RESONANT ABSORPTION IN THE SOLAR CORONA

R. Erdélyi
School of Mathematical and Computational Sciences, Univ. St. Andrews,
North Haugh, St. Andrews, KY16 9SS, Scotland
Armagh Observatory, College Hill, Armagh, BT61 9DG, N. Ireland

ABSTRACT

The effect of an equilibrium flow on resonant absorption of linear Alfvén waves in coronal loops is studied in compressible viscous MHD.

We find that the efficiency of the resonant absorption process strongly depends on both the equilibrium parameters and the characteristics of the resonant waves.

We show that an equilibrium steady flow can significantly influence the resonant absorption of Alfvén waves in coronal magnetic flux tubes. The presence of an equilibrium flow may therefore be important for coronal heating via resonant absorption of Alfvén waves.

A parametric analysis also shows that the resonant absorption can be strongly enhanced, even up to total absorption of the incoming wave.

Key words: MHD (Alfvén) waves; coronal heating

1. INTRODUCTION

The solar atmosphere is a highly non-uniform plasma which is a natural media for MHD waves. MHD waves might play an important role in explaining the observed high temperatures in the solar corona. These waves can be generated by the granular motions (e.g. foot-point motion) and a part of their kinetic energy can be transformed into heat in the magnetic loops.

When the plasma is non-uniform a continuous spectrum of Alfvén and slow waves may exist in ideal MHD and can lead to resonant absorption.

Let us shortly recall the concept of resonant absorption and continuous spectra of Alfvén and slow waves in ideal MHD given in a recent review by Goossens & Ruderman (1996). We are concerned about the linear 1-dimensional stationary state of a driven problem in cylindrical geometry, where all perturbed quantities oscillate with the frequency of an external driver, \( \omega \). If an external driving is turned on at some instant, both free and forced oscillations are excited in the inhomogeneous system. After a transient time the free oscillations decay due to phase mixing, and only the forced oscillations remain. The full set of the ideal MHD equations can be transformed into two first order coupled differential equations in such an equilibrium state. We can describe the wave propagation inside the flux tube by these two first order linear coupled DE's,

\[
\begin{align*}
D \frac{d(\xi_r)}{dr} &= C_1 \xi_r - C_2 r P_1, \\
D \frac{dP_1}{dr} &= C_3 \xi_r - C_4 P_1,
\end{align*}
\]

where

\[
D = \rho(c^2 + v_A^2)(\omega^2 - \omega_A^2)(\omega^2 - \omega_C^2).
\]

Here \( \xi_r \) denotes the radial component of the Lagrangian displacement and \( P_1 (P_1 = p_1 + B \cdot B_1 / \mu) \) is the perturbed total pressure. The coefficients \( C_1, C_2, \) and \( C_3 \) depend on the equilibrium quantities and on the frequency \( \omega \), an they take the form

\[
\begin{align*}
C_1 &= Q \omega^2 - 2m(c^2 + v_A^2)(\omega^2 - \omega_C^2)T/r^2, \\
C_2 &= m^2/(r^2 + k^2), \\
C_3 &= D \left\{ \rho(\omega^2 - \omega_A^2) + r \frac{d}{dr} \left[ \frac{1}{\mu} \left( \frac{B \cdot B_1}{r} \right)^2 \right] \right\} + Q^2 - 4(c^2 + v_A^2)(\omega^2 - \omega_C^2)T^2/r^2,
\end{align*}
\]

where

\[
T = \frac{f_B B_\phi}{\mu},
\]

and

\[
Q = 2m^2 B_\phi^2 / \mu r.
\]

The notations have their usual meaning. In particular: \( c \) is the sound speed, \( \omega_A \) is the local Alfvén frequency, \( v_A \) is the Alfvén velocity, and \( \omega_C \) is the local cusp frequency. The squares of these quantities are defined as

\[
\begin{align*}
c^2 &= \gamma p / \rho, \quad v_A^2 = B^2 / (\mu \rho), \quad \omega_A^2 = f_B^2 / \mu \rho, \\
\omega_C^2 &= c^2 \omega_A^2 / (c^2 + v_A^2),
\end{align*}
\]
where \( f_B = \vec{k} \cdot \vec{B}, \ \vec{k} = (0, m/r, k) \).

Equations (1) define an eigenvalue problem with \( \omega^2 \) as eigenvalue parameter when they are supplemented with boundary conditions. In a driven problem \( \omega \) is prescribed. Since the Alfvén frequency \( \omega_A \), the cusp frequency \( \omega_C \) depend on the radial coordinate \( r \). Equations (1) have mobile regular singularities, defined by

\[
\omega^2 = \omega_A^2(r), \quad \omega^2(r) = \omega_C^2(r).
\]

These two equations define the Alfvén resonance point and the slow resonance point, respectively. The continuous eigenspectra lead to two important phenomena, namely resonant absorption and phase mixing. If the inhomogeneous plasma is driven externally at a frequency \( \omega \) that falls in the band of the continuous eigenfrequencies of the system, the amplitude of the oscillations excited in the system peaks at the resonance point where the frequency of the local field line (\( \omega_A \) for Alfvén waves and \( \omega_C \) for slow cusp waves) matches the frequency \( \omega \). (See Fig. 1)

![Figure 1. Schematic picture of the resonant absorption of Alfvén waves.](image)

At the position where the resonant conditions \( \omega^2 = \omega_A^2(r_A) \) or \( \omega^2 = \omega_C^2(r_C) \) is fulfilled the global external wave motion will be locally in resonance on a particular magnetic surface. The resonance causes energy to build up at the resonant magnetic surface (i.e. at the resonant position \( r = r_A \) and/or \( r = r_C \)) at the expense of the global motion. In ideal MHD the resonantly accumulated energy will be infinite at the resonant position(s).

Eigenfunctions that correspond to driven frequencies in the continuous spectra in Eqs. (1) are improper in the sense that they contain a non-square-integrable singularity at the resonant point \( r_A \) and/or \( r_C \), respectively, where the resonant conditions \( \omega^2 = \omega_A^2(r_A) \) and \( \omega^2 = \omega_C^2(r_C) \) are matched in ideal MHD.

In order to compute the absorption of wave energy and the heating of a plasma we have to include dissipative effects in the MHD equations. If there is even a small amount of dissipation (i.e. viscosity or electrical resistivity) the resonant waves are not anymore characterized by infinite spatial singularities at the ideal resonant position. The inclusion of nonideal effects removes the singularity of the governing equations, and both the energy density and the spatial gradients become large but finite values. In this case the energy which has been transferred to the resonant magnetic surface can be converted into heat.

Resonant absorption of Alfvén waves up to now was almost exclusively studied in static equilibrium states. However, observations reveal both upwards and downwards mass-flows in the magnetic flux tubes along their longitudinal axes. Typical speeds are \( 5 - 50 \) km/sec (see e.g. Doyle et al. 1997). Steady equilibrium flows (velocity fields) change the properties of the MHD waves. The most obvious effect of an equilibrium flow is the Doppler-shift of the Alfvén and slow continua. For the heating by resonant absorption this Doppler-shift is important because the frequency range where the mechanism operates is changed.

In addition to this Doppler-shift an equilibrium flow can have more subtle effects on the absorption of waves which are hidden in the equations. A systematic study of resonant absorption of MHD waves in stationary equilibrium models is needed to assess the effects of an equilibrium flow on resonant absorption.

Erdélyi & Goossens (1996) studied the effect of an equilibrium mass flow on the resonant absorption of \( p \)-mode acoustic waves in cylindrical magnetic flux tubes in compressible viscous MHD. They showed that the presence of an equilibrium flow can be important for resonant absorption of acoustic \( p \)-modes.

In this paper our attempt is to study the effect of a velocity shear on the rate of resonant absorption of Alfvén waves in coronal loops in linear viscous MHD. The viscosity is described by its classical tensorial form, i.e.

\[
\pi_{\alpha\beta} = -\eta \left( \frac{\partial u_\alpha}{\partial x_\beta} + \frac{\partial u_\beta}{\partial x_\alpha} - \frac{2}{3} \delta_{\alpha\beta} \nabla \cdot \vec{v}_l \right) - \\
-\eta_0 \delta_{\alpha\beta} \nabla \cdot \vec{v}_l, \quad (8)
\]

where \( \delta_{\alpha\beta} \) is the Kronecker delta. The two viscosity coefficients are assumed to be constant.

Our numerical results indicate that an equilibrium mass flow can significantly influence the absorption of Alfvén waves. The presence of an equilibrium flow is therefore important for the power loss of Alfvén waves as well.

The present paper is organized as follows. In Section 2 we refer to the basic set of linear differential equations that govern linear waves about a stationary, 1D equilibrium state in viscous MHD and we also refer to the applied numerical method for solving these equations. Section 3 gives a short description of the model of coronal loops applied. In §4 we show and discuss the numerical results. Section 5 summarizes the main conclusions of the recent numerical simulations.
2. VISCOUS MHD EQUATIONS GOVERNING ALFVÉN WAVES

Resonant absorption of Alfvén waves is studied in viscous MHD in coronal magnetic flux tubes. The coronal loops are approximated by cylindrical axisymmetric plasma columns. The cylindrical plasma columns are 1-dimensional which means that in a system of cylindrical coordinates (r, φ, z) with the z-axis coinciding with the axis of symmetry of the plasma column the equilibrium quantities depend on r only and are independent of φ and z. Gravity is ignored. The cylindrical geometry closely resembles that of magnetic loops observed in the solar corona which have a large aspect ratio ε = L/πR with L the length of the loop and R the radius. Although we have an infinitely long loop we have taken ε = 20, a typical value for coronal loops.

The linearized equations of viscous MHD are used to determine the resonant absorption, i.e.

\[ \frac{\partial p_1}{\partial t} = -\nabla \cdot (\rho_0 v_1 + p_1 v_0), \]

\[ \rho_0 \left[ \frac{\partial v_1}{\partial t} + (v_0 \nabla) v_1 + (v_1 \nabla) v_0 \right] + p_1 (v_0 \nabla) v_1 = -\nabla p_1 + \frac{1}{\mu} (\nabla \times B_0) \times B_1 + \frac{1}{\mu} (\nabla \times B_1) \times B_0 - \nabla \cdot \mathbf{F}, \]

\[ \frac{\partial v_0}{\partial t} + v_0 \cdot \nabla T_1 = -\rho_0 v_1 \cdot \nabla T_0 - (\gamma - 1) \rho_0 T_0 \nabla \cdot v_1, \]

\[ \frac{\partial B_1}{\partial t} = \nabla \times (v_1 \times B_0) + \nabla \times (v_0 \times B_1), \]

\[ \frac{\partial p_1}{\partial t} = \frac{p_1}{\rho_0} + \frac{\dot{T}_1}{T_0}, \]

where the ratio of the specific heats, \( \gamma \), is taken to be 5/3. These equations allow compressible displacement for describing resonant absorption of Alfvén waves. These equations also reduce the study of resonant absorption to the study of linear displacements about an ideal stationary equilibrium state that are excited by external forces. The linearization around the ideal stationary equilibrium yields a good approximation for the description of resonant absorption with a characteristic time scale \( t_{\text{char}} \) that is long enough compared to the characteristic time necessary for the Alfvén waves travelling through the system.

Because the equilibrium variables depend on r only and we assume that a stationary state is reached after an initial transient phase, this allows us to carry out a Fourier analysis in the direction of the ignorable coordinates and in time, as given by with respect to the ignorable spatial coordinates \( \phi \) and \( z \) as

\[ f_1(r; \phi, z; t) = f_1(r; t) \exp(\text{i} m \phi + \text{i} k z), \]

where \( f_1 \) denotes any of the perturbed quantities. The azimuthal wave number \( m \) is an integer, and \( k \) is the longitudinal wave number. In this asymptotic state all the perturbed quantities oscillate harmonically with the frequency of the externally imposed driving oscillation. The temporal behaviour of a perturbed quantity \( f_1 \) is then

\[ f_1(r; t) = f_1(r) \exp(-\text{i} \omega t), \]

where \( \omega \) is the real frequency of the external driver.

2.1. MODEL OF CORONAL LOOPS

The aim of the present poster is to include an equilibrium mass flow in the mathematical idealization of coronal loops and to see how this effects the resonant absorption of Alfvén waves. Poedts et al. (1990) and Erdélyi & Goossens (1995) studied resonant absorption of Alfvén waves in 1-dimensional models of coronal loops in resistive and in visco-resistive MHD. Here we use the same equilibrium models but we include an equilibrium flow, \( v_0(r) = (0, v_{0\phi}(r), v_{0z}(r)) \), along the magnetic field lines. The equilibrium mass flow is incompressible, and we take the two component of the equilibrium flow are proportional to the local Alfvén speed. The components of this equilibrium flow are determined by the local Alfvén speed,

\[ v_{0\phi}(r) = f \times (1 - r^2) \times v_{A\phi}(r), \]

and

\[ v_{0z}(r) = f \times (1 - r^2) \times v_{A\phi}(r), \]

where the plasma flow strength, \( f \), is an input parameter, and \( v_{A\phi}(r), v_{A\phi}(r) \) are the azimuthal and longitudinal component of the Alfvén speed vector at position \( r \). The parabolic factor of the expression for the equilibrium mass-flow, i.e. \( 1 - r^2 \), is introduced in order to make sure that there is a smooth transition to the boundary of the coronal loop loop so that there is no need worry about a boundary layer. In the computations the equilibrium flow is taken in its dimensionless form in the unit of the Alfvén speed at the axis of the coronal loop, i.e. measured in \( v_{A\phi}(0) \).

The stationary mass flows in coronal loops are probably more complicated than those specified by the above expressions. In addition equilibrium mass flow are in reality 2-dimensional or even 3-dimensional quantities. The main attempt here is to obtain an insight in the effect of mass flow on the resonant absorption of Alfvén waves in coronal loops.

We also assume that the plasma around the coronal loop is uniform and static, so that in the region surrounding the coronal loop the behaviour of the plasma can be adequately described by the equation of ideal MHD approximation.

3. NUMERICAL RESULTS

3.1. MODEL I

Resonant Alfvén waves are investigated in two classes of equilibrium models of coronal loops. The analytical profiles of the equilibrium quantities are given by

\[ B_{0z} = 1 \]
\[ J_{ax} = j_0 (1 - r^2) \nu \]  
(17)

\[ \rho_0 = 1 - (1 - d)r^2 \]  
(18)
in dimensionless units (see also Poedts et al. 1989, 1990). The constants in the equilibrium profiles are free parameters. \( j_0 \) denotes the current density at the axis, with \( j_0 = 2k_1/\eta_0 \) (where \( k = \nu^{-1} \) is the inverse aspect ratio, \( k \sim 0.05 \) in coronal loops), and \( \eta_0 \) is the safety factor on the axis, \( q(r) = r B_{0y}/B_{0z} \); \( \nu \) determines the electric current density and the shear of the equilibrium magnetic field since \( q(1)/q(0) = \nu + 1 \); and finally \( d \) is the density at the plasma boundary. In the vacuum we can determine an analytical solution for the perturbed magnetic field, and we connect this analytical solution to the numerical solution with the aid of the boundary conditions.

Following Poedts et al. (1990), Erdélyi & Goossens (1995) we derive a measure of the dissipated energy by using the equation for the change of electromagnetic energy over the volume \( V_p \) of the plasma:

\[
- \int_{V_p} \nabla \cdot (E^*_1 \cdot B_1) \, dV = \int_{V_p} \rho_1 \nu^* \frac{\partial \nu^*}{\partial t} \, dV + \\
+ \int_{V_p} \left[ \nu^* \cdot \nabla p - \nu^* \cdot (J_1 \times B_0) + B_1 \cdot \frac{\partial B_1^*}{\partial t} \right] \, dV + \\
+ \int_{V_p} \eta J_1^2 \, dV + \int_{V_p} \nu^* \cdot (\nabla \cdot \mathbf{u}) \, dV,
\]  
(19)

where \( E_1 \) and \( J_1 \) are the Eulerian perturbations of the electric field and the electric current density. The \( \nu^* \) denotes the complex conjugate. The physical interpretation of this last equation is straightforward: the inflow of the electromagnetic energy (i.e. the left hand side) produces a rise of the kinetic energy of the plasma \( K \) (the first integral term on the right hand side), a change in the potential energy of the plasma \( W_p \) (the second integral term on the right hand side), and heat by Ohmic and viscous dissipation (\( OD \) and \( VD \)).

In our model the left hand side term is related to the incoming driving Alfvén waves which are substituted by power emission of an antenna in the vacuum region around the coronal loop. After some algebra we obtain an energy balance equation

\[ P_{ant} = K + W_p + W_v + OD + VD. \]  
(20)

The power emitted by the external antenna produces a rise of the kinetic energy of the plasma (\( K \)), a change of the potential energy of the plasma (\( W_p \)), a rise of the magnetic energy of the vacuum (\( W_v \)), and heat by Ohmic (\( OD \)) and viscous (\( VD \)) dissipation.

In ideal MHD a coronal loop excited periodically by an external driver (Alfvén waves) within the range of the ideal Alfvén current and energy accumulation without dissipation about the singular magnetic surface where the driving frequency \( \omega \) equals the local Alfvén frequency. Infinite dissipation causes the system to attain a stationary state where the energy dissipation in the neighbourhood of the resonant position (surface) just balances the energy inflow from the external driver.

After reaching a stationary state in the energy balance equation the real and imaginary parts of the energy terms are related to terms \( \cos^2(\omega t) \) or \( \sin^2(\omega t) \) - real parts- and \( \cos(\omega t) \sin(\omega t) \) - imaginary parts- in real notation. Integration over one period \( P_p \) of the external driver gives \( P_{ant}/2 \) when the integral is proportional to \( \cos^2(\omega t) \) or \( \sin^2(\omega t) \) and zero when the integral is proportional to \( \cos(\omega t) \sin(\omega t) \). The real parts of the energy balance equation correspond to cumulative or persistent effects, while the imaginary parts correspond to non-persistent effects.

In stationary state all physical quantities oscillate with the frequency of the driver and the dissipative energy balance is give by

\[ \text{Re}(P_{ant}) = OD + VD. \]  
(21)

The efficiency of resonant absorption of Alfvén waves can be expressed with the aid of the fractional absorption, \( f_a \) defined by

\[ f_a = \frac{\text{dissipation rate}}{\text{total power input}} = \frac{VD + OD}{|P_{ant}|}, \]  
(22)

where \( VD \) is the total viscous dissipation rate and \( OD \) is the ohmic dissipation rate.

The first equilibrium model (Model I) has a parabolic current profile, \( \nu = 1 \), so that the ratio \( g(1)/g_0 = \nu + 1 \). We choose \( g_0 = 0.5 \) so that the current density on the axis, \( j_0 = 0.2 \). We take a constant density profile (i.e. \( d = 1 \)). The azimuthal wave number is \( m = 2 \). In the longitudinal direction the wave vector \( k_z \), is quantized so that \( k_z = n k \) where \( n \) is the longitudinal (toroidal) wave number and \( k = \pi/L \) is a quantization factor to allow integral number of half-wavelengths along the coronal loops with \( L \) the length in the \( z \) direction. \( L = \epsilon r, \) with \( \epsilon = 20 \) the aspect ratio of the loop. In the recent model the longitudinal wave number is \( n = 1 \).

For this choice of parameters the local Alfvén frequency is parabolic and monotonic decreasing from the axis of symmetry to the edge of the coronal loop (see Figure 2).

Computational results of resonant absorption of Alfvén waves for Model I are plotted in Fig. 3. In Fig. 3 we have plotted the absorption rate of resonant Alfvén waves as a function of the flow strength parameter, \( f \) and the driving frequency \( \omega \), of the impinging Alfvén waves. The range of flow strength parameter, \( f \), reflects the characteristic values of the observed Doppler-shifts in coronal regions.

Fig. 3 shows that the flow has a very determinant effect on the resonant absorption of Alfvén waves in Model I. Even a small equilibrium mass-flow can drastically influence the absorption of the incident waves. The increasing flow -up to \( c_\alpha \cdot 10\% \) of the local Alfvén speed- can decrease the absorption rate very strongly. Absorption in an equilibrium state with a sufficiently strong equilibrium mass flow \( (f > 0.1) \) results in negative absorption rate. When the absorption rate becomes negative it refers to the
3.2. MODEL II

Fig. 5 shows the absorption rate of Alfvén waves as a function of the flow strength parameter, $f$, for another class of equilibrium models of coronal loops (Model II in Poedts et al. 1989). Model II is characterized by the same type of equilibrium profiles as in Model I, e.g. by Eqs. (16) - (18) but with (i) $\nu = 2$; (ii) the safety factor at the axis of the loop is $q_0 = 0.8$; (iii) and $d = 0.25$, so that the profile of the current density is more peaked than in Model I and the density profile is parabolic. This is a more realistic density profile in view of the observations of Foukal 1978. The aspect ratio is the same as in Model I (i.e. $\epsilon = 2$) and we now consider wave numbers $m = 1$ (poloidal wave number), and $n = 1$ (toroidal wave number). In Model II the local Alfvén frequency is increasing from the axis towards the boundary of the loop.

Computational results of resonant absorption of Alfvén waves of this type of models of coronal loops are given in Figure 5.

Fig. 5 shows, for a broad range of values of realistic flow strength parameters $f$, that the coupling between the resonant Alfvén waves and the flux tube is very efficient. The equilibrium mass-flow does not really affect the absorption rate so strongly as in case of Model I. A total absorption of 100% at $f = -0.19$ might again reflect a discrete eigenvalue of the system (see Goossens & Hollweg 1993).

4. CONCLUSIONS

The effect of an equilibrium flow on the heating of coronal loop plasmas by means of resonant absorption of linear Alfvén waves is considered in compressible viscous MHD. Coronal loops are modelled by cylindrical, one-dimensional, inhomogeneous vertical flux tubes surrounded by a uniform magnetized
plasma. The compressibility of the plasma has been taken into account which made it possible to describe the energy transfer from the external driver of the plasma.

Resonant absorption of Alfvén waves in coronal loops was studied extensively by Poedts et al. 1989, 1990; Erdélyi & Goossens 1995 in static equilibrium states. The present paper extends these numerical studies by taking into account stationary equilibrium flows. Equilibrium mass flows are considered as incompressible bulk motions along the magnetic field lines.

To solve the linear, driven system with an equilibrium background flow a numerical procedure called FEM (Finite Element Method) has been used. A parametric analysis is carried out of the absorption rate of resonant Alfvén waves for two classes of models of coronal loops. The efficiency of the absorption process depends on the equilibrium parameters and on the characteristics of the driver.

Resonant absorption of Alfvén waves is drastically influenced in Model I. We found that for rather small velocity shears the absorption rate can drop to zero or might become even negative. A negative value of the absorption rate reflects that the driving wave extracts energy from the coronal loop.

We also found that an anti-parallel background flow can enhance the absorption rate up to total absorption. This enhanced peak of the absorption rate might reflect the existence of an eigenmode of the coronal loop.

The influence of an equilibrium mass-flow is less prominent for Model II. The absorption rate of resonant Alfvén waves does not show a particular change due to an equilibrium background flow.

However, an anti-parallel background flow enhances the absorption rate up to the value of total absorption. This 100% absorption might, again, reflect a discrete eigenmode of the system.

To obtain a better physical insight into the variation of the absorption rate as function of the flow strength parameter $f$, we might have to solve the eigenvalue problem of coronal loops in viscous MHD in stationary equilibrium states.

The presence of an equilibrium flow may therefore be important for resonant absorption of Alfvén waves in coronal loops.

ACKNOWLEDGMENTS

R.E. would like to thank M. Kéray for patient encouragement and M. Goossens for fruitful discussions. This work was partly done during R.E. was a Research Fellow in Center for Plasma-Astrophysics, K.U.Leuven, Belgium. R.E. also thanks the financial support from PPARC (Particle Physics and Astronomy Research Council) of United Kingdom.

REFERENCES

Goossens, M., & Ruderman M.S. 1996, Physica Scripta, T60, 171