SOLAR ASYMMETRIES FROM SOHO/MDI SPLITTING DATA

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ABSTRACT

We have analyzed time changes in centroid frequencies and multiplet frequency splittings of solar oscillations determined with the Michelson Doppler Imager instrument (MDI) on SOHO. The data were divided into five consecutive 72-day sets covering the period from 1 May 1996 through 25 April 1997. We have detected a significant trend in the a4 and a5 frequency splitting coefficients, which reflects a decrease in the P3 distortion and an increase in the P1 distortion. This is the same trend seen in the BBDO data (Libbrecht & Woodard, 1990) between the previous activity minimum (1986) and the high activity phase two years later. There is no apparent trend in the a2 coefficient in the SOHO/MDI data. The centroid frequencies, as already reported by Kosovichev et al. (1998), exhibit small nonmonotonic variations. The relative differences among the solar radii inferred from the f-mode frequencies from the five sets (at most 6 x 10^-6 or 4 km) are formally significant, but again there is no monotonic trend during the observed period. We find no apparent evidence for asymmetries in the deep interior. However, a more complete picture of the solar cycle awaits further data on the Sun's interior-particularly from SOHO.

Key words: Sun: interior — Sun: oscillations.

1. Introduction

Asymmetries in the fine structure of the p-mode spectrum of solar oscillations vary systematically through the solar cycle, Kuhn (1988, 1989) and Libbrecht & Woodard (1990). The changes are clearly associated with the surface temperature bands reported by Kuhn, Libbrecht, & Dicke (1988). As well, Woodard & Libbrecht (1991) found a strong correlation between oscillation frequency changes and solar surface magnetic variations from monthly averages of their data. One conspicuous feature is the persistent manifestation of a P3 distortion in the splittings. The origin of this latter behavior, as well as the temporal variation of the frequencies is unclear. However, the data to date have an incomplete coverage of the cycle. In addition, we have no basic understanding of the correlation among the various manifestations of solar asphericities and the variation of the mode multiplet frequencies.

Oscillation centroid frequencies have also been shown to vary through the solar cycle. This has been demonstrated for both modes of moderate ℓ, Libbrecht & Woodard (1990), and low ℓ modes (Anguera-Gubau et al. 1992) and (Elsworth et al. 1994). Dziembowski & Goode (1996) showed that all these changes are consistent with a near surface perturbation. That is, there is no evidence for solar cycle variations of the frequencies arising from deeply buried changes.

The SOHO/MDI instrument has the capability of following various forms of the asphericities. In particular, Kuhn et al. (1998) studied the asphericities in brightness, and they detected a significant hexadecupole (P1) distortion. The same instrument measures oscillation frequencies.

Many of the results reviewed here were presented in Dziembowski, et al. (1998).

2. SOLAR ASPHERICITIES FROM SOHO/MDI FREQUENCY DATA

The data treated here are five 72-day sets covering (1) 1 May 1996- 11 July 1996, (2) 12 July 1996 - 21 September 1996, (3) 22 September 1996 - 2 December 1996, (4) 3 December 1996 - 12 February 1997, and (5) 13 February 1997 - 25 April 1997. They contain centroid frequencies νt,n and 36 splitting coefficients aₖ. Each of the sets has data for 1348 p-mode multiplets (covering ℓ from 2 through 184) and 33 f-mode multiplets. For each p-mode multiplet, the splittings are defined by

$$\nu_{t,n,m} - \bar{\nu}_{t,n} = \sum_{k=1}^{36} a_k P_k^m(m),$$  \hspace{1cm} (1)

where P are orthogonal polynomials (see Ritzwoller & Lavelle 1991 and Schou et al. 1994). The remaining symbols in equation (1) have their usual meanings.


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This representation ensures that the $a_{2k}$ are a pure probe of the spherical structure while the $a_{2k}$ – the even-$a$ coefficients – are a pure probe of the distortion described by corresponding $P_{2k}(\cos \theta)$ Legendre polynomials. We have
\[
\int_0^{2\pi} \int_{-1}^1 |Y^m_\ell|^2 P_{2k}(\cos \theta) d\phi = C_{k,\ell} P_{2k}'(m),
\]
where
\[
C_{k,\ell} = (-1)^k \frac{(2k-1)!!}{k!} \frac{(2\ell+1)!!}{(2\ell+2k+1)!!} \frac{(\ell-1)!}{(\ell-k)!}.
\]
In a representation of the splitting in terms of $LP_k(\frac{\pi}{2})$, where $L = \ell$ or $\sqrt{\ell(\ell+1)}$, which was the standard some years ago, the association between $\alpha_{2k}$ and $P_{2k}(\cos \theta)$ was only valid asymptotically for $\ell \gg k$. The antisymmetric (about the equator) part of distortion is undetectable by means of seismology based on mode eigenfrequencies.

We separate the analysis of p-modes and f-modes because they have vastly different properties in the outer layers. Our inferences from the f-modes only concern possible variations in the seismic solar radius ($R_\odot$). In this Section, we only consider even-$a$ coefficients and centroids of the p-modes.

The even order splitting coefficients in the five data sets were fit to the following formula (Dziembowski and Goode 1991, 1996)
\[
a_{2k,\ell,n} = a_{2k,\ell,n,rot} + C_{k,\ell} \frac{\gamma_k}{I_{\ell,n}},
\]
where the first term on the r.h.s. represents the effect of centrifugal distortion which we calculate following the treatment of Dziembowski & Goode (1992). Since our plan is to study temporal variations in the asymmetry, we, therefore, eliminate the constant distortion of rotation from the $\gamma$‘s in equation (4). The \( I_{\ell,n} \) denote mode inertia. They are evaluated assuming all modes have the same value for the radial displacement at the base of solar photosphere. Adopting a value of $10^4$ for the displacement, we have $I_{0,19} \approx 1$. The inertia decreases with increasing $\ell$ or $n$. The same expression has been used to represent centroid changes, however, in this case we can determine only a relative $\gamma_0$. We choose the first set (“SET 1”) of the five as the reference.

The $I_{\ell,n}^{-1}$ factor accounts for most of the $\ell$ and frequency dependence in the splitting coefficients. This was given by Libbrecht & Woodward (1990) as evidence that the asphericity responsible for the even-$a$ coefficients is localized well above the lower turning point of modes in the sample i.e. close to surface. Dziembowski & Goode (1991) were able to determine a slight, but significant $v$ dependence of the $\gamma_1$ and $\gamma_2$ coefficients from the same data. In general, we should also expect an $\ell$-dependence of the $\gamma$‘s. We shall return to the utility of this information later. First, we present the results of fitting constant $\gamma$‘s to each of the five sets of the even-$a$ coefficients from the SOHO/MDI instrument.

In Figure 1, we show the first 12 $\gamma$‘s. We see that the errors rapidly increase with the order, $k$. We will focus on the first four coefficients in the expansion. The changes found in some higher order $\gamma$‘s are formally significant, but we are reluctant to interpret them. We do not see a significant trend in either $\gamma_0$ or $\gamma_1$. The frequency differences have already been presented by Kosovichev et al. (1998) in a different form. The fact that frequencies in the SET 1 are systematically lower than in the remaining sets has been already reported (Kosovichev et al. 1998). However, they reported no apparent solar cycle variation.

The absolute values of $\gamma_2$ and $\gamma_3$ are much larger than those for $\gamma_0$ and $\gamma_1$. The first term which describes the $P_0$ distortion was dominant in 1996. The corresponding distortion has been found by Kuhn et al. (1998) in their intensity measurement with the SOHO/MDI instrument. $P_3$ was the dominant distortion in their measurement, and was consistent with centrifugal stretching. In particular, if we represent their result for the asymmetry in terms of the fractional radius change in the Sun, we have
\[
\frac{\delta R}{R_\odot} = [-5.41 \pm 0.46] P_2 + [1.43 \pm 0.56] P_4 \times 10^{-6}.
\]
Roughly, a seismically determined differential rotation, using the rotation law in Dziembowski & Goode (1992) implies a value of
\[
\frac{\delta R}{R_\odot} = [-6.6 P_2 + 0.6 P_4] \times 10^{-6}.
\]
Thus, after removing the part of the asymmetry due to rotation, only the $P_3$ distortion is significant at the $1\sigma$ level. We stress that we removed the effect of centrifugal distortion from our $\gamma$‘s. The effect is 0.037 $\mu$Hz for $\gamma_1$ and about ten times smaller for $\gamma_2$ and is completely negligible for $\gamma_3$ and higher order terms. Note that the size of centrifugal stretching removed from $\gamma_1$ is smaller than $\gamma_0$ or $\gamma_3$. In detail, we find that the $P_3$ term is dominant in SET 5. This set was obtained contemporaneously with that of Kuhn et al. (1998), but they did not report for a $P_3$ distortion. This might be due to the relatively larger noise level in the intensity data. The relative accuracy in determining the asymmetry serves to emphasize the broad utility of the seismic data.
3. COMPARISON WITH EARLIER RESULTS

The trend seen in $\gamma_2$ and $\gamma_3$ would seem to be the same as at the beginning of the rising phase of the previous activity cycle. We note that the data from the previous rising phase covered a similar frequency range, but they were restricted to lower $\ell$ values. The maximum value of $\ell$ in the BBSO data (Libbrecht & Woodard, 1990) is 140, while in LOWL data (Tomczyk, private communication) it is 100. Thus, to make a meaningful comparison, we have to examine the behavior of the $\gamma$'s with changes in the $\ell$ composition of the data—in particular, the maximum value of $\ell$ in the truncated sets. Fig. 2 shows an example. We show there the effect of truncation of selected data sets on the inferred values of $\gamma_2$ and $\gamma_3$. The values are the largest in the two sets selected, but the patterns are the same for the other original sets. We see that there is a difference in the behaviors—the value of $\gamma_2$ is insensitive to $\ell_{\text{max}}$, whereas the value of $\gamma_3$ monotonically increases with it. The behavior of $\gamma_1$ is similar to that of $\gamma_3$, but the values are much smaller. These facts must be taken into account when comparing $\gamma$'s obtained from various data sets. They may be also important for understanding the origin of the asphericity reflected in p-mode frequencies.

In Figure 3, we present mean values of the $\gamma$'s from various sources which includes BBSO, LOWL and SOHO/MDI. We compare them with the monthly averages of smoothed sunspot numbers. Clearly, the BBSO of 1988 and 1989 give the largest magnitudes of the $\gamma$'s and this corresponds to the first half of the previous sunspot maximum. Unfortunately, we are missing splitting data from the rising phase (1987 or so), as well as from the declining phase.

The errors in the BBSO and LOWL $\gamma$'s are about twice as large as those in SOHO/MDI, but they are still relatively small and the errors would not be visible in Fig. 3. The relatively small errors in the low-order $\gamma$'s encourages a search for the $\nu$ dependence.

The similarities in the values of $\gamma_1$ and $\gamma_2$ between BBSO86 and LOWL data has already been observed. We note that the sets are from last two activity minima. The values of $\gamma_3$ derived from LOWL data were divided into four quarters and showed a barely significant decreasing trend from the value of 0.22 to 0.20 mHz. The mean value from SET 1 measurements taken about one year and half later was 0.13 mHz.

It decreased during next three-quarters of a year to 0.08 mHz, as seen in Fig. 1.

The most clear evidence of the onset of the rising phase of activity during this three-quarter of a year period is the systematic increase of $\gamma_3$ from SOHO/MDI seen both in Figures 1 and 3. In the upper panel of Fig. 3, we see that the values in the data fill the gap in $\gamma_3$ between BBSO86 and BBSO88. Still, even SET 5 is closer to BBSO86 than to BBSO88, but the increase in $\gamma_3$ is evident.

In $\gamma_1$, we also observe an evolution toward the values it has in the Sun's more active state. This evolution presents the weakest case. One should note, however, that temporal changes in $\gamma_1$ during the rising phase of the previous cycle were also lagging behind those in the higher order $\gamma$'s.

4. THE NEAR SURFACE ASPHERICITY

The relatively small errors in the low-order $\gamma$'s encourages a search for the $\nu$ dependence. The $\gamma_\nu(\nu)$ functions were determined by means of a least-square fit to the data assuming a three-term Legendre polynomial dependence.

In Figure 4, we compare the $\gamma_\nu(\nu)$ inferred from SETs 1, 3, and 5 of the SOHO/MDI data with those inferred by Dziembowski & Goode (1996) from the BBSO data and by Dziembowski et al. (1997) from the LOWL data which were combined into a single set. One can see that the $\gamma(\nu)$ function determined from LOWL and SOHO data partially fill the gap between the BBSO datasets from 1986 and 1988. The form of the $\gamma(\nu)$ function yields information about the
sources of the solar distortion reflected in the even-$a$ coefficients. The most direct inference is about localization. The relatively weak dependence tells us that the perturbations resides within a few megameters of the photosphere.

Asphericities, as expressed by the $\gamma$'s, imply that there is a non-radial perturbation of the Sun's mechanical equilibrium. The connection to magnetic fields is obvious in times of high activity, but we should also consider the possible role of the Reynolds's stress. The stress can arise from rotation, meridional circulation and convective velocities. The rotational shape distortion is essentially time-independent, and so, we can quantify and remove it. The effect of meridional circulation is most likely negligible, as can be estimated from the recent measurement of its velocity by Giles et al. (1998). However, the convective flows can be both non-symmetric and time dependent. For instance, the 'thermal shadow' of a deeply buried magnetic field could be reflected in global asymmetries in the near-surface velocity field. Kuhn & Stein (1996) have argued that magnetically induced entropy fluctuations near the base of the convection zone can cause the sound-speed asphericities near the surface, which were detected with time-distance seismology by Duvall et al. (1996).

Thus, even though the cause of the asymmetries may be deeply buried, the source of the $\gamma$'s are large scale structures near the surface. The most likely candidates are near surface convection (where convective velocities are the largest) or magnetic fields (with magnetic pressure being the closest to the gas pressure near the surface).

5. ASPHERICITIES IN THE INTERIOR

It would be of great interest to discover asymmetries seated in the deep interior. Unfortunately, a preliminary analysis of the SOHO/MDI data does not give us much reason to hope at this time. Although, that may change with new data from the rising phase of the cycle. The residuals of the $a_{2k,\ell,n,res}$, after removing the centrifugal term and the $\gamma$ term, do not exhibit a visible dependence on the position of the inner turning point. It is well-known that p-modes preferentially sample the region just above their inner turning points.

In Figure 5, we plot the quantities

$$\gamma_{k, res} = \frac{a_{2k,\ell,n,res}C_k}{I}$$

for $k=2$ and 3 for SETs 1 & 5 (here we dropped the $\ell$ and $n$ subscripts) against the parameter $\lambda$ which determines the position of the lower turning point. We see no trend in the distribution of the residuals about the zero-line, except, perhaps for some preponderance of positive values above $\lambda = 1.8$ which corresponds to values of the fractional radius above 0.85. The region of special interest is the vicinity of the bottom of the convection zone at $r \approx 0.7R_\odot$ which corresponds to $\lambda \approx 1.05$, and is where the solar dynamo is believed to operate. We do not see any feature indicating either a $P_4$ or a $P_6$ distortion in this region. The same is observed in corresponding plots for the remaining sets. Finally, there is no apparent evidence of an interior $P_2$ distortion beyond that due to centrifugal stretching.
6. VARIATIONS IN THE SUN’S SEISMIC RADIUS

Schou et al. (1997) showed that the f-mode ridge in the SOHO/MDI $k$-$\omega$ diagram is so precisely determined that a constant offset between the true frequencies and their model counterparts became apparent. They determined that the seismic radius is about 300 km smaller than the model radius and emphasized that the f-mode data open the possibility of determining the cycle dependence of the solar radius. We have already seen small variations in the seismic radius with the five SOHO/MDI data sets we have. Each of the sets contains data for 33 f-modes. We only made use of their centroid frequencies. Separately, for each set we determined corrections to the standard solar model solar radius. The corrections were evaluated from

$$\frac{\Delta R}{R_\odot} = \frac{2}{3} \left( \frac{\Delta \nu_{\ell,0}}{\nu_{\ell,0}} \right),$$

(6)

where $<>$ denotes the average weighted by the inverse square of the measurement errors. We used a standard solar model of Christensen-Dalsgaard et al. (1996). The results are given in Fig. 6. The differences among the sets are bigger than the formal errors. The maximal relative variation of the solar radius during the observed period was about $6 \times 10^{-6}$ which corresponds to approximately 4 km. However, only subsequent data will tell us about the reality of the apparent radius minimum from SET 3 which coincides with the minimum of the Sun’s activity (see Fig. 3).

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