A COMPARISON OF TWO APPROXIMATIONS FOR THE COOLING OF HOT POST-FLARE LOOPS

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ABSTRACT

A comparison of two simple approximations for the time evolution of the temperature of hot post-flare loops is discussed. Both these estimates suppose constant density in the loop during cooling (static case of cooling). The first approximation (Cargill et al. 1995) supposes that it is possible to split the cooling into a pure conductive phase, when the radiation is neglected and a pure radiative phase, when the conduction is neglected. The criterion for determining which phase takes place at a given temperature is characteristic cooling times \( \tau_c \) and \( \tau_r \). In the second approximation (Švestka 1987) conduction and radiation are taken into account at the same time. We have modified the formula for thermal conduction taken by Švestka to be the same as in first approximation and used the same formulas for radiative losses. The comparison showed that for static cooling in the phase when radiation dominates the thermal conduction is really unimportant, but in the phase when conduction dominates we can not neglect radiation without considerable impact upon results.

Key words: post-flare loops; cooling.

1. INTRODUCTION

It is generally believed that the post-flare loops cool due to heat conduction and radiation. Heat conduction transports thermal energy from the coronal parts of the loops to the lower colder and denser areas, where part of the energy is radiated away from the loops and the rest is given back to the coronal part as a stream of evaporated particles from the transition region or chromosphere.

The time evolution of the temperature and density along a loop is governed by the full set of hydrodynamic equations, see for example Klimchuk & Mariska (1988). Unfortunately, it is not possible to obtain an exact solution of such a complicated problem analytically and thus it is necessary to use numerical methods and computer simulations. These give quite a good approach to the real situation but their disadvantage is the complexity of the numerical codes and typically long computing times which prevents the use of such methods when the cooling of many different loops is investigated. This occurs for example when the concept of coronal heating by nanoflares is examined (Cargill 1994). However, under certain simplifying assumptions, one can derive straightforward analytical expressions which are useful for estimating the role of individual cooling mechanisms (for their review see Cargill 1994 and Cargill et al. 1995). On the other hand, to estimate typical cooling times in post-flare loops, Švestka (1987) has used an approximate form of the energy-balance equation and computed the temperature variations in the static loop due to both conductive and radiative cooling simultaneously. His approach was then used by Schmieder et al. (1995, 1996) to analyze the post-flare loops observed by SXT-Yohkoh instrument. Since several questions arise concerning the methods and results of computations of the loops cooling (van Driel-Gesztelyi et al. 1996; Švestka, 1996), we compare in this paper various approximate solutions and demonstrate their relevance to post-flare loop studies.

2. DESCRIPTION OF THE COMPARED APPROXIMATIONS

Both compared approximations suppose fully ionized plasma and the dominance of the magnetic pressure over the plasma pressure, so that the mass flow is parallel to the magnetic field and the flux tube is considered to be rigid. The cross-sectional area is taken as uniform along the loop. Due to the presence of the magnetic field the thermal conduction perpendicular to the field lines can be neglected against the parallel conduction. The heat flux along the magnetic field lines \( F_c \) is given by the formula

\[
F_c = \kappa_0 T^5 \frac{\partial T}{\partial s},
\]

where \( T \) and \( s \) are temperature and coordinate along the loop, respectively, and \( \kappa_0 = 10^{-6} \) (CGS units are used).

The radiation losses in the coronal parts of the loops are supposed to be optically thin and for fully ionised plasma, they can be expressed as \( n^2 P_{rad} = n^2 \gamma T^n \), where \( n \) is the electron density and \( \gamma \) and \( \alpha \) are given for different temperature intervals to fit the real temperature dependence of the radiative loss function (Cook et al. 1989; Cargill 1994):

\[
P_{rad} = 2 \times 10^{-23}, \quad T > 10^6 \text{K},
\]

\[
P_{rad} = 3.5 \times 10^{-7} T^{-2.5}, \quad 10^6 < T < 10^6 \text{K},
\]

\[
P_{rad} = 3.5 \times 10^{-22}, \quad 10^5 < T < 10^6, \quad (2)
\]

\[
P_{rad} = 1.1 \times 10^{-27} T^{1.1}, \quad T < 10^5.
\]

For \( T > 10^5 \text{K} \) this quite complicated temperature dependence is often expressed by only one approximate
Figure 1. A comparison of results of different approximations for the cooling of different hot post-flare loops. At the beginning of the cooling of all these loops, thermal conduction is dominant. The greater the ratio $\tau_0/\tau_0$, the more dominant is the conductive mechanism at the beginning of cooling. The dashed line is the first approximation (Cargill et al. 1995), the solid line is the second corrected approximation and the dotted line is the second approximation (Svestka 1987).

The equation which is particularly convenient when deriving analytic formulas for cooling (Priest 1984; Cargill 1994)

$$P_{rad} \approx 6.32 \times 10^{-20} T^{-1/2},$$

where $\alpha = -1/2$ and $\chi = 6.32 \times 10^{-20}$.

2.1. The First Approximation

This approach (Cargill et al. 1995) is based on the analytical solutions of the simplified energy equation (Antiochos & Sturrock 1976; Antiochos 1980). The most fundamental simplifications are that the velocity of plasma along the loops $v_j = 0$, so that $n = \text{const}$ and that either the conductive or radiative term on the right hand side of the energy equation is omitted. Now the solution for pure static conduction is (Antiochos & Sturrock 1976):

$$T(t) = T_0 \left(1 + \frac{t}{\tau_0}\right)^{-2/5},$$

where $T_0$ is the initial temperature and $\tau_0$ is the characteristic conductive time for $T_0$ given by equation

If the conductive term in the energy equation is omitted the pure static radiative solution is (Antiochos 1980):

$$T(t) = T_0 \left[1 - (1 - \alpha) \frac{t}{\tau_0}\right]^{1/(1-\alpha)},$$

where $\tau_0$ is the characteristic radiative time (Eq.(6)) for the temperature at the beginning of the pure radiative cooling.

It follows from the formulas for the thermal flux in the loop (Eq.(1)) and the radiative losses (Eq.(3)) that the conduction is much more important than the radiation at high temperatures, while the radiation is much more dominant at low temperatures in the loop. In this approximation the cooling is split into two phases. At high temperatures the loops cool only due to thermal conduction (the radiation is neglected), while at low temperatures only due to radiation (the thermal conduction is neglected). Of course, for every loop which cools from a high temperature there must exist a temperature region where both mechanisms are roughly equally important. As a criterion of their importance at a temperature $T$ the characteristic conductive and radiative cooling times
Figure 2. A comparison of results of different approximations for the cooling of different hot post-flare loops. At the beginning of the cooling of these two loops radiation is dominant. The lower the ratio $\tau_{r0}/\tau_{e0}$, the more dominant the radiative mechanism is at the beginning of the cooling. The dashed line is the first approximation (Cargill et al. 1995), the solid line is the second corrected approximation and the dotted line is the second approximation (Švestka 1987).

$\tau_c(T)$ and $\tau_r(T)$ are used (Cargill et al. 1995):

$$\tau_c = \frac{3 k_B n L^2}{\kappa_0 T^5/2} , \quad \tau_r = \frac{3 k_B T^{1-\alpha}}{n \chi} ,$$

where $L$ is the semilength of the loop and $k_B$ is the Boltzmann constant. The criterion for the change of the cooling mechanism is the temperature when $\tau_c(T) = \tau_r(T)$. If $\tau_c(T) < \tau_r(T)$ resp. $\tau_c(T) > \tau_r(T)$ the model supposes only conductive resp. only radiative cooling.

2.2. The Second Approximation

In this approximation (Švestka 1987) it is supposed, as in the previous one, that $v_0 = 0$, so that $n = const$. If these simplifications are introduced into the energy equation it is clearly visible that the greatest problem is caused by the partial derivatives with respect to $s$ in the term describing thermal conduction. To solve this problem Švestka suggested replacing them by simple estimates: $\partial T/\partial s \sim T/L$ and $\partial/\partial s \sim 1/L$. When all these simplifications are substituted into the energy equation we obtain

$$- 3 n k_B \frac{dT}{dt} = \kappa_0 \frac{T^{7/2}}{L^2} + n^2 \chi T^\alpha .$$

This is an ordinary differential equation first order for temperature. We solved this differential equation numerically by method Runge–Kutta (e.g. Press et al. 1989).

3. MODIFICATION OF THE SECOND APPROXIMATION

When we compare the total cooling times given by both approximations, computed for loops for which conduction is the more important mechanism at the onset of cooling (see Figure 1), it is clearly visible that Švestka’s total cooling times (given by the second approximation) for all such loops are only roughly 50% of the times given by the first approximation (Cargill et al. 1995). When we make the same comparison for loops for which radiation is dominant at the beginning of the cooling (see Figure 2), it is apparent that the difference between these two approximations is much smaller and is probably decreasing, with decreasing ratio $\tau_{r0}/\tau_{e0}$.

To see why, we can take equation (7) with either the radiative term only (we used the one given by Eq.(3)) or the thermal conduction term only. We obtain two simple ordinary differential equations of first order which can be solved analytically. The solution of the first equation, where only radiation is taken into account, is the same as the formula (5) for static radiative cooling. It is not surprising because Švestka (1987) and Antiochos (1980) used the same simplifications of the energy equation. The solution of the second equation, where only thermal conduction is taken into account, is slightly different from the solution obtained by Antiochos (Eq.(4)). The only difference is the numerical constant in the formula for $\tau_{e0}$ which is now

$$\tau_{e0} = \frac{6 k_B n L^2}{5 \kappa_0 T^{5/2}} .$$

The ratio $\tau_{e0}/\tau_{r0} = 2.5$, so that the effect of thermal conduction in the second approximation (Švestka 1987) is overestimated in comparison with the first approximation (Cargill et al. 1995) and these two approximations of cooling of the hot post-flare loops are not directly comparable. This overestimation of thermal conduction can be compensated by the choice of a suitable constant $a$, which can be used to multiply the conductive term in the equation (7) to make the temperature gradient along the loop the same as in the first approximation case. After making some simple calculations we obtain relation $a = 6/b$ between the constants $b$ in the formula for characteristic conductive time in Cargill’s approximation (here $b = 3$) and $a$ which is the constant for multiplying the conductive term in equation (7), so that if

$$\tau_c = 6 \frac{n k_B L^2}{\kappa_0 T^{5/2}} .$$
then the second corrected approximation is

\[-3nk_B \frac{dT}{dt} = \alpha k_0 \frac{T^{7/2}}{L^2} + n^2 \chi T^2.\]  

(10)

By this correction we obtained an equation, where both the cooling mechanisms are included in the same way as the first model. Therefore the solution of this equation allows us to discuss the influence of neglecting radiation, resp. conduction in pure conductive, resp. radiative phase of the first approximation.

4. CONCLUSIONS

We corrected the conductive term in Švestka’s differential equation (7), which includes simultaneously conductive and radiative terms in such a way that its solutions can be compared with the first approximation (Cargill et al. 1995) for cooling of hot post-flare loops. The comparison of these two simple models (see Figure 1, 2) showed that in this static frame, the effect of conduction on the time evolution of temperature is negligible if \(\tau_c \geq \tau_r\), so that the pure radiative cooling describes the evolution of temperature in this temperature region adequately. In the phase of cooling when thermal conduction dominates, so that \(\tau_c < \tau_r\), we have found considerable discrepancies between these two approximations. They result from neglect of the effect of radiation in the conductive phase in the first approximation.

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