multiplet method. Variations of the frequencies averaged over \( l \) and over 0.2 mHz frequency intervals are shown in Fig. 8 relative to the first 72-day observing interval. The results show variations of the order of 0.02 \( \mu \text{Hz} \) for the modes above 2 mHz. These changes are likely to be related to near-surface variations resulting from solar activity. However, no apparent solar cycle variations of the internal structure have been detected yet.

![Figure 8. Averaged (over \( l \)) frequency differences for five 72-day observing intervals during the 360-day run. These intervals are: (1) 05/01/96-07/11/96, (2) 07/12/96-09/21/96, (3) 09/22/96-12/02/96, (4) 12/03/96-02/12/97, (5) 02/13/97-04/25/97.](image)

References