Linear Force-free Magnetic Field over an Active Region with Due Regard to Coronal Magnetic Field

V. I. Abramenko, V. B. Yurchishin

Crimean Astrophysical Observatory, 334413, Nauchny, Crimea, Ukraine

Abstract. A numerical code for the reconstruction of the linear force-free magnetic field in a bounded domain (e.g. a rectangular box, \( \Omega \)) is presented. The Dirichlet boundary value problem for the Helmholtz equation is solved for the \( B_z \) component specified at the \( \Omega \) boundary. Chebyshev’s iteration method with optimal rearrangement of the iteration parameters sequence was used. The solution is obtained as for the positive-definite, as well as for the non-sign-definite difference analogue of the differential operator \( \nabla^2 u + \alpha^2 u \). Specifying two scalar functions \( B_x \) and \( B_y \) on the intersection of the lateral part of the \( \Omega \) boundary with a selected plane \( z = \text{const} \) and using \( B_z \) inside \( \Omega \), we have found \( B_x \) and \( B_y \) throughout \( \Omega \). The root-mean-square deviation of the analytic solution \( \mathbf{B} \) from the calculated \( \mathbf{B}' \) does not exceed 1.0%. The routine is applied here to the AR NOAA 7216. For that the \( B_z \) component of the photospheric field was used, the available information about coronal magnetic fields taken as complementary data. It is shown that the coronal magnetic field of the AR 7216 (4 July 1992 at about 00:35 UT, N12, E17) was non-linear force-free with positive \( \alpha \) decreasing with height. The electric current density in coronal loops decreases with height. The sense of the twisting of the coronal field (clockwise) is untypical for active regions in the northern hemisphere.

1. Introduction

It is well known that in the modeling of the linear force-free field (LFFF) above an active region (AR) it is impossible to calculate the magnetic field uniquely using only the \( B_z \) component of the magnetic field \( \mathbf{B} \) at the photosphere (Chiu and Hilton 1977). One should have some additional information. The use of the transverse field components at photospheric level is not desirable due to their non-force-freeness (Klimchuk et al. 1992). So it is very desirable to use in the modeling the \textit{a priori} information about the coronal magnetic fields, especially when taking into account the fact that just coronal magnetic structures seem to be force-free or current-free.

We propose a numerical routine (Section 2) allowing to calculate the LFFF using \textit{a priori} information about the coronal magnetic fields (say, potentiality of external loops of an AR). The application of the routine to the AR NOAA 7216 (Section 3) allowed us to construct a reliable model of its coronal magnetic field.
2. Mathematical Formulation

The algorithm renders it possible to calculate the linear force-free field \( \mathbf{B} \) in a bounded volume (a rectangular box, \( \Omega \), with a boundary \( G \)), provided that \( B_z \) is specified on \( G \). So, for \( B_z \) we have to solve the Dirichlet boundary-value problem (DBVP) for the Helmholtz equation:

\[
\nabla^2 B_z + \alpha^2 B_z = 0,
\]

(1)

\[
B_z|_G = \psi
\]

(2)

We proposed to solve this DBVP using Chebyshev's iteration method (Samarsky & Nikolaev 1978) with optimal rearrangement of the iteration parameters sequence (Abramenko & Yurchishin 1996). Two kinds of routines were devised: (a) for the positive-defined operator \( \hat{A} \) – the difference analogue of the differentiation operator \( Au \equiv \nabla^2 u + \alpha^2 u \) on our grid (for the case \( \alpha^2 < \delta \)); (b) for the non-sign-defined \( \hat{A} \) (for the case \( \alpha^2 > \delta \)). Here \( \delta \) denotes the minimum eigenvalue of the 3D Laplacian on our grid. This allowed us to calculate the LFFF not only under the restriction \( \alpha^2 < \delta \), as in most existing LFFF routines (see, e.g., the review by Sakurai 1989), but also in the case of \( \alpha^2 > \delta \), except for one requirement: \(|\alpha|\) must not coincide precisely with any eigenvalue of the 3D Laplacian. So, it becomes possible to broaden \( \Omega \) without reducing \(|\alpha|\).

As soon as \( B_z \) is calculated throughout the volume of integration \( \Omega \) one can calculate \( B_x \) and \( B_y \) in the same volume. For that \( B_x \) and \( B_y \) must be specified at the intersection \( C \) of the lateral sides of the \( \Omega \) with a plane \( z^* = \text{const} \). The 2D modification of the same Chebyshev’s iteration scheme can be used (for details see Abramenko & Yurchishin 1996).

The code ensures a good enough accuracy of the LFFF reconstruction: in the test, the root-mean-square deviation of the analytic solution from the calculated one does not exceed 1.0%.

3. Application to the AR NOAA 7216

The method was applied to the AR NOAA 7216 observed on 4 July, 1992. The vector-magnetogram of this large, strong bipolar active region was obtained with the Huairou videomagnetograph of Beijing Astronomical Observatory, China (Wang et al. 1996) in the photospheric spectral line Fe i 5324.19 Å. The magnetogram was recorded at the period of best seeing conditions at 00:29 – 00:33 UT (Figure 1). According to our H\(\alpha\) observations there were no regular large-scale vortex structures around the main spots. Soft X-ray coronal loops by YOHKOH/SXT at 00:29 UT were used here. They are shown by dotted lines in Figure 2.

We started our coronal field modeling from the potential approach. We made sure that external X-ray loops of the AR practically coincide with the potential lines of force (Figure 2a). This allowed us to make a reasonable assumption that at some distance (corresponding to the height of the external loops) from the main spots the field becomes potential. So, we can use this fact in the LFFF-modeling as an additional a priori information, namely: we can specify the potential field values \( B_z(\text{pot}) \) as boundary conditions at the lateral
and top sides of the $\Omega$. For $B_x, B_y$ calculations we made only one assumption: as a boundary condition, we took $B_x(\text{pot})$ and $B_y(\text{pot})$ at the frame $C$ at the height $z^*$ equal to the height of $\Omega$.

Table 1. Parameters of force-free loops: maximal height above $z = 0$ plane, magnetic field strength and electric current density.

<table>
<thead>
<tr>
<th>$\alpha$ (arcsec$^{-1}$)</th>
<th>$h$ ($10^3$ km)</th>
<th>$B$ (Gauss)</th>
<th>$j$ (A • km$^{-2}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.000</td>
<td>100</td>
<td>3.5</td>
<td>0.0</td>
</tr>
<tr>
<td>0.001</td>
<td>90</td>
<td>5.7</td>
<td>0.6</td>
</tr>
<tr>
<td>0.005</td>
<td>40</td>
<td>27.0</td>
<td>15.0</td>
</tr>
<tr>
<td>0.007</td>
<td>15</td>
<td>150.0</td>
<td>115.0</td>
</tr>
</tbody>
</table>

We have chosen a set of positive $\alpha$ and run several LFFF models (negative values of $\alpha$ yielded unacceptable fit with observed X-ray loops). Some of them are shown in Figure 2(b-d). One can see that the stronger the $\alpha$, the shorter and lower X-ray loop coincides in the best way with the LFFF model. At every panel of Figure 2 the observed loop showing the best fit with the model is denoted
Figure 2. X-ray loops of the AR NOAA 7216 (dotted lines, the same in all panels) and lines of force of modeled linear force-free magnetic field (solid lines) by open circles; their parameters are shown in Table 1. The higher the loop the weaker the electric current in it. Moreover, the density of electric current
decreases with height more rapidly than in any linear model. These results allow to suggest that the coronal magnetic field above a stable, large, strong bipolar active region seems to be force-free, but not linear force-free. This coincides qualitatively with results obtained by Klimchuk et al. (1992) and by Schmieder et al. (1996).

4. Conclusions

A new numerical code for the LFFF modeling in a bounded domain above an active region was applied here to the AR NOAA 7216. The code uses the $B_z$-component of the magnetic field at the photosphere as main data and information about coronal fields as supplementary data. In the considered case of AR 7216, the potentiality of external X-ray coronal loops was used. The potential $B_z$ substitution at the non-photospheric sides of the considered volume as boundary conditions secured a good modeling of the coronal field of the AR 7216.

It is shown that the coronal magnetic field of the AR 7216 on 4 July, 1992 at about 00:29 UT was a non-linear force-free field with positive $\alpha$ decreasing with height, slowly turning into a potential field at a height of $10^5$ km. Electric currents in coronal loops were anti-parallel to the magnetic field and their density decreased with height. The total coronal twisting of this large, quiet, vortexless active region was weak ($\alpha = 0.001$ to $0.007$ arcsec$^{-1}$), but the sense of twisting (right-handed) was nontypical for active regions of the northern hemisphere (Seehafer 1990).

References

Samarsky, A. A., & Nikolaev, E. S. 1978, Methods of Solution of Grid Equations, Moscow: Nauka, 588
Seehafer, N. 1990, Solar Phys., 125, 219