DAMPING AND FREQUENCY SHIFT OF THE SOLAR F-MODE DUE TO THE INTERACTION WITH TURBULENT CONVECTION

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ABSTRACT

We present observations which show that the frequency of the high-degree f-mode is substantially lower than the frequency given by the simple dispersion relation, \( \omega^2 = gk \), and that the linewidth grows with the wavenumber \( k \). We attempt to explain this behavior as a result of interaction with granulation which we model as a random flow.

Because of buffeting from the random flow the f-mode speed and consequently frequency are reduced. Additionally, a random flow introduces the negative imaginary part of frequency. This negative imaginary part represents the damping of the mean field, i.e. the generation of random field at the expense of the mean field energy. The linewidth is proportional to the magnitude of the imaginary part of the frequency.

We apply an analytical perturbation technique and numerical methods to estimate the linewidth and the frequency shift, and show that the results are consistent with the properties of the f-mode obtained from the high-resolution Michelson Doppler Imager (MDI) data from SOHO.

Key words: convection — Sun: interior — Sun: oscillations

1. INTRODUCTION

The solar f-mode of high angular degree, \( l \), is recognized as a surface gravity wave satisfying asymptotically (at \( l \to \infty \)) the dispersion relation \( \omega^2 = gk \), where \( \omega_0 \) and \( k \equiv (l + 0.5)/R \) are the frequency and wavevector, \( g \) is the surface gravity, and \( R \) is the solar radius. The previous observations of the high-degree f-mode by Libbrecht et al. (1990), Rhodes et al. (1991), Fernandes et al. (1992), & Bachmann et al. (1995) have shown that the frequency of this mode is substantially lower than follows from the parabolic dispersion relation. An even stronger frequency decrease is observed for solar acoustic (p) modes in the high-frequency range. Kosovichev (1995) has argued that the p-mode frequencies may be decreased due to turbulent pressure in the upper convective boundary layer, but the turbulent pressure does not affect the frequencies of the f-mode which is essentially incompressible and independent of the hydrostatic structure of the Sun. Therefore, the f-mode frequency reduction cannot be explained by changes in the solar structure, but requires the introduction of new physics into the standard model of solar oscillation.

Brown (1984) first suggested that eigenfrequencies of solar oscillations can be depressed by large fluctuating velocity fields in the upper convection zone. This dynamical effect of solar convection should work for both f- and p-modes. Theoretical models of this effect for the f-mode have been proposed by several authors. Murawski & Roberts (1993a), Murawski & Roberts (1993b), and Murawski & Goossens (1993) showed that the f-mode frequency is reduced by granulation, modeled as a random velocity field that is located in the convection zone. A strong (\( \sim 1\% \)) blueshift of the f-mode frequency is reported by Ghosh et al. (1995) for the case of shearing velocity field in the solar photosphere. Perturbation theory has been recently used by Gruzinov (1998) to show that the low-degree f-mode frequency shift due to turbulence is positive and unobservably small, whereas for \( l > 500 \) the frequency shift is proportional to the squared Mach number of convection near the solar surface.

In this paper we present results of new observations of the high-degree f-mode using the high-resolution data from the Michelson Doppler Imager (MDI) instrument on board Solar and Heliospheric Observatory (SOHO) (Scherrer et al. 1995). These observations have allowed us to obtain accurate measurements of the f-mode frequency shift in the range \( l = 700-1800 \), and also detect a dramatic increase in the f-mode linewidth at high \( l \). We also present a generalization of the Murawski-Roberts model to the case of a complex frequency of the f-mode, and explain the observed frequency reduction and wave damping of the f-mode.

In this generalization, a random flow causes the appearance of the negative imaginary part of frequency. This negative imaginary part represents the damping of the mean field, i.e. the generation of random field at the expense of the mean field energy. The linewidth is proportional to the magnitude of the imaginary part of the frequency. We apply an an-


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alytical perturbation technique and numerical methods to estimate the imaginary part and the frequency shift, and show that the results are consistent with the properties of the f-mode obtained from the high-resolution MDI data.

2. OBSERVATIONS

![Figure 1. The solar oscillation power as a function of angular degree l and cyclic frequency, ν ≡ ω/2π, obtained from the MDI high-resolution data. The lowest ridge is the f-mode. The black points on the diagram show the frequencies of f and p₁ modes determined from the power spectrum.](image)

The power spectrum of the solar oscillations of high degree has been obtained by applying a 3D Fourier transform to a 8-hour long time series of Doppler velocity images in the MDI high-resolution field of view. The pixel size of the MDI high-resolution field is approximately 0.63. This allows us to observe the oscillation modes up to l = 5000, or k = 7 Mm⁻¹. The power spectra corresponding to modes of the same angular degree l, but different azimuthal order m were corrected for the rotational frequency shift and then averaged together. The m-averaged power spectrum shown in Fig. 1 displays a set of mode ridges the lowest of which corresponds to the f-mode. This mode becomes dominant in the spectrum above l = 1000. It also shows a dramatic increase in linewidth at high l. The oscillation power drops above l = 2500.

We have determined the mean frequencies and linewidths for both f and p₁ modes in the range l = 700 – 1800 by fitting a Lorentzian profile to the power spectrum. Fitting m-averaged spectra may lead to a small systematic shift in the measured mean frequencies (Bachmann et al. 1995). However, for our work this error is insignificant because this shift is much smaller than the convective frequency shift discussed in this paper.

The difference between the observed and theoretical frequencies of f and p₁ modes, scaled with the factor Q ≡ I₁(ν)/I₀(ν), where Iₙ(ν) is the mode inertia, and the observed linewidth are shown in Fig. 2. This scaling factor takes into account the difference in the reaction of modes of different l to near surface perturbations, revealing the frequency dependence of the frequency change. The similar trends of the scaled frequency differences for f and p₁ modes suggest that, at least partly the frequency decrease of both f and p₁ is caused by the same physical mechanism. Since f-mode frequencies are practically insensitive to variations in the solar structure and to non-adiabatic effects, this suggests that the main cause of the frequency depression is related to dynamical effects of convection. The additional decrease of the p₁ frequencies is probably caused by the turbulent pressure effect (Kosovichev 1995).

The linewidth of the f-mode sharply increases with the frequency (or angular degree, l). The linewidth of the p₁ mode increases less dramatically with frequency; however, it shows a dependence similar to the f-mode on the angular degree l (Fig.3), which is approximately \( l^2 \).

These results suggest that the physical mechanisms for the frequency decrease and increase of the linewidth are similar for the f and p₁ modes. In the next Section, we present a theoretical model for the f-mode; a generalization of this model for p modes is currently under development.

3. THEORETICAL MODEL

3.1. Random Dispersion Relation

The solar plasma below the visible layers is a dynamic environment, supporting convection which reveals itself principally on two scales of motion, a large scale supergranulation and a much smaller scale granulation. (There are possibly other convective patterns, such as mesogranulation, which we here ignore.) Su-
pergranules have a horizontal scale of 30 Mm and flows of 0.1-0.4 km s\(^{-1}\); granules are found to cover a range of scale from 0.2 Mm to 2 Mm, with a preference for 1 Mm. See Simon et al. (1991) for a discussion of granulation. Flows in granules, at 1-3 km s\(^{-1}\), are stronger than those in supergranules.

In this paper, we employ the random f-mode model of Murawski & Roberts (1993a). In this model the photospheric plasma is modeled by incompressible fluid equations with weak random velocity field settled below the photosphere-chromosphere transition. Assuming a binary collision approximation (Howe 1971) the random dispersion relation can be written as follows (Murawski & Roberts 1993a)

\[
L(\omega, k) = \frac{4}{\pi} \lambda \sigma^2 \omega^2 k \int_{-\infty}^{\infty} \frac{\xi e^{-\lambda^2 (k-\xi)^2}}{L(\omega, \xi)} d\xi, \tag{1}
\]

where \(\sigma^2\) and \(\lambda\) are the variance and correlation length of the convective velocities, respectively, \(\xi\) is the integration variable, and \(L(\omega_0, k)\) is a static dispersion relation, i.e.

\[
L(\omega_0, k) \equiv \omega_0^2 - gk = 0. \tag{2}
\]

An approximate formula for the imaginary part of frequency can be derived by calculating the corresponding correction to the roots, \(\omega_0\), of the static dispersion relation (2). Thus suppose that

\[
\omega = \omega_0 + \sigma^2 \omega_1 \tag{3}
\]

and

\[
L(\omega, k) \simeq L(\omega_0, k) + \frac{\partial L}{\partial \omega} \sigma^2 \omega_1. \tag{4}
\]

Neglecting the real correction to the frequency, (4.10) from Howe (1971) leads to an approximate formula for the imaginary part of \(\omega\), i.e.

\[
\text{Im}(\omega) \simeq -\frac{2 \lambda^2 \sigma^2}{\sqrt{g}} k^2 e^{-2 \lambda^2 k^2} \int_0^{\pi} e^{2 \lambda^2 k^2 \cos \phi} d\phi. \tag{5}
\]

The integral in this equation has been computed by the use of the 8th point Gauss quadrature method.

The linewidth \(\Gamma\) is expressed through the imaginary part of the frequency, \(\Gamma = -2 \text{Im}(\omega)\) (e.g. Osaki 1990).

Dispersion relation (1) has some physical consequences. The dependence \(\omega(k)\) differs from the static (non-random) dispersion relation (2). The random velocity field introduces a correction to the wave frequency. This correction is described by the real part of \(\omega\). Due to scattering by random flow, the energy of the coherent field that is associated with the f-mode is partially transformed into the random field. The mathematical manifestation of this phenomenon corresponds to an appearance of the imaginary part of \(\omega\) (eq. 5).

![Graph](https://via.placeholder.com/150)

**Figure 4.** The observed (diamonds) and model (a) f-mode frequencies relative to the static frequency, \(\nu_0\), and (b) the linewidths, \(\Gamma\), as a function of angular degree \(l\) for five different sets of the convection flow parameters: \(\sigma = 1\) km s\(^{-1}\), \(\lambda = 1\) Mm (solid curve), \(\sigma = 0.9\) km s\(^{-1}\), \(\lambda = 1\) Mm (long dashes), \(\sigma = 0.1\) km s\(^{-1}\), \(\lambda = 1\) Mm (dotted), \(\sigma = 2\) km s\(^{-1}\), \(\lambda = 1\) Mm (dashed), \(\sigma = 1\) km s\(^{-1}\), \(\lambda = 0.2\) Mm (dash dot), and \(\sigma = 1\) km s\(^{-1}\), \(\lambda = 2\) Mm (dash dot dot dot).

### 3.2. Numerical Results and Comparison with Observations

We have solved the dispersion equation (1) numerically for various values of the convective velocity variance, \(\sigma^2\), and the correlation length, \(\lambda\) and compared the results with the MDI experimental data.

The main problem associated with solving the transcendental dispersion relation (1) is to perform the
integration in the limit as \( \text{Im}(\omega) \to +0 \) along the path of integration in complex \( z \)-space to pass around the pole of the integrand. For this purpose, we split the integration function into real and imaginary parts and used the 8th point Gauss-Hermite method to perform improper integrals. As the path of integration we have chosen an integrand along the straight line \( \text{Im}(z) = -\lambda \). The transcendental equation has been solved by the ZANLYT routine from the IMSL library.

Figure 4 compares numerical results for a set of \( \sigma \) and \( \lambda \) with the observed frequency depression and linewidth of the f-mode. The effect of the random flow is to reduce the real part of the f-mode frequency and to increase the linewidth, and this effect grows with the value of the angular degree, \( \ell \), or the wavevector, \( k \). The numerical curves that were obtained for the case of the typical solar conditions \( \sigma = 1 \text{ km/s} \) and \( \lambda = 10^5 \text{ km} \) fits quite well to the observational data for both the frequency reduction and linewidth (Fig. 4a).

The numerical results show that the magnitude of the frequency decrease depends mainly on the velocity variance, \( \sigma^2 \). The change in the correlation length of convection, \( \lambda \), affects the functional form of this depression. Also, at very small correlation lengths (\( 0.2 \text{ Mm} \)) the frequency depression becomes too small compared to the current data.

The effect of the random flow on the linewidth, \( \Gamma \), is also interesting. Fig. 4b shows that the theoretical \( \Gamma \) attains maxima at values of \( \ell \) which correspond to the wave number \( k_m \sim 2\pi/\lambda \). That means that f-modes experience the strongest damping when their wavelength becomes comparable with the correlation length of the random flow.

It is worth mentioning that real and imaginary parts of \( \omega \) are coupled; the f-mode damping which corresponds to the negative imaginary part of \( \omega \) enhances the speed reduction and vice versa. As a consequence of that, a low value of the flow amplitude \( \sigma = 1 \text{ km/s} \) was sufficient to fit the observational data while for the case when \( \text{Im}(\omega) \) was neglected, \( \sigma = 3 \text{ km/s} \) was required to match the random and observational data (Murawski & Roberts 1993a).

4. SUMMARY AND CONCLUSIONS

The observed frequencies of the solar f-mode fall off below the expected frequencies of the standard f-mode for high spherical degree and the linewidth grows with degree. The model presented in this paper shows that the random velocity field is a possible explanation for the frequency reduction and mode damping at high degrees. The frequency reduction corresponds to a decrease of the real part of the f-mode frequency with the wavevector \( k \) (or angular degree, \( \ell \)). The mode damping is associated with a negative value of the imaginary part of the frequency, which is proportional to the linewidth. The linewidth attains its maximum at the wavevector \( k \) that is inversely proportional to the correlation length \( \lambda \). This corresponds to the maximum of the f-mode damping rate due to the interaction with the random convective flows.

We have derived a dispersion relation for the f-mode in a simple model atmosphere containing a random velocity field. An asymptotic solution to this equation has been found for the case of small magnitude of the random flow and wavenumber. Numerical computations for arbitrary flows and wavevector indicate that the effect of the random flow is to decrease the frequency of the f-mode and broaden line profiles. This reduction is revealed by the real part of \( \omega \). The negative imaginary part of this frequency represents the damping of the coherent f-mode field due to scattering by random flow. The f-mode damping is a result of the generation of random field at the expense of the coherent field.

Our simple model of the solar atmosphere and random velocity field is able to give similar results to those observed by the MDI instrument. The overall agreement of the model frequency shift with the experimental data offers encouragement for the further studies of the influence of the convection on the f-mode frequencies. Of course, the solar atmosphere is far more complex than uniform and incompressible and flows in the photosphere are not horizontal. However, frequency shifts for more realistic models will be more difficult to determine analytically. It is likely that a purely numerical approach will be necessary. The present semi-analytical model may then serve as a building block towards implementing the basic ingredients that make up the observed f-mode frequencies.

One of us (K.M.) expresses his thanks to the organizers and sponsors of the SOHO meeting for the invitation and financial support.

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