Predicted Starspot Distributions on Pre-MS Stars

Th. Granzer¹, K.G. Strassmeier¹, M. Schüssler², and P. Caligari²

Abstract:
We present an application of the magnetic flux-tube model of Caligari (1995, 1991) to pre-main-sequence stars. This model was originally designed as an explanation for sunspots. The primary goal is to derive the emerging latitude of flux tubes for a large parameter volume including stellar mass, age, and rotational period. Since the original model worked well in explaining certain sunspot features, the next step would be to compare our model predictions with observed starspot distributions obtained by Doppler imaging techniques.

1. Introduction

We study here the behaviour of a thin magnetic flux tube as it rises through the convective layer of a star. A flux tube in pressure equilibrium with its surrounding has a lower gas pressure because its magnetic field contributes \( p_{\text{mag}} = B^2/2\mu \) to the total pressure; \( \mu \) is the magnetic permeability. If we prescribe \( T_{\text{flux tube}} \approx T_{\text{surrounding}} \) and use the perfect gas law \( p_{\text{gas}} = \mathcal{R}\rho T/\tilde{\mu} \), we see that the flux tube must have a density deficit, leading to magnetic buoyancy; here \( \tilde{\mu} \) is the mean molecular weight. In a superadiabatic stratified layer this density deficit grows as the tube rises adiabatically. In a subadiabatic zone however this initial density deficit can be compensated for as the tube rises, leading to a flux tube in mechanical equilibrium characterized by \( \rho_{\text{flux tube}} = \rho_{\text{surrounding}} \).

The only place in a star’s convective region with a subadiabatic stratification is the overshoot layer, the region where convective elements penetrate the radiative core. This zone is characterized by large gradients in the rotation rate and thus can be the location of the strong \( \Omega \) effect needed to produce strong toroidal magnetic fields. In our model we place an axisymmetric flux tube at the bottom of this overshoot layer and perturb it with a non-axisymmetric undulatory excitation. Depending on its initial field strength, magnetic flux, and other parameters, this disturbance will either fade away or grow non-linearly (Ferriz-Mas & Schüssler 1993, 1994); this non-linear growth is usually called Parker’s instability. If the amplitude of the undulatory disturbance grows sufficiently so that the peak of the flux tube reaches out of the (thin) overshoot layer, the superadiabatic stratification in the convection zone will amplify the buoyancy force on the elevated part of the flux tube. This part rises rapidly through the convective layer, while the main part of the flux tube still remains anchored inside the overshoot layer. When the former reaches the photosphere it forms stellar spots.

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In the case of the Sun, the magnetic flux within the sunspots is an average of $10^{22} \text{Mx}$. Accordingly, we focus here only on flux tubes with $10^{22} \text{Mx}$. We track the evolution of the flux tube numerically until the “thin-tube” approximation breaks down. The thin-tube approximation specifies that the tube’s diameter be negligible compared to the pressure scale height, its radius of curvature, and the wavelength of any undulatory excitation. In all cases considered it was the outward drop of the pressure scale height that caused the thin-tube approximation to become invalid. Nevertheless the evolution of the flux tubes could be followed up to $r \approx 0.97 R_\odot$, justifying the assumption that the endpoints of the calculations coincide with the emerging latitude of the flux tube.

2. Numerical Computations

Calculations were carried out for three stellar masses, 0.6, 1.0, and 1.5 $M_\odot$. The stellar models were calculated with an updated version of the Kippenhahn code. New opacities (Alexander & Ferguson 1994), a new equation of state (Gautschy, private comm.), and an improved treatment of the overshoot layer were implemented. As the centrifugal acceleration at the bottom of the convective zone is always less than 2.2% of the gravitational acceleration, we neglect the rotation in the calculation of the stellar models. For each mass, three stellar models at different stages of evolution were considered: H: shortly after the star left the Hayashi track (i.e., 5% of the star’s mass is in the radiative core), T: at a “transition” stage (i.e., 0.1% of the star’s luminosity is produced by nuclear reactions), and one model as the star reaches the main sequence: Z (i.e., 1% depletion of the core’s initial H). The stellar parameters are summarized in Table 1 (see also Fig. 1). In contrast to Caligari (1995), we calculated models for stellar masses other than 1 $M_\odot$ and included the overshoot layer in our calculations even in the very young H models. Note that the 1.5 $M_\odot$ model is almost entirely radiative when it reaches the main sequence, so that the flux tube evolution cannot be calculated.

### Table 1. Stellar parameters of calculated models

<table>
<thead>
<tr>
<th>Sym.</th>
<th>$M/M_\odot$</th>
<th>Age/$10^6$yr</th>
<th>$L/L_\odot$</th>
<th>$R/R_\odot$</th>
<th>$T_{\text{eff}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>H</td>
<td>0.6</td>
<td>7.52</td>
<td>0.172</td>
<td>1.06</td>
<td>3620 K</td>
</tr>
<tr>
<td>H</td>
<td>1.0</td>
<td>2.05</td>
<td>1.12</td>
<td>1.88</td>
<td>4330 K</td>
</tr>
<tr>
<td>H</td>
<td>1.5</td>
<td>0.831</td>
<td>3.98</td>
<td>3.1</td>
<td>4640 K</td>
</tr>
<tr>
<td>T</td>
<td>0.6</td>
<td>13.4</td>
<td>0.107</td>
<td>0.91</td>
<td>3470 K</td>
</tr>
<tr>
<td>T</td>
<td>1.0</td>
<td>8.44</td>
<td>0.47</td>
<td>1.25</td>
<td>4280 K</td>
</tr>
<tr>
<td>T</td>
<td>1.5</td>
<td>6.12</td>
<td>1.68</td>
<td>1.78</td>
<td>4940 K</td>
</tr>
<tr>
<td>Z</td>
<td>0.6</td>
<td>636</td>
<td>0.065</td>
<td>0.568</td>
<td>3870 K</td>
</tr>
<tr>
<td>Z</td>
<td>1.0</td>
<td>145</td>
<td>0.69</td>
<td>0.91</td>
<td>5510 K</td>
</tr>
<tr>
<td>Z</td>
<td>1.5</td>
<td>46.4</td>
<td>4.73</td>
<td>1.46</td>
<td>7055 K</td>
</tr>
</tbody>
</table>

The properties of the overshoot layers of the various stars are summarized in Table 2. For each model, different rotation rates are applied, ranging from
Figure 1. Pre-main-sequence tracks for the three masses considered. The labelled dots represent calculated models (0.6, 1, 1.5 $M_\odot$). **H**: 5% of mass in radiative core, **T**: 0.1% of the star’s luminosity is generated by nuclear reactions, **Z**: ZAMS.

0.25 $\Omega_\odot$ to 63 $\Omega_\odot$. All stars are assumed to rotate rigidly in their outer convection zone. This seems justified since the differential rotation of the present Sun does not significantly influence the properties of rising flux tubes (Caligari 1995); for more rapidly rotating stars, an even smaller amount of differential rotation in the convective zone is indicated by observations (e.g., Hall 1991) and also predicted by theoretical models (e.g., Kitchatinov & Rüdiger 1995).

We first performed a linear stability analysis (Ferriz-Mas & Schüssler 1993, 1994); we perturb an originally axisymmetric flux tube with a small undulatory excitation, $\delta \vec{r} \times \exp(i m \phi_0 + i \omega t)$. Then we linearize the tube’s equations of motion with respect to the amplitude $\delta \vec{r}$ of the disturbance. We find nontrivial solutions of the linearized equations of motion only for certain, usually complex $\omega$. A negative imaginary part of $\omega$ yields an exponentially growing disturbance with a characteristic growth time of $1/\omega$ days. Fig. 2 shows the results of the linear stability analysis. The white areas correspond to stable flux tubes, whereas the coloured ones indicate unstable flux tubes. The contour lines give the growth time of the instability, and the colour codes the order of the undulatory disturbance with the highest growth rate (Parameter $m$ in the formula above). Numerical calculation shows that during most of its evolution the flux tube stays within the overshoot layer and any undulatory disturbance grows indeed on time scales like those derived from the linear analysis. In order to match the activity cycle of a star (e.g., 2 x 11 yr for the Sun) with the time scale of the evolution of a flux tube, we focus on flux tubes with a growth time of some fraction of the activity cycle. According to Caligari (1995) we used growth times of 300 days. As the average distance between lines of equal
Table 2. Parameters of the overshoot layer

<table>
<thead>
<tr>
<th>Sym.</th>
<th>$M/M_\odot$</th>
<th>$\rho_{\text{hot}}/\rho_1$</th>
<th>$\rho_{\text{hot}}/\rho_e$</th>
<th>$\rho_{\text{hot}}/\rho_1$</th>
<th>$\rho_{\text{hot}}/\rho_e$</th>
<th>$\rho_{\text{hot}}/\rho_1$</th>
<th>$\rho_{\text{hot}}/\rho_e$</th>
<th>$\delta(r_{dt})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>H</td>
<td>0.6</td>
<td>75.7</td>
<td>0.103</td>
<td>86.5</td>
<td>0.28</td>
<td>203.9</td>
<td>0.8 $\times 10^{-8}$</td>
<td></td>
</tr>
<tr>
<td>H</td>
<td>1.0</td>
<td>93.6</td>
<td>0.071</td>
<td>175.7</td>
<td>0.31</td>
<td>382.1</td>
<td>3.11 $\times 10^{-8}$</td>
<td></td>
</tr>
<tr>
<td>H</td>
<td>1.5</td>
<td>265.8</td>
<td>0.123</td>
<td>223.0</td>
<td>0.27</td>
<td>427.2</td>
<td>9.48 $\times 10^{-8}$</td>
<td></td>
</tr>
<tr>
<td>T</td>
<td>0.6</td>
<td>184.5</td>
<td>0.291</td>
<td>41.7</td>
<td>0.27</td>
<td>84.55</td>
<td>1.36 $\times 10^{-8}$</td>
<td></td>
</tr>
<tr>
<td>T</td>
<td>1.0</td>
<td>391.5</td>
<td>0.453</td>
<td>35.8</td>
<td>0.27</td>
<td>104.4</td>
<td>3.66 $\times 10^{-8}$</td>
<td></td>
</tr>
<tr>
<td>T</td>
<td>1.5</td>
<td>725.9</td>
<td>0.579</td>
<td>32.5</td>
<td>0.24</td>
<td>105.5</td>
<td>38.4 $\times 10^{-8}$</td>
<td></td>
</tr>
<tr>
<td>Z</td>
<td>0.6</td>
<td>249.6</td>
<td>0.627</td>
<td>9.8</td>
<td>0.24</td>
<td>31.7</td>
<td>3.57 $\times 10^{-8}$</td>
<td></td>
</tr>
<tr>
<td>Z</td>
<td>1.0</td>
<td>439.4</td>
<td>0.704</td>
<td>15.1</td>
<td>0.3</td>
<td>46.0</td>
<td>54.4 $\times 10^{-8}$</td>
<td></td>
</tr>
</tbody>
</table>

The parameters of the flux tubes obtained from the linear analysis are used to study the evolution of a flux tube with a code first described by Caligari (1995). The flux tubes which we use in our calculations start in mechanical equilibrium which requires $\rho_i = \rho_e$ and a slightly increased rotation rate $R_i^2(\Omega_i^2 - \Omega_e^2) = v_A^2$ (see Caligari 1995 for details). Subscripts $i$ and $e$ refer to parameters inside and outside the tube, respectively; $v_A$ is the Alfvén speed, $v_A = B_0/\sqrt{4\pi \rho_i}$.

Flux tubes in stars with small radiative cores are subject to the “pole-slip instability”, meaning that the part of the flux tube that is anchored in the overshoot layer slips towards the stellar pole, in analogy with a rubber band on a polished sphere. The use of a cylindrical coordinate system means that the calculations break down at that point, which accounts for the sudden stops of the trajectories in the 0.6 $M_\odot$ T models. Fig. 3 shows the trajectories of the peaks of the flux tubes through the convective layer.

All of these plots were combined to Fig. 4, showing the most likely emerging latitudes versus rotation rate of the three different masses. For the very youngest models (H) the calculations were limited to axisymmetric ($m = 0$) cases to avoid the pole-slip of the anchored part. One can see from the stability diagrams in Fig. 2 that the $m = 0$ mode is indeed the dominant one for higher latitudes.

We deduce from observations of the Sun that the probability that a flux tube can form in the overshoot layer decreases rapidly with latitude. We have therefore indicated the latitudes that are reached by flux tubes starting high in the overshoot layer (i.e., $\varphi \geq 40^\circ$) with dashed bars in Fig. 4. Note also that the thickness of the bar indicates the density of emerging flux tubes.
Figure 2. Stability diagrams of the stellar masses at three different evolutionary stages (top row : H, middle row T, bottom row Z) all with $\Omega = \Omega_\odot$ and, from left to right with 0.6, 1.0, and 1.5 $M_\odot$. Coloured areas indicate unstable flux tubes at the bottom of the convective layer. The parameter $m$ is the order of the undulatory disturbance with the highest growth rate (see insert in each panel). Note the similarity between the Z model with $M = 0.6$ $M_\odot$ and the T model with $M = 1.5$ $M_\odot$. Since the $M = 1.5$ $M_\odot$ model at the ZAMS is almost entirely radiative, no diagram is given there. (H : 5% of mass in radiative core, T : 0.1% of the star’s luminosity is generated by nuclear reactions, Z : ZAMS).

3. Results and discussion

As can be seen from Fig. 4, the emerging latitude of the flux tubes initially grows with rotation rate in agreement with the results of Schüssler et al. (1996). Schüssler & Solanki (1992) pointed out that the strong polward deflection of flux tubes in rapidly rotating stars is due to an interplay between the buoyancy and the Coriolis force. Since the latter works against the equatorial component of the radially outward directed buoyancy force, the remaining force is almost parallel to the axis of rotation.

The second most important fact appears to be the age of the star or, more precisely, the depth of the convective layer. The deeper the convective layer, the bigger is the polward deflection of the flux tubes. Once again the Coriolis force plays a vital role. As the flux tube rises it conserves its angular momentum, apart from frictional losses. The further out it reaches, the more it must lose angular velocity, leading to a proper motion contrary to the rotation of the
Figure 3. Trajectories of the peaks of the flux tubes as they rise through the convective layer. Only models with $\Omega = \Omega_\odot$ are shown. The graphs hardly differ for the very young (=H) stars, therefore only the solar model is displayed. Note that these cross-section graphs are positioned equal to the appropriate stability diagrams in Fig. 2. $r/R$ is the fractional radius of the star.

surrounding material. This motion gives rise to an inward-directed Coriolis force which balances the equatorial component of the buoyancy force. Since the magnitude of the Coriolis force depends on the velocity of the tube compared to its surroundings, its contribution grows with the depth of the tube’s starting point.

If we compare the three masses we see that the differences in the stability diagrams for the youngest models (H) are negligible. As a consequence, the spot patterns for these three models look very alike. This picture changes at the T stage, where we have important differences in both the spot distribution and the stability diagrams. We see the latitude of starspots increase with decreasing mass, due to the increasing depth of the convective layer. For the main-sequence models (Z) we find similar behaviour to that of the T models—once again correlated to the depth of the convective layer. The model with $M = 1.5 \ M_\odot$ is missing in Fig. 4, since its radiative core reaches up to 0.99$R_*$.

Note that both the stability diagram and the spot pattern for the $M = 0.6 \ M_\odot$ Z model and the $M = 1.5 \ M_\odot$ T model are similar. Both stars have almost the same depth of the convective layer. The model with $M = 1 \ M_\odot$ at an age of about 10 Myr has the same depth of the convective layer as the $M =$
Figure 4. Emerging latitudes $\varphi$ of the flux tubes vs. stellar rotation rate $\Omega$. The top panel shows the youngest models ($H$: 5% of mass in radiative core), the middle panel the transition model ($T$: 0.1% of the star's luminosity is generated by nuclear reactions), and the bottom graph the ZAMS model ($Z$). The thickness of the bars indicates the density of emerging flux tubes. Dashed areas are only reached by flux tubes that were originally located at latitudes $\varphi \geq 40^\circ$ in the overshoot layer.
0.6 \( M_{\odot} \)-Z model and the \( M = 1.5 \, M_{\odot} \) T model. Their stability diagrams are also very much alike. This leads one to believe that the depth of the convective layer is an almost more important factor in determining a star’s spot pattern than its mass.

Fig. 4 T shows that for rapidly rotating stars with \( M = 0.6 \, M_{\odot} \) flux tubes can only emerge at intermediate latitudes if they start at high latitudes in the overshoot layer. This is because in these cases the initial field strengths are so high, that the buoyancy force exceeds the Coriolis by far and governs the rise of the flux tubes.

We know from Doppler images of young stars that polar spots appear to be a relatively common feature. But in our model only the very young stars (H) show emerging flux tubes almost at the poles. Once again we want to mention that our picture describes the development of the flux tube only to a point close to the outburst in the photosphere. We cannot give any clue about possible migrations of the spots after their formation. Still we recommend to compare our models with observations as soon as the latter are available in a statistically sufficient number.

4. Further Work

Doppler imaging can provide star spot maps for comparing our models with real stars. Since Doppler imaging gives us nothing more than a snapshot of a spot distribution, we must use a serial of images of the same star to average out seasonal variations. Doppler images are not easy to obtain, so at the moment we do not have a reasonable data base for constructing average spot diagrams for a given rotation period and stellar mass. On the other hand observations focus not on pre-main-sequence stars, but merely on bright evolved stars, and so it is important to extend the modelling to evolved stars.

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References