SOME COMMENTS ON PHASE INVERSION

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ABSTRACT

The method of phase inversion have been proposed and tested for simple cases by Gough, Merryfield and Toomre (1991, 1993, 1998) for detection of inhomogeneity in media by observing wave propagation. We discuss some of the difficulties that are encountered with the procedure in practice, and what might be done to overcome them in transferring the technique to the solar case, such as in the study of horizontal inhomogeneity in the solar cavity along the equator using the MDI sectoral-mode data.

Key words: helioseismology; Sun: aspherical structure

1. STUDY OF BACKGROUND INHOMOGENEITY

Let us consider wave propagation in inhomogeneous medium governed by the model equation

$$\frac{d^2\Psi(x)}{dx^2} + \kappa^2(x)\Psi(x) = 0.$$  

How can we infer $\kappa^2(x)$ from observation of $\Psi(x)$? Gough, Merryfield & Toomre (1991) proposed a method based on a separation of the waveform $\Psi(x)$ into an amplitude part and a phase part, using a Hilbert transform. The Hilbert transform of $\Psi(x)$ is defined as

$$\tilde{\Psi}(x) \equiv \frac{1}{\pi} P \int_{-\infty}^{\infty} \frac{\Psi(\xi)}{\xi - x} d\xi,$$

where P denotes Cauchy’s principal value. It can be shown that by defining the amplitude function

$$A(x) \equiv \sqrt{\Psi(x)^2 + \tilde{\Psi}(x)^2},$$

and the phase function

$$\phi(x) \equiv -\tan^{-1}[\tilde{\Psi}(x)/\Psi(x)],$$

$\Psi(x)$ is appropriately separated into the two components:

$$\Psi(x) = A(x) \cos \phi(x).$$

By comparing this expression with the asymptotic form of $\Psi(x)$:

$$\Psi(x) \sim \frac{1}{\sqrt{\kappa(x)}} \exp \left( i \int x \kappa(x') dx' \right),$$

one obtains the following relation

$$\frac{d\phi}{dx} \approx \kappa(x).$$

In practice, the Hilbert transform of $\Psi(x)$ is obtained through Fourier decomposition of $\Psi(x)$, exploiting the fact that the Hilbert transform of $\cos kx$ and $\sin kx$ are $-\sin kx$ and $\cos kx$ respectively.

The amplitude-phase decomposition provides us with a recipe for investigating inhomogeneity in media by observing the propagation of the wave.

2. 1D ARTIFICIAL WAVE TRAIN DATA

The formulation given in the previous section is concerned with the case where only a single wave component (with a single frequency) is present. However, in the cases of wave phenomena in the sun, we cannot observe such a single component. Gough, Merryfield and Toomre (1991, 1993, 1998, hereafter GMT) therefore considered the case of multiple waves with similar frequencies. When there are more than one wave components, they interfere with each other (beating) and complicate the signal. The interference must somehow be removed, and GMT pointed out that this can be done in the following way, provided that there is a well defined group velocity $v_g$ i.e. in the frequency range covered by the waves the dispersion relation is well approximated by a linear formula. In this case $\phi'(x,t)$ (the prime denotes spatial derivative) contains a component that travels at this group velocity $v_g$, so we may write

$$\phi'(x,t) = \kappa(x) + f(x - v_g t).$$


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would mean that the wave is strictly sinusoidal, implying that there is no inhomogeneity in the medium. There are also observational effects such as mode leakage arising from incomplete masking (broadening in k space), but at this stage we do not discuss these effects.

In the numerical experiment of GMT, only the broadening due to the inhomogeneity was present in the artificial data. When the excitation and damping are present, Fourier components for a given frequency are ‘contaminated’ by the broadening in w-space. We shall have to disentangle them to extract the signature of the inhomogeneity. Eventually, some kind of decomposition of Fourier components into single-frequency waves might be required. Ironically, if such decomposition is possible at all, we may no longer need to consider the multiple-wave case as in GMT, because then we shall be able to extract information about a single wave component scattered by the inhomogeneity. In any case, we would like to note that whatever is causing the line asymmetry (see, e.g., Nigam et al 1998) will add another complication to any such decomposition.

Figure 1 shows an example of \( \phi' \) derived from MDSI 10-day sectoral data, including wave components in the range \( l = 90 - 110, \nu = 3095 - 3145 \) (\( \mu \)Hz). We are looking for a time-independent background pattern with an easily identifiable moving pattern on the top of it, which we might remove later to recover \( \kappa \). Unfortunately we do not find such features here. The direct cause of this has been found to be the lack of a well defined group velocity, because the concentration of power around the ridge is not strong enough. We have attempted to circumvent this problem by averaging (merging frequency and/or wavenumber bins), or by analysing the lower-frequency part (where the line width is narrower) to no avail. The difficulty is basically the inability to tell if a picture is noisy and blurred yet basically ‘correct’, or is simply badly contaminated. Generation of artificial datasets for the purpose of careful study of the effects of excitation and damping is under way.

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REFERENCES


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