ON THE PRECISION OF TIME/DISTANCE MEASUREMENTS

F. Baudin & S.G. Korzennik

Harvard Smithsonian Center for Astrophysics, 60 Garden St, Cambridge MA 02138, U.S.A.

ABSTRACT

"Time-distance" analysis is an emerging tool for local helioseismology, pioneered by Duvall et al. 1993. Like other seismological analysis techniques, time-distance analysis allow us to infer internal physical properties of the Sun from measurements made at the surface by solving an inverse problem. In the case of time-distance analysis, the observed quantities are travel times of acoustic waves from one point of the surface to another (or between group of points). These travel times are estimated by fitting a wavelet to the correlation function computed from time-series of velocities averaged along annuli of given radius. The precision with which these measurements can be made is a crucial parameter of the inverse problem, and controls the achievable trade-off between resolution and uncertainty magnification. In the work presented here, we investigate the precision of these measurements, and how this precision varies with some of the parameters of the analysis.

Key words: Sun; local seismology; uncertainty estimation.

1. INTRODUCTION

The key parameter measured in time-distance analysis is the correlation time (or travel time) between time series corresponding to surface manifestation of wave packets at different locations on the solar disk. These correlation functions can be gathered to form a time-distance map where the propagation of the waves is seen as ridges. The visibility of these ridges can be very good, or not, as shown in Fig. 2. Let us compare these maps and in particular how they are built.

A correlation function can be computed in various ways. For example, it can be computed between the time series corresponding to a given point on the surface and the time series made from a surrounding annulus (see Fig. 1). In that case, the correlation time is the travel time for the distance $\Delta$ corresponding to path n°1 in Fig. 1. The same computation can be done using only part of the surrounding annulus (corresponding to North, West, South and East quadrants), giving information on the travel time as a function of direction. Another way to proceed is to use the time series corresponding to an annulus, filter out the inward or outward propagating waves, and correlate the resulting time series. This yields the travel time for the distance $2\Delta$ (ie path n°2) in Fig. 1.

As the surface of the annulus is larger than that of the center (ie one pixel), the corresponding time series is less noisy due to spatial averaging, and so is the correlation function corresponding to path n°2. Other parameters can influence the level of noise in the correlation function, like the length of the time series used. On the other hand, the computation of time-distance map can be performed for different positions of the central point. Hence, the resulting maps can be averaged to yield a better signal to noise ratio. Of course, averaging (in space or in time) involves a loss of resolution (in the respective domains). This defines a trade-off between resolution and signal to noise ratio, illustrated in Fig. 2.
2. ANALYSIS

2.1. Data used

For this work, we have used images from the Michelson Doppler Imager on board SoHO (Scherrer et al. 1995). Time-distance maps were computed from sub-images extracted from MDI full disk Dopplergrams, located at the disk center, with a size of 40°. This region did not show any sign of activity during the period we used (May 1st 1997).

2.2. Uncertainty computation

For each distance \( \Delta \), the correlation time is measured by fitting a wavelet to the correlation function:

\[
W(t) = A e^{-(t-t_0)^2} \cos\left(\frac{2\pi(t-t_0)}{T}\right)
\]  

(1)

where \( A \), \( T \), and \( w \) are the amplitude, period and width of the fitted wavelet, and \( t_0 \) corresponds to the maximum of the envelop of the wavelet, while \( t_0 \) is the correlation time, or physically speaking, the travel time. The algorithm used is a non-linear least squares fit. In order to estimate the precision on this determination, we derive the 1\( \sigma \) value on the fitted parameters from the inverse of the Hessian matrix (Press et al. 1992). A typical variation of uncertainty versus distance is shown in Fig. 3.
2.3. Simulations and statistics

In order to check the reliability of our uncertainty estimation, we have performed some numerical simulations. We have computed a large number (N~10000) of realisations of a wavelet model with additive Gaussian noise (of various variance). We have then fitted each realisation, yielding an ensemble of value of the correlation time, ensemble which standard deviation is equal to the averaged uncertainty given by the fitting algorithm. Nevertheless, if this confirms the reliability of the uncertainty estimation in the case of Gaussian noise, this may not be the case for real data as the noise distribution may not be Gaussian. To investigate this, we have fitted a real time-distance map, and computed the residuals from the fit. Their histogram (corresponding to the fit of 49 wavelet of 21 minutes long, yielding 1029 values) shows a misleading Gaussian distribution (Fig. 4). Indeed, when averaging the the residual of each fit and plotting the result versus time, the result shows a clear oscillatory behaviour. Thus, the noise in our fits cannot be considered as normally distributed in the real data, but made of spurious oscillatory signals (as seen in the modulation seen in Fig. 2). This can be the cause an improper estimation of the uncertainty in the fitting process.

3. PRECISION

3.1. Precision versus time resolution

In order to study the influence of the length of the time series used to compute the time-distance maps, we have computed several maps, for a given path (n°2 of Fig. 1) and a given spatial averaging (~3.3°). The time series have a length ranging from 250min to 900min.

Fig. 6 shows the time measured for a given distance (Δ=6.5°) versus the length of the time series, and the associated uncertainty. For length shorter than 250min, our fitting algorithm did not converge, nor it did in some occurrences for larger time series. Above 250min and up to ~600min, the measured travel time shows a quite large dispersion, while the corresponding uncertainty values have a similar behaviour. Then, for longer time series, both travel time and uncertainty converge to a stable value. Uncertainty stabilizes around ~1s, value slightly smaller than the ones for time series shorter than ~600min. For correlation functions computed using other paths (Fig. 1), the results are similar. The required length is somewhat larger (~350min) to get a sufficient signal-to-noise ratio, but then, the uncertainty on travel time is comparable to those of Fig. 6.

3.2. Precision versus spatial resolution

A very efficient way to improve the signal to noise ratio in the time-distance maps is to average several maps corresponding to different centers. However, this gain leads to a loss of resolution in space. We have computed an ensemble of individual time-distance maps, using time series of 900min and corresponding to path n°2 of Fig. 1. We have built a sequence of averaged maps with increasing averaging area in order to study the effect of this parameter. As in the case of time resolution, an insufficient averaging leads to maps of too low signal to noise ratio to be fitted. Averaging on a surface of at least ~4°² is necessary (in our implementation). Once the averaging is sufficient, uncertainties do not vary drastically, showing only a slight decrease when averaging increases. The results are very similar when using other paths to compute the time-distance maps.
4. CONCLUSION

We have computed some maps of time anomalies for a small area on the Sun. It is made of the variations of the travel time at a given distance $\Delta$ for different locations over a small area of the solar disk (Fig. 8). These variations are supposed to be due to physical variations in the subphotospheric layers. The typical standard deviation of these variations is $\sim 6s$. Compared to the uncertainty estimated, this is significantly larger. However, points raised in section 2.3 and Fig. 2 prevent to draw a final conclusion about the significance of observed time anomalies. Uncertainty derived from the Hessian matrix may be underestimated because of the particular distribution of perturbations in time-distance maps. More work is necessary for a more reliable fitting.

In a future work, we will also extend our range of investigation about time and spatial averaging (sections 3.1 and 3.2) in order to confirm these results on larger scale and to check the possibilities of the time-distance analysis on large scale, as used by Giles et al. 1993, who reached a precision of the order of a tenth of second.

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REFERENCES


Figure 6. Results of the fit versus the length of the time series used. Both travel time and uncertainty converge to a stable value for time series longer than $\sim 600$ min

Figure 7. Variation of uncertainty on travel time versus space averaging. When the fit found a solution, uncertainty values keep similar, except for a slight decrease for increasing averaging. Fit did not converge for smaller averaging area because of poor signal to noise ratio.
Figure 8. Map of time anomalies for a small area of the solar disk. These correspond to travel time differences for different locations on the Sun, and are expected to be due to some physical variations in the subsurface layers, as they are larger than typical uncertainty values. However, these ones may be underestimated (see text)