INITIAL SOI/MDI HIGH-DEGREE FREQUENCIES AND FREQUENCY SPLITTINGS

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ABSTRACT

We present the first high-degree $p$-mode frequencies and frequency splittings obtained from the Full-Disk Program of the SOHO Solar Oscillation Investigation/Michelson Doppler Imager experiment. The frequencies and splittings which we present here were computed from power spectra obtained during the 1996 SOI/MDI Dynamics Run. Specifically, a time series of Dopplergrams which covered 60.75 days was converted into sets of zonal, tesseral, and sectoral power spectra covering the degree range of 0 through 1000. These sets of power spectra were then analyzed in two different ways to yield both frequencies and rotationally-induced frequency splittings. First, estimates of the frequency splittings were employed to compute average power spectra for all degrees up to $\ell = 1000$. Estimates were then made of the frequencies, frequency uncertainties, widths, peak power densities, and background power densities of a total of 13,864 separate peaks in these 1,001 average power spectra. A total of 2,554 of these peaks were isolated enough in their respective spectra to be fit as single $p$-modes. However, for the remaining 11,310 peaks (mostly those above $\ell = 200$), the individual $p$-mode peaks and their spatial sidelobes were located so close together in frequency that they appeared as ridges rather than as isolated modal peaks in the average power spectra. For these "ridge-fit" cases we were forced to correct the raw frequencies.

In our second analysis forty sets of these power spectra were also processed to yield estimates of the rotational splitting coefficients for individual $p$-mode ridges for every 25th degree between $\ell = 25$ and 1000. For $\ell$ between 25 and 175 we will compare these Full-Disk program splittings with the previously-published splittings from the 1996 SOI/MDI Medium-$\ell$ Program. We will also compare these splittings with ground-based estimates of solar photospheric differential rotation in order to demonstrate that such high-degree frequency splitting coefficients will also have to be corrected for the systematic errors which are introduced when the individual modes blend into ridges.

Key words: solar oscillations; frequencies.

1. INTRODUCTION

Since becoming operational in April of 1996, the SOHO SOI/MDI experiment has been operated in three different ways for the study of solar oscillations: 1) reduced-resolution Dopplergrams have been obtained in the Medium-$\ell$ Program, 2) 1024x1024 pixel resolution Dopplergrams have been obtained in the Full-Disk (FD) Program, and 3) 1024x1024 pixel high-resolution images have been obtained during limited campaign runs. In this paper we present the first high-degree $p$-mode frequencies and frequency splittings which have ever been obtained from the SOHO SOI/MDI FD Program.

The MDI Medium-$\ell$ Program was designed to trade off spatial resolution for temporal coverage in the study of solar low-degree and intermediate-degree $p$-mode oscillations. Specifically, the Medium-$\ell$ Program was designed to obtain a nearly uninterrupted time series of Dopplergrams for the entire duration of the SOHO mission. This goal of a nearly 100 % duty cycle was achieved by pre-processing the full-resolution images onboard the spacecraft so that only much smaller, reduced-resolution images had to be transmitted to the ground. As described in more detail by Scherrer et al. (1996), it is the reduced spatial resolution which limits the MDI Medium-$\ell$ Program to the study of $p$-modes having degrees up to 300.

1.1. Development of Frequency Estimation Techniques for Medium-$\ell$ Program

Because many of the power spectral peaks in the Medium-$\ell$ power spectra were located far enough apart from one another that these individual peaks corresponded to individual $p$-modes in many cases, we were able to develop two different methods of frequency estimation for use on those spectra. As was described in detail by Rhodes et al. (1997), these two methods consisted of the so-called "mean-multiplet" and "averaged-spectrum" methods. In the
“mean-multiplet” method, which was developed by Schou (1992), the peaks are assumed to have a symmetric Lorentzian shape and a maximum-likelihood method is employed to determine the parameters of the Lorentzian profiles. In addition, in this method the peaks in the entire set of $2\ell + 1$ zonal, tesseral, and sectoral power spectra for a given degree are fit simultaneously so that the effects of overlapping peaks can be included in the fits. The resulting $2\ell + 1$ frequencies for a given p-mode multiplet are then averaged together to yield a single frequency for that mode.

In the “averaged-spectrum” method each one of the separate $2\ell + 1$ power spectra at a given degree is first shifted in frequency by an estimate of the rotationally-induced frequency shift for that azimuthal order, $m$. Then, these shifted power spectra are averaged together to create a single, so-called “$m$-averaged” power spectrum. For the Medium-$\ell$ Program this averaging process was repeated for every degree between one and 300.

The results of a systematic comparison of the frequencies and frequency errors which resulted from the use of these two peak-fitting techniques on both 60-day and 144-day subsets of the SOHO mission were recently published in Rhodes et al. (1997). Differences in the radial profile of sound speed inside the sun as inferred from inversions of these independent frequency datasets were also contained in that article. Additional scientific results based upon both sets of MDI Medium-$\ell$ frequencies have been published by Kosovichev et al. (1997, 1998a, 1998b).

2. MDI FULL-DISK PROGRAM

In the MDI FD Program the entire visible hemisphere of the Sun is imaged onto the instrument’s 1024x1024-pixel CCD detector. However, in contrast to the Medium-$\ell$ Program, which reduces the spatial resolution of these full-disk images by binning them with a set of Gaussian weights in a square 5x5-pixel grid, the FD retains the original resolution of these full-disk images when they are transmitted to the ground. The approximately 2 arcsec pixel size of each FD image allows spherical harmonic modes to be measured in principle for $\ell \leq 1500$. On the other hand, the large number of pixels in each FD image requires a much higher telemetry rate (160 kbps) for complete transmission than is required for the reduced-resolution images of the Medium-$\ell$ Program (5 kbps). Consequently, the FD images can only be transmitted to the ground when the SOHO spacecraft is being tracked by one of the stations of the Deep Space Network. This requirement in turn means that nearly-continuous time series of FD observations can only be obtained during limited portions of each year of SOHO operations. In particular, the FD Program only operates for one two- or three-month interval per year and then for one additional three-day long run in each of the other months of the year. The annual two- or three-month FD observing runs are known as the MDI Dynamics Runs. Most of the high-degree results which we will present in this paper were obtained during the 1996 Dynamics Run.

There are several different types of images which can be transmitted to the ground during one FD observing run. We have employed three of these different types of images in our past processing: full-disk Dopplergrams, full-disk line-intensity maps, and full-disk line-depth maps. We have in fact produced frequencies from all three of these different observables. However, most of the results we will present in this paper were generated from the processing of several different time series of Dopplergrams.

2.1. 1996 MDI Dynamics Run

The 1996 Dynamics Run began on May 23 and ended on July 24. The power spectra which we have been working with were computed from a time series of 87,480 minutes (or 90.75 days) in duration. The original duty cycle of this time series was 95.3%. After a small number of short gaps were filled, the duty cycle of the final time series we employed was 97.5%. This time series of Dopplergrams was processed using the MDI software pipeline at Stanford. For the present time an upper degree limit of 1000 was selected and a total of 1,001 sets of one-dimensional power spectra were computed. For $\ell = 0$ only a single $m = 0$ spectrum was generated, but for all non-zero degrees a set of $2\ell + 1$ zonal, tesseral, and sectoral power spectra were created. Hence, the final product of the pipeline processing was a total of 1,002,001 one-dimensional power spectra, in which the power density was a function of frequency ranging from 0 to 8333.1425 µHz.

2.2. Application of Averaged-Spectrum Method To Full-Disk Observations

We then processed these 1,001 sets of power spectra in one of two ways, depending upon whether our final product was to be a set of fitted parameters for the different power spectral peaks or a set of rotationally-induced frequency splittings. Initially, we were interested in determining the frequencies, widths, power densities, and the associated uncertainties of as many p-mode peaks as we could measure. Because so many of the peaks in the high-degree power spectra which we computed from the FD observations were not visible as isolated p-modes, we employed only the “averaged-spectrum” method of frequency estimation for these spectra. Consequently, we needed to generate a set of $m$-averaged power spectra similar to those which we had employed in our fitting of the MDI Medium-$\ell$ Dopplergrams.

We computed these 1,001 $m$-averaged power spectra using a set of Legendre splitting coefficients (the so-called “$a$-coefficients” which we will return to shortly) which were derived by averaging together the splitting coefficients obtained for the different $p$-mode multiplets of a given degree into a single set of $a$-coefficients for that degree. The coefficients which were averaged in this process came from the results of the Medium-$\ell$ observations for all degrees between $\ell = 1$ and $\ell = 134$. For higher degrees we simply used the values for $\ell = 134$.

The 1,001 $m$-averaged one-dimensional power spectra which resulted from the entire MDI 1996 Dynamics Run are displayed in a two-dimensional $\ell$-$\nu$ diagram.
in Figure 1. Each of the one-dimensional spectra has been plotted so that its frequency axis increases to-ward the top of the figure, while the spectra for the different degrees are displaced horizontally. The $\xi$ value increases from 0 at the left to 1000 at the right. The observed power density is scaled such that the strongest peaks are shown in white.

2.3. Differences Between Mode-Fitting and Ridge-Fitting

Due to the widely-varying signal-to-noise ratios of the peaks in different portions of the $\ell$-$\nu$ plane, the averaged-spectrum fitting procedure did not always converge to yield a suitable set of modal parameters. In fact, as will be shown in Figure 3a, isolated $\ell$-mode peaks could be measured only for degrees less than about 220, with the exception of the fundamental mode, which has been detected as a set of isolated peaks up to $\ell = 296$.

In addition, even for degrees for which some individual modal peaks can be identified at low frequencies, these peaks become blurred together as the frequency is increased. Thus, at high frequencies for both low- and intermediate-degrees and at all frequencies for $\ell \geq 300$ the mean-multiplet fitting technique can no longer be applied to a set of isolated peaks which are separated by low levels of background power. Instead, the individual peaks become so closely spaced in frequency that they begin to overlap with the spatial sidelobes of the peak at adjacent degrees which are present in the spectrum of interest. This overlap of peaks and sidelobes produces a so-called “ridge” of observed power which must be fit instead of a set of isolated peaks. In such cases individual modal peaks can no longer be resolved in the $m$-averaged power spectra and the fitted frequency represents the frequency of the centroid of the ridge rather than that of the targeted mode. Hence, any asymmetry with respect to the target mode frequency (e.g. an asymmetry in the leakage matrix and/or the power density distribution) will introduce a systematic shift in the resulting ridge-fit frequency away from the “true” modal frequency.

We illustrate these effects in Figure 2. In Figure 2a we show the $\ell = 150, n = 2$ multiplet along with the fitted profile from our “averaged-spectrum” fitting routine as the solid line. It is evident that when the peaks begin to coalesce into a ridge of observed power the varying background power between the peaks is not adequately described by the fit of a single Lorentzian profile even when a linearly-sloping background is included in the fitted profile. In Figure 2b we show the observed multiplet for $\ell = 260, n = 2$ with the fit from the above procedure superimposed upon it. Clearly, in this case the individual peaks have coalesced into a single, broad ridge of power. The location of $\ell = 260, n = 2$ modal peak is not identifiable in this ridge as was the case for the $\ell = 150, n = 2$ peak.

This blending of the observed peaks into a set of broad ridges is the cause of the principal difficulty which we face in the processing of the observations of the MDI FD Program. The fact that we are forced to fit broad peaks, which are not symmetric in shape due to several instrumental and physical reasons, is what causes our initial estimates of the high-degree $\ell$-mode frequencies and frequency splittings to be systematically incorrect. Furthermore, since we do not have isolated modes whose frequencies we can definitely identify with a set of sharp, isolated peaks for a given degree, we have an inherent uncertainty in assigning the proper degrees to the peaks in a given spectrum. Hence, we will always have more uncertainty in the assignment of the frequencies of the high-degree modes than we do in the case of the low- and intermediate-degree modes.

2.4. Development of Frequency-Correction Methods for Ridge-Fits

The problems inherent in the fitting of broad ridges rather than sharp, isolated peaks of observed power were first addressed in the literature by Libbrecht and Kaufmann (1988). These authors proposed a multiple-linear-regression technique which they then used to correct the frequencies of high-degree $\ell$-modes which they had measured using power spectra computed from observations obtained at the Big Bear Solar Observatory (BBSO). They described how such an approach was necessary to correct for the inherent uncertainty in the image scale of their set of solar observations and for the variations in the observed power density as a function of degree which were caused by both instrumental and solar effects. Libbrecht and Kaufmann (1988) showed how both of these problems resulted in a so-called “frequency pulling” of the measured ridge-fit frequencies away from the “true” solar frequencies for the high-degree modes.

Subsequently, Korzennik (1990, section 5.2) in his analysis of 1988 Mt. Wilson Observatory (MWO) high-degree observations, recast the equations used by the BBSO group and refined their multiple-regression model. Due to our familiarity with Korzennik’s multiple-regression approach, we began our FD processing by employing Korzennik's numerical codes. When we did so, we found that programs were inadequate to correct the more accurate MDI observations. Specifically, his correction codes employed polynomial fits to various numerical functions which were numerically unstable and which introduced artificial “wiggles” into the frequency corrections. These problems resulted in a set of poorly-corrected high-degree frequencies which we were forced to abandon.

Consequently, we have developed an alternative frequency-correction procedure for use on the MDI FD spectra. This technique is also a multiple-linear-regression approach; however, it is one which has been substantially revised to improve its numerical stability. In this technique we fit the low- and intermediate-degree power spectral peaks with both a set of narrow fitting ranges and with a set of broad fitting ranges for our Lorentzian profiles. The frequencies which we compute using the narrow fitting ranges are our so-called “modal” frequencies. The frequencies which we obtain from the use of the wide fitting range are our so-called “ridge-fit” frequencies.

In Figure 2c we show the $\ell = 75, n = 4$ multiplet along with the fit obtained using our “narrow fitting range” and the alternative fit obtained for this multiplet using our “wide fitting range.” While it is
not evident at the scale of this figure, the frequencies computed for the two different fits for the same multiplet were in fact slightly different. When we repeated these two types of fits for all of the cases for which we could identify both single modal peaks and broad ridges, we obtained two sets of frequencies. We subtracted these two sets of frequencies on a peak-by-peak basis to obtain a set of “modal” minus “ridge-fit” frequency differences. This set of frequency differences looks very similar to the sample of “modal” minus “ridge-fit” frequencies which Korzennik (1998) presents elsewhere in these Proceedings.

Once we had computed these frequency differences, we then employed them as the dependent variable in our multiple-regression model. In this model we used the frequency, the degree, the logarithm of the full-width at half-maximum, and the derivative of the observed power density with degree of the observed velocity-based power spectral peaks as the independent variables. We then used least-squares techniques to obtain a set of regression coefficients for this model. Lastly, we employed the coefficients of this regression model to compute a set of frequency corrections which we then applied to each raw, ridge-
3. INITIAL HIGH-DEGREE FREQUENCIES FROM 1996 DYNAMICS RUN

Our initial table of MDI FD frequencies consists of the set of 2,554 modal frequencies which is supplemented by the set of corrected ridge-fit frequencies for the 11,110 remaining cases for which we could not obtain such a modal fit. This combined set of frequencies is superimposed upon the MDI FD power spectra in Figure 1. These frequencies are plotted as a set of 13,564 individual black dots; however, at the scale of this figure these dots all appear to be blended together into solid ridges. In Figure 3a we illustrate the portion of the $\ell$-$\nu$ plane where we were able to identify modal frequencies and the remainder of the plane where we could only compute the ridge-fit frequencies.

3.1. Comparison of MDI FD and MWO Modal-Fit Frequencies

In Figure 3b we show the comparison of our modal-fit frequencies with a set of 1,855 simultaneously-obtained modal-fit frequencies obtained from the MWO 60° Solar Tower. The mean difference of these two sets of modal frequencies was equal to $-22.5 \text{ nHz}$ with a standard deviation of 238 nHz. Hence, our MDI and MWO modal frequencies agreed to within 0.1 standard deviation. As Figure 3b illustrates, there was no measureable frequency dependence to these modal frequency differences. Nor was there any discernable degree dependence to these differences. These results attest to the validity of the averaged-spectrum method of frequency estimation and to the accuracy of both the MDI and MWO sets of observations.

3.2. Comparison of MDI Ridge-Fit Frequencies with Other Datasets

We now will compare our MDI high-degree ridge-fit frequencies with several other sets of observed frequencies. In Figure 4 we show the comparison between our frequencies and those from the 22-25 June 1993 run of the NSO High-L Helioseismometer (Bachmann et al., 1995). The frequency differences in the sense $\text{MDI} - \text{HLH}$ is shown in Figure 4a, while the degree dependence is shown in Figure 4b. Figure 4b shows evidence for a clear image scale error in one or both of these datasets.

In Figure 5 we show the similar comparison between our corrected MDI high-$\ell$ frequencies and a corresponding set of corrected ridge-fit frequencies which we computed from a simultaneous ground-based dataset which was obtained during the 1996 Dynamics Run at the MWO 60° Solar Tower. The frequency dependence of the differences between these two datasets is shown in Figure 5a, while the degree dependence is shown in Figure 5b. These two plots indicate that the simultaneous MWO frequencies agree much more closely with the MDI frequencies than do the 1993 HLH frequencies. Furthermore, the slope of the degree dependence of the MDI-MWO frequency differences of Figure 5b is so much smaller than the slope of the differences shown in Figure 4b, that we believe it is most likely the HLH measurements which were reduced using the incorrect image scale.

We also compared the one-sigma frequency errors computed from our MDI ridge-fits with the corresponding one-sigma statistical errors quoted by Bachmann et al. (1995) for the 1993 HLH ridge-fits. We found the HLH errors to be as many as 70 times larger than our MDI errors for corresponding modes. This comparison also showed that the HLH errors were much greater than could be explained simply by the different durations of the HLH and MDI time series. Since the one-sigma frequency errors are used in structural inversions to weight the different frequency estimates, the relatively small sizes of the MDI errors in comparison to the HLH errors indicate that the MDI frequencies will provide much tighter constraints on the solar internal thermodynamic structure than have been provided previously by the HLH.
Figure 3. (left) $L$-$v$ coverage of our MDI modal and ridge-fit frequencies. The modal fits are shown as small dots, while the ridge-fit frequencies are shown as large squares. (right) The frequency dependence of the differences between our MDI modal frequencies and a set of simultaneously-observed MWO modal frequencies.

Figure 4. (left) The frequency dependence of the differences between our corrected set of MDI ridge-fit frequencies from the 1996 SOI Dynamics Run and the corrected ridge-fit frequencies from the June 22-25 1993 observing run of the NSO High-L Helioseismometer. (right) The degree dependence of $v_{\text{MDI}} - v_{\text{HLH}}$. The slope of these differences is indicative of an image scale error, most likely in the HLH observations.

Figure 5. (left) The frequency dependence of the differences between our corrected set of MDI ridge-fit frequencies and a simultaneous set of ground-based frequencies obtained during the 1996 Dynamics Run at the MWO 60' Solar Tower. (right) Degree dependence of $v_{\text{MDI}} - v_{\text{MWO}}$. Both of these comparisons show that the MWO observations agreed much more closely with the MDI frequencies than did the HLH frequencies.

observations.

3.3. Comparison of MDI and Theoretical High-Degree Frequencies

We now illustrate the scientific value of these new 1996 MDI high-degree frequencies by comparing them with the computed frequencies from several recent solar models. First, in Figure 6a we compare our MDI high-degree frequencies with a set of theoretical frequencies computed for the same modes using the Standard Solar Model S of Christensen-Dalsgaard et al. (1996). What is most intriguing about this figure is the fact that the major disagreement between the observed and the theoretical frequencies does not continue to increase without limit with increasing frequency, but instead the observed frequencies come back into reasonable agreement with those from the model above frequencies of 6000 $\mu$Hz. We believe that this figure indicates that the disagreement near 5000 $\mu$Hz is most likely due to a combination of an
inadequate treatment of the atmospheric layers in Model S, although the neglect of non-adiabatic influences on the theoretical frequencies as described by Rhodes, Ulrich and Simon (1977) may also be partly responsible.

In Figure 6b we compare our MDI 1996 Dynamics Run frequencies with a set of frequencies computed from a Los Alamos solar model. These theoretical frequencies were provided to us by Dr. Joyce Guzik and include non-adiabatic frequency corrections. These corrections are known to become too large for frequencies above 4000 μHz, (that is they are known to over-correct the raw frequencies above this frequency; Guzik, 1997). Nevertheless, we find it interesting that these corrections do decrease the discrepancies between the observed frequencies and those of Model S which were shown in Figure 6a for those frequencies where they are thought to be valid (i.e. below 4000 μHz).

In Figure 7a we show the frequency dependence of the scaled differences between the observed MDI and theoretical Model S frequencies. In contrast to Figure 6a, in which we employed the unscaled frequency differences, in Figure 7a we have scaled the observed minus theoretical frequency differences by the relative mode inertia. This scaling serves to reduce the magnitude of the frequency differences and it tends to minimize the effects of the different degrees of the modes. In Figure 7b we present a comparison with an alternative solar model which diminishes the discrepancies shown in Figure 7a. Here we show the scaled differences between our MDI FD frequencies and the new set of theoretical frequencies which have been computed using the so-called Gas-Gamma Model (GGM), (see Rosenthal et al. 1998a, and 1998b for a discussion of this new model). These scaled frequency differences are about two times smaller than the scaled (MDI - Model S) differences. Nevertheless, there are still systematic differences between the MDI FD frequencies and the GGM frequencies which need to be removed. Some of these remaining differences may be due to the neglect of non-adiabatic effects in the computation of the GGM theoretical frequencies.

3.4. Radial Structural Inversion of MDI High-Degree Frequencies

As a demonstration of the scientific usefulness of the MDI high-degree p-mode frequencies, we show in Figure 8 the results of a one-dimensional structural inversion which we computed from a set of 3,673 combined modal and ridge-fit frequencies. The entire radial profile of the square of the difference in the sound speed inferred from the observed frequencies and the standard solar model S of Christensen-Dalsgaard et al. (1996) is shown here in Figure 8a. For much of the solar interior this profile looks very similar to earlier profiles obtained from inversions of our Medium-f frequencies, such as those shown in Figure 10 of Rhodes
et al. (1997).

What is unique about this inversion is the sharp drop below the surface that is shown at the right-side of the panel. In Figure 8b we show this sub-surface portion of the inverted sound speed difference profile on an expanded horizontal scale. It is clear from Figure 8b that our new high-ℓ frequencies are going to make it possible to refine the solar model in the shallow sub-photospheric layers where it is certainly in error. It will be interesting to repeat this inversion using kernels computed from the GGM instead of Model S.

3.5. Comparison of MDI Velocity and Intensity Frequencies

As we mentioned earlier, the MDI FD program is also capable of obtaining observations of intensity oscillations in addition to time series of Dopplergrams. For some of the brief, three-day FD runs simultaneous velocity and intensity observations have been obtained. We have computed raw, uncorrected ridge-fit frequencies for the July 21-23, 1996, portion of the 1996 Dynamics Run. For this run we employed three simultaneous sets of m-averaged power spectra. We employed our ridge-fitting code on all three sets of power spectra. In Figure 9a we show the differences in the frequencies which we computed from the velocity-derived and intensity-derived power spectra. This figure shows the most pronounced frequency dependence of any of the different types of frequency differences that we have computed to this time. These velocity-intensity frequency differences are larger than the changes which are introduced into our fitted frequencies when we allow for the effects of the asymmetric shapes of the observed power spectral peaks.

3.6. Comparison of p-Mode Properties at Two Heights in the Solar Atmosphere from Simultaneous MDI-MWO Observations

In contrast to Figure 5a, in which we compared our simultaneous MDI and MWO ridge-fit frequencies over a limited frequency range, here in Figure 9b we compare these two sets of frequencies over their entire frequency range. We do so to illustrate the systematic frequency differences which exist at high frequencies in the MDI and MWO datasets. We believe that these differences are entirely due to the difference in the height-of-formation of the photospheric Ni I line used by the MDI instrument and of the chromospheric Na II lines employed by the Magneto-
Optical Filter at MWO. We believe that it is not a coincidence that the MDI-MWO frequency differences are the largest where the MDI-Model S frequency differences were also the largest in Figure 9a. Furthermore, when we binned the frequency differences shown in this figure in 100-$\mu$Hz wide bins, we could see clear evidence for an oscillation in these frequency differences for frequencies above 6200 $\mu$Hz. These binned MDI-MWO frequency differences are presented by Rhodes et al. (1998). We believe that these oscillations may tell us something about the relative atmospheric heights of the source of the $p$-modes themselves and of the spectral lines used in our two instruments. While the high-frequency modes do have aliases of frequencies from above the Nyquist frequency complicating their profiles, the exact same temporal sampling was used in space and on the ground for the two sets of observations which yielded the frequency differences of Figure 9b. Hence, we would have expected any alias-induced alterations of the ridge-fit frequencies to be the same for both sets of frequencies.
4. INITIAL HIGH-DEGREE ROTATIONAL SPLITTINGS FROM 1996 DYNAMICS RUN

In addition to obtaining the first corrected high-degree $p$-mode frequencies from the MDI FD Program, we have also modified the rotational splitting analysis software which Kozennik (1990) developed for the 1988 MWO observations to employ the MDI unaveraged power spectra as the input spectra. We have employed this code in two different ways thus far. First, to obtain a quick overview of the degree dependence of the Legendre splitting coefficients for the modes accessible to the FD program, we computed a single set of fifth-order $a$-coefficients for a set of 42 different degrees ranging between 0 and 1000. Such a single set of splitting coefficients for a given degree is known as a set of "n-averaged" splitting coefficients. While such averaged splitting coefficients are no longer the state-of-the-art for use in rotational inversion software, we show the entire set of 42 n-averaged splitting coefficients in Figure 10a in order to compare them with similar non-averaged coefficients obtained from the simultaneous MWO observations in Figure 10b and with the non-averaged MDI coefficients which are shown in Figure 10c.

Next, in order to illustrate that all of the MDI and MWO high-degree splitting coefficients are in need of systematic corrections due to frequency-pulling effects of the p-mode ridge positions, we have also included in Figure 10d the set of odd $a$-coefficients which we computed from the Medium-ell observations which were also acquired during the 1996 Dynamics Run. Also, in order to show the systematic differences between the FD coefficients and the Medium-ell coefficients, we have included lines in the panels which show the value of the corresponding coefficients obtained from the rotation of magnetic features in the photosphere by Snodgrass (1983). It is clear from this comparison that the Medium-ell coefficients agree reasonably well with the surface rotation rate coefficients, but that the higher-degree ridge-fit splittings are all systematically displaced away from the surface values. The change in the behavior of the $a_3$, $a_5$, and $a_7$ splitting coefficients between degrees of 120 and 200 is caused by the blending together of the p-mode peaks into ridges in each of the $2l+1$ separate spectra which are employed in the measurement of the splittings.

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REFERENCES

Christensen-Dalsgaard, J., Däppen, W., and the GONG team, 1996, Science, 272, 1286
Guzik, J., 1997, private communication
Kozennik, S. G. 1998, these Proceedings
Schou, J. 1992, On the Analysis of Helioseismic Data, PhD. Dissertation, Aarhus University