THE SAN FERNANDO OBSERVATORY VIDEO SPECTRA-SPECTROHELIOGRAF

STEPHEN R. WALTON and GARY A. CHAPMAN
San Fernando Observatory, California State University, Northridge, Northridge, CA 91330–8268, U.S.A.

(Received 16 October, 1995; in revised form 27 December, 1995)

Abstract. We describe recent work in the development of the San Fernando Observatory (SFO) Video Spectra-Spectroheliograph (VSSHG), a spectrum-based instrument for the measurement of the solar Stokes profiles. Its most important features are: simultaneous measurement of Stokes \( I \) plus one of Stokes \( Q, U, \) or \( V \); spatial sampling of 0.5 arc sec; spectral sampling of 8.8 m\( \AA \); and time sampling of one minute (for one pair of Stokes profile) to three minutes (for all four profiles). Routine data processing is carried out using a moments technique; tests of this technique show it to be reasonably accurate. Sample data are shown and briefly discussed: a longitudinal magnetogram and Dopplergram of NOAA 5573 observed on 17 August, 1989, and a vector magnetic field map and Dopplergram of NOAA 6659 observed on 10 June, 1991.

1. Introduction

Recent advances in detector and computer technology have made it possible to observe the complete solar Stokes vector on a routine basis. Several such instruments are now in regular use throughout the world, and more are under development. These instruments overcome the well-known difficulties of calibration and saturation which limit magnetographs using the Babcock technique, and (at least in principal) the data collected allow recovery of the entire vector magnetic field over the solar photosphere.

Such observations can be carried out in one of two ways: either by a sequence of images at different wavelengths and polarizations (Hagyard, Gary, and West, 1982; Zirin, 1985; Title, Tarbell, and Topka, 1987), or a sequence of spectra at different positions and polarizations. Two recently constructed spectrographic instruments are the Spectromagnetograph (SPM) at the Kitt Peak Vacuum Tower Telescope (Jones et al., 1992) and the Advanced Stokes Polarimeter of the High Altitude Observatory (Lites et al., 1991). The instrument described herein, the San Fernando Observatory (SFO) Video Spectra-Spectroheliograph (VSSHG) is a spectral scanning instrument similar in concept to the SPM.

The VSSHG is based on the spectral approach. This has the advantage that all spectral elements for one line on the Sun are obtained at the same time. Thus, there is no mixing of spectral and spatial information. Furthermore, there is no electronic switching of polarization that could alias seeing changes. The spatial resolution of the images may be lower than for the filter method, although this will depend on scanning speed, image stability, and selection of raw spectra. However,
the (generally) poorer spectral resolution and small number of wavelength samples in a filter-based instrument can lead to significant errors in the recovery of the magnetic field, particularly the azimuth of the transverse field (Lites, Martinez Pillet, and Skumanich, 1994).

Once Stokes spectra are obtained, the vector magnetic field must be extracted. Arguably, the best technique is to fit analytic Stokes profiles to the observed spectra (Skumanich and Lites, 1987; Ruiz Cobo and del Toro Iniesta, 1992). However, this is a very compute-intensive procedure, as it involves a nonlinear least-squares fit containing many free parameters. Thus, several workers (for example, Rees and Semel, 1979; Ronan, Mickey, and Orrall, 1987) have explored alternative techniques for extracting the magnetic field which may be less accurate but are significantly faster. We have adopted a moments technique for our routine data processing, which is fast and quite accurate (Cauzzi, Smaldone, and Balasubramaniam, 1993).

In Section 2 of this paper, the VSSHG is described. We detail and analyze our data reduction procedures in Section 3. Section 4 discusses some recent VSSHG results. We summarize in Section 5.

2. Instrument Description

Since the telescope and spectroheliograph used at SFO have been previously described in some detail (Mayfield et al., 1969; Richter, 1985), we simply summarize the most important features here. We use an echelle grating that gives a dispersion of about 2 mm Å⁻¹ at the 6302.5 Å line. The spectrum is masked and passed through a 15° Wollaston prism purchased from Karl Lambrecht. The spectrum is focused directly onto a CCD in a Cohu camera that produces standard RS-170 composite video. This signal is recorded by a Sony 3/4" videocassette recorder with a signal-to-noise ratio of 49 dB in its black and white mode.

The camera is oriented such that each video scan line crosses the two oppositely analyzed spectra of the same point on the entrance slit and the solar image. The spectroheliograph slit is then mechanically scanned across the solar image. The resulting spectra have a spatial scale of 0.51" per pixel along the slit, and a spectral scale of 8.75 mÅ per pixel perpendicular to the slit. Our spectral resolution of approximately 30 mÅ is set by the entrance slit width. The spectroheliograph slit scan rate is 2.3" per second of time, which gives an undistorted output image when six video frames are co-added to produce each spectrum analyzed. This scan rate is adjustable according to the required signal to noise. A higher signal-to-noise ratio would require more video frames to be summed for each line of the output images, and a correspondingly slower scanning speed.

We obtain two orthogonal polarizations at any one time; that is, any of the three possible pairs of Stokes sums $I \pm Q$, $I \pm U$, or $I \pm V$. A typical active region scan of an area 220" E–W by 150" N–S requires less than one minute of
spectroheliograph scanning. Changing from one polarization to another requires simple rotation of a quarter-wave and/or a half-wave plate, and takes only a few seconds. Thus, relatively rapid time sequences are possible. We typically obtain all three Stokes pairs over the course of four to five minutes, usually with spacing such that pairs of spectra are separated by $2\frac{1}{2}$ minutes, one-half the period of the strongest solar $p$-mode oscillations. This relatively high time resolution, combined with the good spectral resolution, is one of the most important capabilities of the VSSHG.

We record several seconds of dark signal from the camera before each spectral scan; these video frames, when digitized, are used later as dark calibration images for the CCD camera. Similarly, when an observer is at the telescope, defocused spectra are obtained in clean continuum for use as flat-field calibrations. The spectrograph grating is rotated so that clean continuum just to the red of $\lambda$6302.5 illuminates the same pixels as are used for observation, and both the telescope and spectrograph are defocused. The same quarter-wave or half-wave plate setting(s) are used for the defocused spectra as for the imaged spectra. We argue in the Appendix that use of these defocused spectra helps to suppress the effects of the telescope polarization. Changes in telescope and spectrograph focusing may lead to different interference fringes in the spectra (though we see no such fringes), and definitely result in any dust on the spectrograph slit affecting the defocused and imaged spectra differently. However, such changes are nearly equivalent to a multiplication of an entire row (spatial position) of the spectrum by a constant near one, and this constant will cancel when the ratio of Stokes profiles is taken for the moments calculation described below.

The recorded spectra are digitized by a Matrox MVP video digitizer and processor operated in a 80386-equipped PC compatible computer that also controls a JVC model 600 U-matic VCR. The video data are digitized from the tape while the tape is in play mode, as commanded from the PC; the tape is backed up 90 frames (three seconds) before the next desired digitization to allow the playback to stabilize. Each summed video spectrum is corrected for the dark and flat-field response of the CCD using the previously recorded calibration frames. A nonlinear least-squares fit of three gaussian line profiles to the three observed spectral lines, the telluric $O_2$ lines at $\lambda$6302.00 and $\lambda$6302.75 and the solar line, is done once for each summed, calibrated spectral image, in a quiet-Sun area. The centers of the fits to the telluric lines are used to assign an absolute wavelength scale to the spectrum. The gaussian fit is also used to remove the blend with the telluric $O_2$ line at 6302.75 Å. Finally, the desired processing scheme is applied to each row of the spectrum, and the processed data are written to magnetic disk as a set of digital images generated one row at a time.

At the beginning of the summer of 1993, we added the capability for unattended control of the entire VSSHG system, including spectrograph scanning, switching of polarization optics, and control of the VCR. This has resulted in an enormous increase in our observing efficiency, since a single person can program a set of scans.
to be taken automatically. A set of scans spaced 15 minutes apart, for example, allows an entire observing day to be placed on a single U-matic videocassette. An autoguiding system located at the entrance slit of the spectrograph, also installed in 1993 and improved in August 1994, keeps the desired region of the Sun centered in the field of view. As of this writing (August 1995), we have over one hundred one-hour videocassettes of spectra, representing approximately 1500 scans of the Stokes profiles of various regions of the solar disk.

3. Data Analysis

3.1. General Formulation

The most sophisticated (and computationally intensive) techniques presently in use for extracting vector magnetic field information from Stokes profiles are least-squares fits of varying degrees of complexity to the profiles.

One technique uses the analytic solution to the equations of transfer of the Stokes profile in the Milne–Eddington approximation (Landi Degl’Innocenti and Landi Degl’Innocenti, 1973; Auer, Heasley, and House, 1977; Skumanich and Lites, 1987) and solves for the magnetic field strength and geometry, line-of-sight velocity, the first derivative of the source function with respect to optical depth, and the filling factor, though the last quantity can only be found reliably if multiple lines are observed simultaneously. A second, more recent, effort is the ‘response functions’ technique (Ruiz Cobo and del Toro Iniesta, 1992; Ruiz Cobo and del Toro Iniesta, 1994). Both of these least-squares techniques take full advantage of the available signal. The response functions method has the additional advantage that it fully inverts the profiles to produce a model atmosphere complete with depth-dependent temperatures, velocities, and magnetic fields, though it assumes unit filling factor. It was recently applied for the first time to actual data (del Toro Iniesta, Tarbell, and Ruiz Cobo, 1994).

Both of these techniques are very computationally intensive. In the case of the VSSHG data, using a least-squares program would require the storage, and transport to a fast computer, of large numbers of digitized spectra. With the six-frame video sum mentioned above, each of our images has over 200 000 image points, each of which would in turn contain all four Stokes spectra of roughly 100 points each. The processing time required for this quantity of data is very large.

For these reasons, many authors have searched for a less expensive means of producing magnetic field maps from Stokes spectra. A comparison of these three techniques for longitudinal fields has been carried out (Cauzzi, Smaldone, and Balasubramaniam, 1993): the center-of-gravity technique (Rees and Semel, 1979); the first derivative approximation (Jeffries and Mickey, 1991); and a numerical simulation of a filter-based (Babcock) magnetograph. This study finds that, for data of sufficient spectral resolution, the center-of-gravity technique is preferred. Of the
three techniques, the comparison shows that only the center-of-gravity technique requires no instrument or model dependent calibration coefficients and proves accurate at high field strengths.

For these reasons, we have chosen to use moments of the Stokes $I$, $Q$, $U$, and $V$ profiles to extract the magnetic field information. Specifically, we calculate the following:

$$Q_2 = \int_{-\Delta \lambda}^{+\Delta \lambda} \lambda^2 Q(\lambda) \, d\lambda$$

(and similarly for $U_2$ from $U(\lambda)$),

$$V_1 = \int_{-\Delta \lambda}^{+\Delta \lambda} \lambda V(\lambda) \, d\lambda$$

and

$$I_0 = \int_{-\Delta \lambda}^{+\Delta \lambda} (I_c - I(\lambda)) \, d\lambda.$$

In these expressions, the subscripts denote the order of the moment of the profile, and $I_c$ is the continuum intensity. $\lambda$ is the wavelength measured from the center of the unshifted line. Note that $I_0$ is actually the line's equivalent width.

We then use the following approximations:

$$B_\parallel = \frac{1}{\Delta \lambda_B} \frac{V_1}{I_0},$$

$$B_T = \frac{1}{\Delta \lambda_B} \left[ \left( \frac{2 \, Q_2}{I_0} \right)^2 + \left( \frac{2 \, U_2}{I_0} \right)^2 \right]^{1/4},$$

$$\gamma = \cos^{-1} \left( \frac{B_\parallel}{\sqrt{B_\parallel^2 + B_T^2}} \right),$$

and

$$\chi = \tan^{-1} \left( \frac{U_2}{Q_2} \right).$$

In these expressions, $B_\parallel$ is the longitudinal component of the magnetic field, $B_T$ is the magnitude of the transverse component, $\gamma$ is the inclination of the field vector to the line of sight, and $\chi$ is the azimuth of the field vector. $\Delta \lambda_B$ is simply the splitting for a normal Zeeman triplet.
\[ \Delta \lambda_B = 4.67 \times 10^{-13} g_L B \lambda^2 , \]  

where \( g_L \) is the Landé \( g \) factor for the line, \( B \) is in gauss and \( \lambda \) is in ångstroms.

The expressions above for the magnetic field are exact when the line profile is given by the Seares formulae. More generally, Equation (4) is correct whenever the \( V(\lambda) \) profile is given by the form

\[ V(\lambda) = \frac{1}{2} [p(\lambda + \Delta \lambda) - p(\lambda - \Delta \lambda)] . \]  

In this expression, \( p(\lambda) \) is any profile function which is symmetric about its maximum value at line center and \( \Delta \lambda = \Delta \lambda_B \cos \gamma \).

The expression \( V_1/I_0 \) is equivalent to a ‘center-of-gravity’ calculation. The center of gravity of a function over some range is defined as the ratio of its first moment to its integral (zeroth moment) over that range. With this definition, the separation between the centers of gravity of the two observed, oppositely circularly polarized profiles would be

\[ \Delta \lambda_{CG} = \frac{(I + V)_1}{(I + V)_0} - \frac{(I - V)_1}{(I - V)_0} , \]  

where the subscripts indicate the appropriate moment of the profile expression in parentheses. If Stokes \( V \) is antisymmetric about line center, then \( V_0 \) is zero. Similarly, \( I_1 \) is zero if Stokes \( I \) is symmetric. In this case, the above expression reduces to \( 2V_1/I_0 \). Some workers (Cauuzzi, Smaldone, and Balasubramaniam, 1993) use this expression, while others (Rees and Semel, 1979) actually use \( V_1/I_0 \) and not \( \Delta \lambda_{CG}/2 \) as their measurement of the magnetic splitting, but still refer to the ‘center of gravity’. A third technique (Jones et al., 1992) measures the splitting in Stokes \( V \) by searching for the zero positions of a convolution of the \( I + V \) and \( I - V \) profiles with a fixed antisymmetric kernel. However, the difference between all these techniques is small. In a comparison of VSSHG and SPM longitudinal field data (Jones et al., 1993; Walton et al., 1993), both instruments gave very similar values, despite the use of the moments for the VSSHG data and the center of gravity for the SPM data.

3.2. SYSTEMATIC ERRORS

In this section, we describe evaluations of systematic errors of the moment technique defined by our Equations (1)–(7) by calculating actual Stokes profiles in the Unno–Rachkovsky solution (Landi Degl’Innocenti and Landi Degl’Innocenti, 1973), and applying the moment technique to these model profiles. We calculated line profiles for the following parameters: source function \( S(\tau) = B_0 + B_1 \tau \), where \( B_0 = 0.4, B_1 = 0.6 \); Voigt parameter \( a = 0.025 \); Doppler width \( \Delta \lambda_D = 0.06 \text{ Å} \); ratio of line-center to continuum opacity \( \eta_0 = 1.99 \); line-center wavelength.
\[ \lambda_0 = 6302.5 \, \text{Å}; \text{ and Landé } g = 2.5. \] These values give a FWHM of the model quiet-Sun profile of 140 mÅ and a central intensity of 0.52 of continuum, comparable to the observed values for the \( \lambda 6302.5 \) line in the quiet Sun. The moments required were calculated by numerical integration of the simulated Stokes profiles from \(-0.3 \, \text{Å}\) to 0.3 Å from line center. The profiles were calculated at field magnitudes of 100 to 2000 G in steps of 100 G, field inclinations from 0° to 180° in steps of 10°, and field azimuths from −90° to +90° in steps of 10°, for a total of 7220 model profiles. Figures 1(a–c) compare the actual and calculated field parameters for the moments technique described in the present work. All three figures are scattergrams with the actual field used in the Unno–Rachkovsky solution along the horizontal axis, and the result of the integral calculations along the vertical axis. Within each set of figures, results for magnitude of transverse field \( B_T \), azimuth angle \( \chi \), and longitudinal field \( B_\parallel \) are shown.

Figure 1(a) shows that the moments technique reproduces the longitudinal field almost exactly, confirming previous work (Cauzzi, Smaldone, and Balasubramaniam, 1993). Linear regression of the actual on the computed field yields a slope of 0.95, corresponding to a 5% underestimate of \( B_\parallel \) with no calibration constant required. Figure 1(b) shows a low-scatter line in the calculation of \( B_T \); the slope is about 1.09.

Figure 1(c) shows that the moments technique recovers the azimuth to only fair accuracy; the scatter around the true value is about ±15°. Examination of the individual calculations shows that the largest disagreement between the calculated and observed azimuth is at small values of the field inclination. Since the moments technique does reproduce the longitudinal and transverse fields reasonably well, it also gives a good value for the field inclination. Thus, one can conclude that if the moments technique yields a large inclination, the calculated azimuth will also be accurate.

### 3.3. Some Practical Considerations

Real data are not noise free, and use of the moments technique needs careful attention in order to minimize the effects of noise in the calculated field.

The first issue is that of the calculation of the integral of noisy data. The noise in integrals of noisy data actually increases with increasing width of the band over which the integral is calculated. A simple example illustrates this point. Consider the measurement of a set of values with mean \( \bar{x} \) and per-point standard deviation \( \sigma \). Averaging \( N \) measurements of \( x \) reduces the noise in the measurement of \( \bar{x} \) by a factor of \( \sqrt{N} \); in other words, the standard deviation in the mean is \( \sigma / \sqrt{N} \). However, if we consider the sum of \( N \) measurements of \( x \), the sum will have a standard deviation of \( \sqrt{N} \sigma \). In particular, summing many values of \( x \) with zero mean generates a random walk away from zero, with a mean distance of \( \sqrt{N} \) times the step size.
Figure 1a.

Figure 1b.
This is a general problem with integral calculations. It is difficult to treat with most standard data smoothing techniques. For example, consider filtering of the data in the Fourier domain before the calculation of the zeroth moment. Any such filtering will leave the value of the zero-frequency component unchanged, but that component is precisely the desired zeroth moment. Data smoothing techniques which average the data over a moving window also have no effect on the sum of the values of the points within the window.

We have found the only effective technique to reduce the noise in our moment calculations is to limit the wavelength range over which we calculate the integrals. The integrals in Equations (1)--(3) are calculated only over that wavelength range for which \( I(\lambda) \) is less than 90% of \( I_c \) (Brants, 1985). These integration limits are found anew for each individual spectrum processed. In addition, Fourier noise filtering is applied to the spectra before the integrals are calculated, with a smooth cutoff centered at 0.2 times the Nyquist frequency. This smoothing substantially reduces the error in finding the limits of integration, even though we argue above that it does not change the integral itself.

A small area of a single VSSHG map of \( B_\parallel \), containing a few hundred quiet-Sun pixels, has a per-pixel standard deviation of about 15 G; the standard deviation of
the corresponding area of a $v_\parallel$ map is about 150 m s$^{-1}$. Noise levels are significantly higher in maps of $B_T$ and $\chi$, due to the need to combine separate maps of $Q_2$ and $U_2$ produced from observations separated by approximately one minute, and due to the inherently lower linear polarization signal. The per-pixel standard deviation in a $B_T$ map is about 200 G. Note also that all of the moment integrals assume unit filling factor in calculating $B_\parallel$ and $B_T$. However, examination of Equation (7) show that the calculation of $\chi$ depends only in integrals of $Q(\lambda)$ and $U(\lambda)$; in other words, only on integrals of the linear polarization profiles. The value of $\chi$ calculated should thus not depend on filling factor.

The effects of the telescope and spectrograph polarizations are not, at present, compensated for. At $\lambda$6302, the largest off-diagonal element of the spectrograph’s Mueller matrix is 0.07 (Richter, 1985), and the diagonal elements are all nearly equal. The telescope Mueller matrix is similar. We argue in the Appendix that these effects do not change the results of many of the moment calculations.

The same software which process the magnetic field maps also produces three other maps from the same spectra. These are:

1. A map of continuous intensity, taken as the average of five spectral pixels centered at a wavelength of 6302.25 Å in the $I(\lambda)$ profile. This is, in effect, a spectroheliogram of 50 mÅ bandpass.

2. A map of line core intensity, taken as the smallest pixel value in the observed $I(\lambda)$ profile.

3. A map of the longitudinal component of the velocity, calculated from the ratio of the first and zero moments of the $I(\lambda)$ profile:

$$v_\parallel = \frac{c}{\lambda_0} \frac{I_1}{I_0}, \quad (11)$$

where $\lambda_0$ is the line-center wavelength and $c$ is the speed of light. (This, incidentally, is a true center-of-gravity calculation.)

The continuum maps are used to align the individual maps of $V_1$, $Q_2$, and $U_2$ to produce maps of the vector magnetic field ($B_T$, $\gamma$, and $\chi$). Thus, the vector field maps are not calculated from simultaneous data, and seeing-induced noise will be present in them. Within a given spectroheliograph scan, of course, the continuum, Doppler, line core, and Stokes moment maps are precisely co-spatial and simultaneous. In particular, we can produce high-quality, saturation-free, simultaneous longitudinal magnetograms, intensity maps, and Dopplergrams. The processing time required is long but not excessive. A single scan of an active region requires about 1$\frac{1}{4}$ hours of processing using our computer with an accessory array processor, or about one minute to process one second of video. We remark again that this processing time on a modest computer produces four maps, each containing about one-quarter of a million pixels.
4. Sample VSSHG Observations

We began observations with the VSSHG in 1989, though only circular polarization measurements were usable before the summer of 1992, when we installed a half-wave plate to allow switching between Stokes $Q$ and $U$. Figures 2(a) and 2(b) are, respectively, the longitudinal magnetogram and Dopplermgram of NOAA 5573 observed on 17 August, 1989. The data were taken at a time of good seeing. Note in particular the fine features visible in the northern part of the image, just north of the main sunspots. The highest field strength observed in the umbra of this region is over 3000 G; the very dark umbra has, however, resulted in poor signal-to-noise ratio there, which is particularly visible in the Dopplermgram. The smallest easily visible features are about 2 arc sec across; careful inspection of the original image also shows small arcs of Doppler shift, presumably due to the Evershed flow, on the leading edge of the smallest pores which are about 1 arc sec across.

This image is one of several analyzed to find the relationship between continuum facular contrast and magnetic field strength (Lawrence, Chapman, and Walton, 1991). We have begun a new analysis of these data, with the goal of extending the previous work into the sunspots.

Figure 3 shows VSSHG observations of NOAA 6659 taken on 10 June, 1991 at 21:54 UT, about four hours before an X12.0 flare. This region was possibly the most prolific flare producer of Cycle 22, producing five X12 flares during its disk passage. A study of the 11 June 00:20 UT flare (Sakurai et al., 1992) called attention to the highly sheared magnetic field configuration along the neutral line. This shear is visible in Figure 3(a). Our simultaneous Doppler image, shown in Figure 3(b), shows two very interesting features. The first is an obvious line of oppositely directed vertical flows separated by the neutral line, which is consistent with the hypothesis that the neutral line represents an ‘arcade’ of magnetic loops with flows along them. The second feature of interest is an isolated red-shifted (bright) feature, embedded within an area of overall blue shift, located at the northern end of the neutral line. This feature is cospatial with the earliest and brightest kernels of the X12.0 flare of 11 June 02:00 UT (Sakurai et al., 1992), and has disappeared in the VSSHG observations of this region on 11 June.

5. Summary and Future Work

The San Fernando Observatory VSSHG has proven to be a reliable and economical instrument for the routine production of vector magnetograms and Dopplergrams of active regions. Several research programs using the data already obtained with the VSSHG are underway.

One program is a study of the long-term evolution of active regions. Some of this work has already been reported (David, 1992; Corbin, 1993; Chapman and Walton, 1993; and Walton, 1993), and it is continuing. We plan a particular emphasis on the
characteristics of emerging flux, as VSSHG observations have been taken which show all the characteristics of emerging flux tubes on scales of several arc seconds (Chapman and Walton, 1993; Corbin, 1993; Walton, 1993), including transverse fields connecting two areas of opposite longitudinal field polarity and upflows and/or downflows which are co-spatial with the footpoints of the apparent flux tube.

Research on the fractal characteristics of the quiet-Sun magnetic field (Lawrence, Ruzmaikin, and Cadavid, 1993; Cadavid et al., 1994; Ruzmaikin et al., 1995) also continues and is being extended to studies of the magnetic field in faculae. The connection between magnetic fields and solar irradiance variations has also been studied (Lawrence, Chapman, and Walton, 1991); more could be done in this field as well.
Several instrument improvements are planned. Replacement of the Cohu camera with a more sensitive detector would allow for better observations of the magnetic field and flows in the deep umbra, and perhaps for improvement in our spectral resolution by allowing use of a narrower entrance slit. A re-imaging system placed before the Wollaston prism will both correct for the slight differential refraction of the two spectra and allow simultaneous observation of the $\lambda 6301$ line, which is important for analyzing stray light effects (Lites, Martínez Pillet, and Skumanich, 1994).
The existing VSSHG spectra can be easily reanalyzed using either of the least-squares techniques (Skumanich and Lites, 1987; Ruiz Cobo and del Toro Iniesta, 1992), and such reanalysis is presently being planned.

Appendix. Effects of VSSHG Instrumental Polarization

In this Appendix, we argue that the combination of our calibration technique and the moment calculations in Equations (1)–(7) also help suppress the effects of the VSSHG instrument polarization.

Let $S(\lambda)$ be the true Stokes vector:

$$S(\lambda) = \begin{pmatrix}
I(\lambda) \\
Q(\lambda) \\
U(\lambda) \\
V(\lambda)
\end{pmatrix}. \quad (A1)$$

For calibration, we observe a defocused, unpolarized, disk center spectrum; its true Stokes vector is $S_C = (I_C, 0, 0, 0)^T$. The system Mueller matrix $M$ operates
on both of these Stokes vectors to produce $S'_\lambda$ and $S'_C$, respectively. After passing through our Wollaston prism, we then observe one pair of $I'_\lambda \pm Q'_\lambda$, $I'_\lambda \pm U'_\lambda$, or $I'_\lambda \pm V'_\lambda$, each of which are then multiplied by the gain of the CCD camera. The flat-field calibration we use means that we divide each of these observed pairs by $I'_C \pm Q'_C$, $I'_C \pm U'_C$, or $I'_C \pm V'_C$, respectively.

We consider one polarization explicitly, since the algebra is the same for all. The VSSHG observes

$$I' + Q' = M_{00}I + M_{01}Q + M_{02}U + M_{03}V + M_{10}I + M_{11}Q + M_{12}U + M_{13}V,$$

(A2)

$$I' - Q' = M_{00}I + M_{01}Q + M_{02}U + M_{03}V - M_{10}I - M_{11}Q - M_{12}U - M_{13}V,$$

(A3)

where we have written the elements of $M$ with a double subscript indicating the row and column, both numbered from zero to three. For the sake of brevity, the Stokes
profiles' functional dependence on $\lambda$ will be implicit. The calibration spectra are given by

$$I'_C + Q'_C = (M_{00} + M_{10})I_C,$$

$$I'_C - Q'_C = (M_{00} - M_{10})I_C.$$  \hspace{1cm} (A4)

$$I'_C + Q'_C = (M_{00} + M_{10})I_C.$$  \hspace{1cm} (A5)

After some algebra, the observed, calibrated, Stokes $I$ profile, denoted $I^{\text{cal}}$, is then given by

$$I^{\text{cal}} \ = \ \frac{1}{2} \left( \frac{I' + Q'}{I'_C + Q'_C} + \frac{I' - Q'}{I'_C - Q'_C} \right),$$  \hspace{1cm} (A6)

$$I^{\text{cal}} \ = \ [(M_{00}^2 - M_{10}^2)I + (M_{00}M_{01} - M_{10}M_{11})Q + (M_{00}M_{02} - M_{10}M_{12})U +$$

$$+ (M_{00}M_{03} - M_{10}M_{13})V]/(M_{00}^2 - M_{10}^2)I_C.$$  \hspace{1cm} (A7)

Because the off-diagonal elements of $M$ are small, we can ignore those parts of the above expression which are the products of such elements, leaving

$$I^{\text{cal}} \ = \ \frac{M_{00}I + M_{01}Q + M_{02}U + M_{03}V}{M_{00}I_C}.$$  \hspace{1cm} (A8)

The observed Stokes $Q$ profile is given by

$$Q^{\text{cal}} \ = \ \frac{1}{2} \left( \frac{I' + Q'}{I'_C + Q'_C} - \frac{I' - Q'}{I'_C - Q'_C} \right),$$  \hspace{1cm} (A9)

$$Q^{\text{cal}} \ = \ [(M_{00}M_{11} - M_{10}M_{01})Q + (M_{00}M_{12} - M_{10}M_{02})U +$$

$$+ (M_{00}M_{13} - M_{10}M_{03})V]/(M_{00}^2 - M_{10}^2)I_C.$$  \hspace{1cm} (A10)

Note the *exact* cancellation of the contribution of $I$ to the numerator of the above expression. In other words, our calibration procedure eliminates crosstalk between $I$ and the polarization measurements. Neglecting the products of off-diagonal elements leaves

$$Q^{\text{cal}} \ = \ \frac{M_{11}Q + M_{12}U + M_{13}V}{M_{00}I_C}.$$  \hspace{1cm} (A11)

The same arguments yield the results

$$U^{\text{cal}} \ = \ \frac{M_{21}Q + M_{22}U + M_{23}V}{M_{00}I_C}$$  \hspace{1cm} (A12)

and
\[ V_{\text{cal}} = \frac{M_{31}Q + M_{32}U + M_{33}V}{M_{00}I_C}. \] (A13)

The denominator, \( M_{00}I_C \), has only a very slow variation with wavelength, since the calibrations are taken with the spectrograph out of focus in an area of clean continuum. It thus has no effect on the value of \( V_{1\text{cal}} \), the first moment of the observed \( V \) profile, and can be dropped. This gives

\[ V_{1\text{cal}} = \int_{-\Delta \lambda}^{\Delta \lambda} (M_{31}Q(\lambda) + M_{32}U(\lambda) + M_{33}V(\lambda)) \lambda \, d\lambda, \] (A14)

\[ V_{1\text{cal}} = M_{31}Q_1 + M_{32}U_1 + M_{33}V_1. \] (A15)

We argue that the profiles of \( Q(\lambda) \) and \( U(\lambda) \) are nearly symmetric about line center, and so their first moments vanish. Then

\[ V_{1\text{cal}} = M_{33}V_1. \] (A16)

The integral (equivalent width or zeroth moment) of the observed \( I \) profile is

\[ I_{0\text{cal}} = M_{00}I_0 + M_{01}Q_0 + M_{02}U_0 + M_{03}V_0. \] (A17)

If \( V(\lambda) \) is anti-symmetric about line center, its integral vanishes. Neither \( Q(\lambda) \) or \( U(\lambda) \) have vanishing integrals, but they are both a few percent at most of \( I(\lambda) \) and their integrals are multiplied by a small off-diagonal element of \( M \), so to a good approximation,

\[ I_{0\text{cal}} = M_{00}I_0. \] (A18)

Finally,

\[ \frac{V_{1\text{cal}}}{I_{0\text{cal}}} = \frac{M_{33}V_1}{M_{00}I_0}. \] (A19)

The ratio of the two diagonal Mueller matrix elements is essentially one for our telescope and spectrograph at \( \lambda6302 \).

Consider now the ratio of the second moment of \( Q_{\text{cal}} \) to \( I_{0\text{cal}} \):

\[ Q_{2\text{cal}} = \int_{-\Delta \lambda}^{\Delta \lambda} (M_{11}Q(\lambda) + M_{12}U(\lambda) + M_{13}V(\lambda)) \lambda^2 \, d\lambda, \] (A20)

\[ Q_{2\text{cal}} = M_{11}Q_1 + M_{12}U_1 + M_{13}V_2. \] (A21)
$M_{12}U_2$ is small compared to $M_{11}Q_2$ and so can be neglected. $V_2$ is also small, since the $V$ profile is largely anti-symmetric. Thus

$$\frac{Q_2^{\text{cal}}}{I_0^{\text{cal}}} = \frac{M_{11}Q_2}{M_{00}I_0}. \quad \text{(A22)}$$

These two diagonal elements of the Mueller matrix are also nearly equal.

References
