HOW SOLAR MODELS FIT THE SOHO OBSERVATIONS?

P. Morel, J. Provost & G. Berthomieu
Département Cassini, UMR CNRS 6529, Observatoire de la Côte d’Azur,
BP 4229, 06304 Nice CEDEX 4, France

ABSTRACT

Solar models are computed with CESAM code (Morel 1997) using different physical assumptions concerning the description of the convection and of the nuclear reactions and screening. The effect of the primeval evolution and of the uncertainties in the age are discussed. The fit with the observations is estimated by comparison with seismic models derived from SOHO observations. Finally we have performed some numerical experiments to mimic the transport of elements in the radiative zone beneath the convection zone resulting in a decrease of the sound speed peak between the Sun and the model.

Key words: Solar modeling; Helioseismology.

1. INTRODUCTION

The models are computed with the following physics. We used the Opal 96 equation of state (Iglesias & Rogers 1996) and the Opal 96 opacities (Rogers et al. 1996) interpolated with Houdek’s routines (Houdek & Rogl 1996). The atmosphere is restored using the Hopf’s $T(\tau)$ law; the connection with the envelope is made at a Rosseland optical depth $r_H \geq 10$. We used the thermonuclear reaction rates of Caughlan & Fowler (1988) for PP+CNO cycles and weak screening. The species $^4$He, $^7$Li and $^7$Be are not assumed at equilibrium in models labeled S13, but in models S9.

The temperature gradient in convection zone is computed according to the standard mixing length theory; the convection zones are mixed by a strong diffusion of coefficient $D_{\alpha} = 10^{13}\text{cm}^2\text{s}^{-1}$.

The diffusion of chemicals is made using the microscopical diffusion coefficients of Michaud & Proffit (1992); all chemicals but $^1$H and $^4$He are trace elements. The change in Z due to the diffusion of heavier elements is taken into account in the model calculation. The models are calibrated at 4.5Gyr for the present day solar radius, luminosity and $Z/X = 0.0245$ ratio (Grevesse & Noels 1993). More details can be found in Morel et al. (1997).

In the following we study the influence on the solar model and its oscillation frequencies of different descriptions of the convection, of updated nuclear parameters and of changing the solar age. Finally the effect on the sound speed of an additional mixing below the convection zone is investigated.

![Figure 1. Comparison of the temperature gradients $\nabla$ of models S13 (dashed) and S13c (full) computed respectively with MLT and CMT convection theories. The adiabatic gradient $\nabla_{\text{ad}}$ (dotted) and the radiative gradient $\nabla_{\text{rad}}$ (dot-dash) are also plotted.](image)

All the theoretical sound speed profiles are compared with the sound speed profiles obtained by inversion using combined LOWL and GOLF data (Turck-Chièze et al. 1997).

The global characteristics of the models discussed in this paper are given in Table 1.

2. CONVECTION

With the convection theory of Canuto & Mazitelli (1991), hereafter CMT, the one mode-model approach of the standard Mixing Length Theory, hereafter MLT, is abandoned for an entire spectrum approach, that leads to a larger convection flux. The models S13 and S13c include the pre-main sequence evolution and are calibrated at 4.5Gyr. For both models the mixing-length is taken as $l = \alpha R_\odot$. S13c has been computed with CMT and S13 with MLT.

For S13c the calibrated value of $\alpha$ comes out of the or-
Table 1. Global characteristics of models. $i$ is the mixing-length, $H_p$ the pressure scale height; $Y_i$ and $Z_i$ are the initial abundances in mass of helium and heavy elements; $Y_0$, $Z_0$ and $R_{ZZ}$ respectively are, at present day, the surface abundances, per unit of mass, of helium and of heavy element and the radius, in solar units at the bottom of the convection zone; $T_c$, $\rho_c$, $Y_c$ and $Z_c$ are the central values at present day respectively of, the temperature in units of $10^7 K$, the density in g cm$^{-3}$, the abundances, per unit of mass of helium and of heavy element. $\Phi_{Ga}$ and $\Phi_{Cl}$ in SNU and $\Phi_{Kas}$ in events day$^{-1}$, are the expected fluxes for the three neutrino experiments namely Gallium, Chlorine and Kamiokande; $\delta\nu_{02}$ and $\delta\nu_{13}$ are the average values in $\mu$Hz of the frequency differences between the radial p-modes of degree $\ell = 0 - 2$ and $\ell = 1 - 3$. $P_b$ is the characteristic spacing period of g modes in minutes.

<table>
<thead>
<tr>
<th>$S9$</th>
<th>$S9n$</th>
<th>$S9nm$</th>
<th>$S9np$</th>
<th>$S9g1$</th>
<th>$S9g2$</th>
<th>$S9e$</th>
<th>$S9E$</th>
<th>$S9f0$</th>
<th>$S9f2$</th>
<th>$S13$</th>
<th>$S13c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$i/H_p$</td>
<td>1.76</td>
<td>1.75</td>
<td>1.75</td>
<td>1.75</td>
<td>1.76</td>
<td>1.77</td>
<td>1.76</td>
<td>1.75</td>
<td>1.75</td>
<td>1.75</td>
<td>1.77</td>
</tr>
<tr>
<td>$Y_1$</td>
<td>0.274</td>
<td>0.274</td>
<td>0.273</td>
<td>0.274</td>
<td>0.273</td>
<td>0.272</td>
<td>0.273</td>
<td>0.272</td>
<td>0.272</td>
<td>0.272</td>
<td>0.274</td>
</tr>
<tr>
<td>$Z_1$</td>
<td>0.0195</td>
<td>0.0195</td>
<td>0.0195</td>
<td>0.0195</td>
<td>0.0196</td>
<td>0.0199</td>
<td>0.0194</td>
<td>0.0191</td>
<td>0.0192</td>
<td>0.0191</td>
<td>0.0195</td>
</tr>
<tr>
<td>$Y_0$</td>
<td>0.245</td>
<td>0.245</td>
<td>0.244</td>
<td>0.245</td>
<td>0.244</td>
<td>0.243</td>
<td>0.246</td>
<td>0.248</td>
<td>0.247</td>
<td>0.248</td>
<td>0.246</td>
</tr>
<tr>
<td>$Z_0$</td>
<td>0.0180</td>
<td>0.0178</td>
<td>0.0180</td>
<td>0.0180</td>
<td>0.0180</td>
<td>0.0180</td>
<td>0.0180</td>
<td>0.0179</td>
<td>0.0179</td>
<td>0.0179</td>
<td>0.0180</td>
</tr>
<tr>
<td>$R_{ZZ}$</td>
<td>0.711</td>
<td>0.712</td>
<td>0.712</td>
<td>0.712</td>
<td>0.711</td>
<td>0.710</td>
<td>0.711</td>
<td>0.712</td>
<td>0.712</td>
<td>0.712</td>
<td>0.710</td>
</tr>
<tr>
<td>$T_c$</td>
<td>1.565</td>
<td>1.569</td>
<td>1.569</td>
<td>1.568</td>
<td>1.571</td>
<td>1.573</td>
<td>1.564</td>
<td>1.562</td>
<td>1.562</td>
<td>1.562</td>
<td>1.561</td>
</tr>
<tr>
<td>$\rho_c$</td>
<td>150.7</td>
<td>151.8</td>
<td>152.2</td>
<td>151.4</td>
<td>153.1</td>
<td>154.4</td>
<td>150.7</td>
<td>150.6</td>
<td>150.6</td>
<td>150.6</td>
<td>149.8</td>
</tr>
<tr>
<td>$Y_c$</td>
<td>0.637</td>
<td>0.638</td>
<td>0.637</td>
<td>0.637</td>
<td>0.643</td>
<td>0.647</td>
<td>0.637</td>
<td>0.635</td>
<td>0.636</td>
<td>0.635</td>
<td>0.639</td>
</tr>
<tr>
<td>$Z_c$</td>
<td>0.0208</td>
<td>0.0208</td>
<td>0.0209</td>
<td>0.0208</td>
<td>0.0209</td>
<td>0.0210</td>
<td>0.0207</td>
<td>0.0204</td>
<td>0.0204</td>
<td>0.0204</td>
<td>0.0208</td>
</tr>
<tr>
<td>$\Phi_{Ga}$</td>
<td>129.0</td>
<td>127.8</td>
<td>126.7</td>
<td>126.7</td>
<td>128.7</td>
<td>129.5</td>
<td>128.7</td>
<td>127.9</td>
<td>128.1</td>
<td>127.9</td>
<td>127.3</td>
</tr>
<tr>
<td>$\Phi_{Cl}$</td>
<td>8.21</td>
<td>7.21</td>
<td>6.98</td>
<td>7.16</td>
<td>7.35</td>
<td>7.50</td>
<td>8.14</td>
<td>7.98</td>
<td>8.02</td>
<td>7.99</td>
<td>7.21</td>
</tr>
<tr>
<td>$\Phi_{Kas}$</td>
<td>0.66</td>
<td>0.55</td>
<td>0.52</td>
<td>0.54</td>
<td>0.56</td>
<td>0.57</td>
<td>0.65</td>
<td>0.64</td>
<td>0.64</td>
<td>0.64</td>
<td>0.55</td>
</tr>
<tr>
<td>$P_b$</td>
<td>35.68</td>
<td>35.47</td>
<td>35.45</td>
<td>35.55</td>
<td>35.22</td>
<td>34.98</td>
<td>35.67</td>
<td>35.69</td>
<td>35.69</td>
<td>35.69</td>
<td>35.73</td>
</tr>
</tbody>
</table>

Figure 2. Normalized frequencies differences $\delta\nu$, between models, computed respectively with CMT and MLT convection theories, as a function of the frequency $\nu$ for modes of degrees $\ell = 0$ to 500. $Q = E_r(\nu)/E_{L\ell=0}(\nu)$ where $E_r(\nu)$ is the energy of the mode ($\ell, \nu$).

Figure 3. Effect of updated nuclear reaction rates. The model S9 (dashed) is computed with the Caughlan & Fowler (1988) reaction rates and the models S9n (full) and S9nm (dot-dash) with the slightly less efficient updated rates (Adelberger et al. 1998). The main effect is a slight increase of the temperature in the core and an increase of the sound velocity there. Likewise the effect of the less efficient screening of Miller (1977) is also an increase of the central temperature (model S9nm).
3. UPDATED NUCLEAR REACTION RATES

Models S9n and S9mn have been calibrated using the updated nuclear reaction rates of Adelberger et al. (1998). In the solar core the revised values give slightly smaller rates for the PP reactions, compared to the values of Caughlan & Fowler (1988). Likewise for the CNO reactions but $^{13}$C$(p, \gamma)^{14}$N, $^{14}$N$(p, \gamma)^{15}$N and $^{15}$N$(p, \gamma)^{16}$O which are slightly more efficient. The updated nuclear reactions rates are, in all, slightly less efficient than previously, the nuclear energy generated is then lowered, from the calibration result an increase of the central temperature and of the sound speed. With the new rates, with respect to the model S9 computed with the same physics, the sound velocity increases in the core and decreases beneath the convection zone, as seen in Fig.3. On the same way the sound velocity in the core is increased if the less efficient Mitler’s screening (Mitler 1977) is used (model S9mn) instead of weak screening. The neutrino fluxes are slightly decreased $\theta_p$ for Gallium from 129.8 SNU to 126.7 SNU, cf. Table1. More details can be found in Brun et al. (1998a).

The changes of the solar structure, induced mainly in the solar core by the updated nuclear reactions rates, do not modify the small frequency separation of low degree $p$ modes $\delta v_{02}$ and $\delta v_{13}$. Contrarily they result in a significant decrease of the characteristic period of the $g$ mode $P_0$ by about 0.6%. Consequently the frequencies of low degree low frequency modes are increased with a maximum around 1.5$\mu$Hz around 300$\mu$Hz, when models S9n and S9 are compared (Fig.4). Similar calculations with models S13n (not presented here) and S13 provide analogous results, but with larger amplitude for the differences.

4. SOLAR AGE

According to the most recent determinations, the solar age is $4.52 \pm 0.04$Gy (Guenther et al. 1992). In order to estimate the effect of the solar age on the solar interior, we consider the following models. The model S9np includes the pre-main sequence evolution and the model S9n (thin full) is initialized at homogeneous zero-age main sequence. They are both calibrated at 4.50Gy. $T_{eff}$ and $L/L_0$ vs. age are plotted panels (a) and (b); the same, panels (c) and (d), but with the abscissae of S9n translated by $+25$My.

Figure 4. Effect of updated nuclear reaction rates on low degree ($\ell = 0$, 1, 2 and 3) low frequency $p$ and $g$ modes. The difference of frequency $\delta v$ between model S9n and model S9 is plotted as a function of the frequency. The $g$ modes are changed according to asymptotic behavior while the $p$ modes are almost unaffected.

Figure 5. Effect of the solar age. The model S9np (full) includes the pre-main sequence evolution and the models S9n (dashed), S9g1 (dash-dot) and S9g2 (dash-dot-dot-dot) are initialized at homogeneous zero-age main sequence; they are calibrated with ages 4.50Gy, 4.50Gy, 4.58Gy and 4.66Gy respectively. The dotted line corresponds to a model initialized at homogeneous zero-age main sequence and calibrated at 4.475Gy.

Figure 6. A consequence of the pre-main sequence evolution. The model S9np (heavy full) includes the pre-main sequence evolution and the model S9n (thin full) is initialized at homogeneous zero-age main sequence. They are both calibrated at 4.50Gy. $T_{eff}$ and $L/L_0$ vs. age are plotted panels (a) and (b); the same, panels (c) and (d), but with the abscissae of S9n translated by $+25$My.
age while it decreases in the core, leading to a better agreement with the observations. One notices that the curves pass through the same point at 0.2 \( R_\odot \). That corresponds to the limit of the CNO burning.

The change of stratification in the solar core modifies the p and g mode frequencies: the small frequency separations of low degree p-mode \( \delta \nu_{12} \) and \( \delta \nu_{13} \), as well as the g mode characteristic period \( P_0 \), decrease significantly with the age cf. Table 1. See Provost et al. (1998) for more details on the sensitivity of these modes to the solar structure.

At the level of accuracy nowadays reached by solar modeling and helioseismology, the choice of the initial model, homogeneous zero-age main sequence or model evolved with pre-main sequence could have an effect. With respect to a solar model evolved with pre-main sequence the question of the definition of the zero-age main sequence i.e. of the age of the Sun remains open. A preliminary discussion has been made by Rossignol (1995).

The central temperature of a homogeneous zero-age main sequence model is close to \( T_\odot \sim 1.37 \times 10^7 \) K then, the abundances of CNO species are not at equilibrium with the thermonuclear reactions and the efficient CNO burning generates a convective core which disappears after about 65 My when the equilibrium is reached. A similar scenario occurs if the pre-main sequence evolution is taken into account, but the convective core appears at \( \sim 25 \) My and disappears at \( \sim 90 \) My.

For two models evolved, the first, from homogeneous zero-age main sequence and the second, with pre-main sequence, when \( \sim 25 \) My is added to the age of the former \( t_{\text{AMS}} \), the profiles respectively of \( T_\odot \) and \( L/L_\odot \) with respect to time, are superimposed after \( \sim 90 \) My as illustrated in Fig. 6. Recall that the hydrodynamical phases of the solar system formation last less than \( \sim 1 \) My (Winkler & Newman 1980), then the onset of a solar quasi-static evolution can be taken at the beginning of the pre-main sequence and the age of a model is then the time \( t_{\text{AMS}} \) lasts from this epoch. Roughly speaking one can say that a homogeneous zero-age main sequence model starts with an advance of about 25 My thus \( t_{\text{AMS}} \sim t_{\text{AMS}} - 25 \) My, then the evolutionary time from the CNO equilibrium to present day is the same if the model is evolved from homogeneous zero-age main sequence or with pre-main sequence. Remark that 25 My is within the error bar of the solar age determination and that this estimate can change by a few million years, depending on the physics.

Using the physics of S9n but with age diminished of 25 My namely, 4.475 Gyr instead of 4.5 Gyr, we have calibrated a model from homogeneous zero-age main sequence; as expected the global characteristics and the sound speed come out almost identical to those of S9np as illustrated in Fig. 5 (dotted). Nevertheless the sound speeds of models S9n initialized at homogeneous zero-age main sequence and S9np evolved with pre-main sequence with the same duration are already close enough.

In conclusion of this section we note that a slightly higher age than usually accepted improves the agreement between the sound speed of the Sun and the solar model.

![Figure 7. Numerical experiments have been made in order to erase the peak in sound velocity beneath the convection zone. The models were computed using an extra mixing with several diffusion coefficient profiles (a), (b), (c) and (d) shown in Fig. 8. S9e(a) (dash-dot-dot-dot), S9E(b) (full), S910(c) (dashed), S902(a) (dot-dash).](image1)

![Figure 8. Profiles of the diffusion coefficients used to simulate an extra mixing in the numerical experiments reported in Fig. 7. \( R_\odot \) and \( \Delta \), in solar radius \( R_\odot \), are respectively the center and the characteristic width of the jump of the diffusion coefficient \( D_T \). The width of the extra mixed zone is respectively of order 0.05 \( R_\odot \) in (a), 0.1 \( R_\odot \) in (c) and (d) and 0.4 \( R_\odot \) in (b).](image2)

5. MIXING IN THE TACHOCLINE

The inversions of the solar angular rotation reveal a transition between the differential rotation in outer convective part and the rigid rotation in inner radiative core. This transition zone, called the tachocline, is located beneath the convection zone at radius \( R/R_\odot = 0.695 \pm 0.005 \) of thickness \( \Delta = 0.05 \pm 0.03 R_\odot \) (Corbard et al. 1998) i.e. close to the characteristic peak observed in the sound speed differences between the Sun and the solar models. There the sound velocity of the models is lower than in the Sun. It is likely that the shear of rotation velocity produces a rotationally induced mixing. Here we described this mixing, in a first approach, by an additional diffusion process, without attempt to specify the physics.
Such a process smooths the gradients of chemical composition just below the convection zone. It results a decrease of the helium abundance and of the mean molecular weight, thus an increase of the sound speed of the model, which becomes closer to the observations. Similar approaches are given in Gabriel (1997) and Brun et al. (1998b).

By numerical experiments, we have investigated the amplitude of the diffusion coefficient $D_T$ and the extension of the mixed zone required to damp the sound speed peak. We illustrate our study by the effects on the sound speed beneath the convection zone in Fig. 7 when one uses additional diffusion coefficients with profiles shown in Fig. 8. After some attempts we found that with $D_T \sim 100 \text{cm}^2\text{s}^{-1}$ the peak of sound speed difference is significantly reduced. This value is comparable to that given in Vaclavir (1998). As a matter of comparison, the diffusion coefficient is around ten time less in this part of the model.

It appears that a width of the mixed zone of 0.1$R_\odot$ at least, is necessary to erase significantly the sound speed difference peak between the Sun and the model, derived from inversion. The peak corresponding to model S9E is almost unchanged, due to a too small region of extra mixing. In our description the effects of medium and large profiles for $D_T$ are hardly distinguishable (models S9E and S9F0). However this preliminary estimate of the width of the mixed zone may be overestimated due to the finite resolution of the sound speed inversion (Elliott et al. 1998) which has to be taken into account in future work.

ACKNOWLEDGMENTS

This work has been performed using the computing facilities provided by the OCA program: “Simulations Interactives et Visualisation en Astronomie et Mécanique (SIVAM)”.

REFERENCES

Adelberger, E. et al. 1998, Rev. Mod. Phys. in press
Brun, S., Turck-Chièze, S., Zahn, J.P. 1998b, these proceedings
Caughlan, G. R., Fowler, W.A. 1988, Atomic Data and Nuclear Data Tables, 40, 284
Elliott, J.R., Gough, D.O., Sekii, T. 1998, these proceedings
Rossignol, S. 1995, stage DEA Imagerie en Sciences de l’Univers, Université de Nice-Sophia Antipolis