NONLINEARLY SELECTED FREQUENCIES IN CORONAL LOOPS

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Abstract. A nonlinear process for the resonant generation of low-frequency fast magnetosonic kink waves in coronal loops is discussed. The efficiency of the process is strongly enhanced due to the existence of a nonlinearly selected frequency produced by a constant frequency difference in the dispersion curves in the short wavelength limit. The kink wave with the selected frequency interacts with high-frequency kink and sausage waves. The efficiency of such interaction does not require coherence in the interactive waves. In a loop of width \(2 \times 10^3\) km, field strength 50 G and number density \(5 \times 10^{15}\) m\(^{-3}\), the nonlinearly selected frequency is of order 46 mHz (period 21.8 s), but this may range through 11 mHz to 184 mHz (periods 86.5 s to 5.4 s) for typical coronal conditions.

1. Introduction

Low-frequency pulsations observed in the coronae of the Sun and flare stars are often associated with MHD oscillations in coronal loops (see, e.g., Aschwanden, 1987; Švestka, 1994; Mullan and Johnson, 1995, and references therein). Typical periods range from several seconds to several minutes. The phenomena seem to be connected with magnetosonic waves trapped in coronal loops (Roberts, Edwin, and Benz, 1984; see also Bray et al., 1991). The smallest periods of the oscillations correspond to the time taken by a fast magnetosonic wave to travel back and forth across a loop diameter. However, the interpretation of coronal pulsations is still far from a complete understanding (see, e.g., Sasidharan et al. 1995).

From another point of view, the MHD oscillations of coronal loops are connected with the problem of coronal heating. Investigations of coronal heating show that resonant absorption is most efficient when the exciting frequency coincides with the global Alfvén mode frequency in the coronal loop (Ofman, Davila, and Steinolfson, 1994). The heating could be even more efficient if a mechanism for the continuous supply of energy in the vicinity of the global mode frequency could be identified.

Here we demonstrate the existence of a frequency selection mechanism in coronal loops that depends upon the nonlinear behaviour of waves. The fast magnetosonic kink mode with the selected frequency interacts nonlinearly with sausage and kink modes at high frequency, resulting in the transfer of energy from the high-frequency part of the spectrum to the low, selected frequency part. The selected frequency is defined by the transversal structure of the loop and depends upon the loop diameter but not the loop length. The effect may increase the efficiency of resonant absorption in the loop.
2. Nonlinearly Selected Frequency

It is established that the fast magnetosonic modes of coronal loops may nonlinearly interact with each other (Nakariakov and Oraevsky, 1995). Due to the small amplitudes of the waves, a main contribution to such interaction is fulfilled by resonant interactions. For waves propagating in a wave guide, the frequencies and wave numbers of the interacting waves must satisfy the resonant conditions

$$\omega_a + \omega_b = \omega_c, \quad k_a + k_b = k_c,$$  \hspace{1cm} (1)

where the indices \(a, b,\) and \(c\) correspond to different interacting waves in a resonant triplet.

We consider the resonant triplets (1) that arise from interactions between the kink and sausage fast magnetosonic waves in a coronal loop. To do this we suppose the loop to be modelled by a straight magnetic slab of strength \(B_0\) in a zero-\(\beta\) (cold) plasma. The density structure \(\rho_0(x)\) is taken to be the so-called Epstein profile, with density varying from \(\rho_0\) at the centre \(x = 0\) of the magnetic slab to \(\rho_\infty < \rho_0\) far from the slab (see also Nakariakov and Roberts, 1995):

$$\rho_0(x) = (\rho_0 - \rho_\infty)\text{sech}^2(x/d) + \rho_\infty.$$  \hspace{1cm} (2)

The slab has a characteristic width of \(2d\). The Alfvén speed is

$$C_A(x) = B_0/(4\pi \rho_0(x))^{1/2},$$

and varies from \(C_{A\infty}\) at the slab centre to \(C_{A0}\) as \(|x| \to \infty\). Since \(\rho_0 > \rho_\infty\) and the field is uniform, \(C_{A0} < C_{A\infty}\); fast magnetosonic waves are trapped within the wave guide defined as the region of high plasma density, low Alfvén speed (Roberts, Edwin, and Benz, 1984).

Consider a wave of angular frequency \(\omega\) and longitudinal wave number \(k\). The dispersion relation for the principal kink wave is (Nakariakov and Roberts, 1995)

$$\sqrt{C_{A\infty}^2 - (\omega/k^2)} = \frac{C_{A\infty} |k| d}{C_{A0}} \left[ (\omega/k)^2 - C_{A0}^2 \right];$$  \hspace{1cm} (3)

the dispersion relation of the principal sausage wave may be obtained similarly,

$$\frac{|k| d}{C_{A0}^2} \left[ (\omega/k)^2 - C_{A0}^2 \right] - 2 \frac{|k| d}{|k| d} = \frac{3}{C_{A\infty}} \sqrt{C_{A\infty}^2 - (\omega/k)^2}.$$  \hspace{1cm} (4)

Dispersion relations (3) and (4) are displayed in Figure 1. For both sausage and kink modes, the speed \(\omega/k\) lies between \(C_{A0}\) and \(C_{\infty}\); the sausage mode has a cutoff, for which \(\omega = kC_{A\infty}\) and \(|k| d = [2C_{A\infty}^2/(C_{A\infty}^2 - C_{A0}^2)]^{1/2}\). It is interesting to note that the difference \(\delta \omega(k)\) in frequency between the principal sausage mode and principal kink mode is close to a constant for large wave numbers \(k\). Squaring (3) and (4), we may obtain biquadratic equations for the phase speeds of the sausage and kink modes, from which we may determine \(\delta \omega(k)\) as \(kd < \infty\). Specifically,
Figure 1. Dependences of frequency on wave number for principal kink and sausage modes of a magnetic slab with the Epstein profile of the plasma density (solid lines). Dashed lines show a possible resonant triplet a, b, c. Dot-dashed lines for $\omega = \pm k C_{A\infty}$ and $\omega = \pm k C_{A0}$ give the upper and lower limits of the modes. The curves are drawn for $C_{A\infty}/C_{A0}^2 = 4$.

$$\delta \omega \equiv \lim_{kd \to \infty} [\omega_{\text{sausage}}(k) - \omega_{\text{kink}}(k)] = \frac{C_{A0} \sqrt{C_{A\infty}^2 - C_{A0}^2}}{C_{A\infty} d}. \quad (5)$$

The fact of approximate parallelism in the dispersion curves for the principal sausage and kink modes in an Epstein density profile has an important consequence for the three wave resonance described by (1). If, in the resonant triplet (1), the frequencies $\omega_b, \omega_c$ (or wave numbers $k_b, k_c$) increase, the frequency $\omega_a$ approaches a constant value, $\omega_{NS}$; this is the nonlinearly selected frequency. For any arbitrary pair $\omega_b, \omega_c$, the third frequency in the resonant triplet (1) has almost constant value, $\omega_a \approx \omega_{NS}$. Thus, any sausage wave with arbitrary large $\omega_c, k_c$ interacts with a kink wave with suitable $\omega_b, k_b$ and with a second kink wave with a frequency and wave number that are close to $\omega_{NS}, k(\omega_{NS})$. The nonlinearly selected frequency $\omega_{NS}$ is given by

$$\omega_{NS} = \alpha \delta \omega = \alpha \frac{C_{A0}}{d} \left(1 - \frac{C_{A0}^2}{C_{A\infty}^2}\right)^{1/2}, \quad (6)$$

where $\alpha$ is a geometrical coefficient. The specific value of $\alpha$ may be deduced from Figure 1 by noting that $\omega_{NS} = \omega_c - \omega_b$, these two frequencies being calculated
according to condition (1) at different wave numbers and then proceeding to the limit \( kd \to \infty \). Since \( \omega_{NS} < \delta \omega \), then \( 0 < \alpha < 1 \), the precise value being determined numerically from Equations (1), (3), and (4). For an increase in the frequencies \( \omega_b \) or \( \omega_c \), frequency \( \omega_a \), rapidly approaches \( \omega_{NS} \), the speed of the approach increasing with increasing \( \rho_0/\rho_\infty \).

3. Discussion

The efficiency of resonant interactions of waves may be strongly decreased by the non-coherence of the interacting waves. Non-coherence may result from an absence of a coherent source of waves and by phase ‘breaks’ in the wave trains, which lead to a detuning of the frequencies and wave numbers from the resonant conditions (1). In a realistic case, one may expect that the interaction (1) is considerable only if there is some mechanism supporting the accumulation of nonlinear effects.

The effect we have discussed leads to an accumulation of the energy in those waves with frequency close to the specific nonlinearly selected frequency \( \omega_{NS} \), given by Equation (6) with the coefficient \( \alpha \) being less than unity. The fact that \( \omega_a \) is not exactly equal to \( \omega_{NS} \) does not matter, because in a realistic situation dispersion curves have a finite width, due to the finite quality of the waveguide. Consequently, sausage and kink waves generated by noise in coronal loops can be expected to generate nonlinearly a kink wave with the frequency \( \omega_{NS} \). This effect may be very efficient because it does not require coherence in the interacting waves.

The parallelism of the dispersion curves is a property of the Epstein profile of the plasma density and is also present in the parabolic profile considered by Nakariakov and Oraevsky (1995). But the property is absent from the step profile (e.g., Edwin and Roberts, 1982), for which dispersion curves tend to the line \( C_{A0} \) as \( k \to \infty \). However, this does not eliminate the discussed transfer of wave energy to the low-frequency part of the wave spectrum, because in realistic conditions the dispersion curves have a finite width. Thus we expect that the nonlinearly selection of oscillations with a frequency \( \omega_{NS} \) is a robust property of coronal loops and waveguides.

We may illustrate the magnitude of the frequencies expected for kink waves generated resonantly through (1). The nonlinearly selected frequency \( \omega_{NS} \) is given by Equation (6). For a coronal magnetic field of 50 G and an electron number density of \( 5 \times 10^{15} \) m\(^{-3} \) withing a loop (see, e.g., Bray et al. 1991), we obtain an Alfvén speed \( C_{A0} \) of 2000 km s\(^{-1} \). For a loop of width \( 2d = 2 \times 10^3 \) km, this produces a frequency \( \delta \omega \) of 0.93 s\(^{-1} \) for \( C_{A\infty} = 3C_{A0} \) and a corresponding cyclic frequency \( \delta \omega/2\pi \) of 0.15 Hz (period 6.7 s). The nonlinearly selected frequency \( \omega_{NS} \) is the fraction \( \alpha \delta \omega \) of \( \delta \omega \); for \( C_{A\infty} = 4C_{A0} \), the geometrical factor \( \alpha \approx 0.15 \), giving a nonlinearly selected cyclic frequency of 46 mHz (period 21.8 s). For a range of loop cross-sections and Alfvén speeds we obtain a corresponding range of
frequencies. With a width $2d$ in the range 1000–4000 km and internal Alfvén speed $C_{A0}$ ranging from 1000–4000 km s$^{-1}$, we obtain a range of nonlinearly selected cyclic frequencies $\omega_{NS}/2\pi$ of 11 mHz to 184 mHz (periods 86.5 s to 5.4 s).

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