CHROMOSPHERIC AND CORONAL HEATING MECHANISMS II

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Abstract. We review the mechanisms which are thought to provide steady heating of chromospheres and coronae. It appears now fairly well established that nonmagnetic chromospheric regions of late-type stars are heated by shock dissipation of acoustic waves which are generated in the stellar surface convection zones. In the case of late-type giants there is additional heating by shocks from pulsational waves. For slowly rotating stars, which have weak or no magnetic fields, these two are the dominant chromospheric heating mechanisms.

Except for F-stars, the chromospheric heating of rapidly rotating late-type stars is dominated by magnetic heating either through MHD wave dissipation (AC mechanisms) or through magnetic field dissipation (DC mechanisms). The MHD wave and magnetic field energy comes from fluid motions in the stellar convection zones. Waves are also generated by reconnection events at chromospheric and coronal heights. The high-frequency part of the motion spectrum leads to AC heating, the low frequency part to DC heating. The coronae are almost exclusively heated by magnetic mechanisms. It is not possible to say at the moment whether AC or DC mechanisms are dominant, although presently the DC mechanisms (e.g., nanoflares) appear to be the more important. Only a more detailed study of the formation of and the dissipation in small-scale structures can answer this question.

The X-ray emission in early-type stars shows the presence of coronal structures which are very different from those in late-type stars. This emission apparently arises in the post-shock regions of gas blobs which are accelerated in the stellar wind by the intense radiation field of these stars.

1. Introduction

The present work is an update of our previous review of the mechanisms which steadily heat stellar chromospheres and coronae (Narain and Ulmschneider, 1990, henceforth called Paper I). In that work we had reviewed most of the papers published up to December 1989. We now continue till the end of 1994. Similar to our previous review in order to limit our scope we do not consider occasional impulsive events like flares, flare heating or the extensive work on coronal magnetic fields in general. Anyway as discussed in Section 7, regular flares are not thought to be an important steady heating process for the solar or stellar coronae. The aim of this review is to concentrate on the persistent heating processes.

Significant developments have occurred since our last review. Books by Ulmschneider et al. (1991) and Spicer and MacNeice (1992) and the Volumes 68 and 70 of Space Science Reviews devote themselves almost exclusively to the topic of chromospheric and coronal heating mechanisms. The great advance is particularly

noticeable in the field of acoustic heating, where now a fairly consistent picture of
the heating of stellar chromospheres exists. Shock dissipation by acoustic waves
can now be considered to be the basic heating process for the chromosphere of all
late-type stars. For rapidly rotating stars magnetic heating is the most important
mechanism. In the field of magnetic heating likewise significant advances have
been made, particularly in methods which study the formation of small-scale struc-
tures. The old question, MHD wave (AC) versus magnetic field (DC) dissipation,
is now seen as simply a question of the spectrum of the photospheric foot point
motions. In addition, reconnection events are thought to generate MHD waves at
heights much above the foot points. Rapid motions lead to wave dissipation (AC
heating), slow motions to magnetic field dissipation (DC heating). As the spectrum
covers a wide range, which by recent observations has been extended towards the
high frequencies, a large number of different AC and DC mechanisms are thought
to operate simultaneously. However, it can not yet be stated with confidence which
magnetic heating mechanism is dominant.

Typical physical parameters in coronal loops are given in Table I of Paper I.
From this table it is seen that one requires mechanical energy fluxes of the order of
$F_M = 4 \times 10^5$, $1 \times 10^6$, and $1 \times 10^7$ erg cm$^{-2}$ s$^{-1}$ with an uncertainty of a factor
of three, for the interconnecting loops, quiet region loops and active region loops,
respectively. Generally in the literature it is assumed (e.g., Withbroe and Noyes,
1997) that active regions require average heating fluxes $F_M$ of about $3 \times 10^7$
and quiet regions of about $4 \times 10^6$ erg cm$^{-2}$ s$^{-1}$ at the temperature minimum.
These values are probably somewhat low as empirical chromospheric emission flux
determinations on basis of the average Sun model C of Vernazza, Avrett and Loeser
(1981) by Anderson and Athay (1989a, b) already requires $1.4 \times 10^7$ erg cm$^{-2}$ s$^{-1}$.
It is generally assumed that active regions have a magnetic field filling factor of
order of 10% and quiet regions of no more than 1% at the bottom of the photosphere
(e.g., Zwaan, 1987). For recent detailed overviews of the magnetic fields of the
solar atmosphere see Moore (1990), Schüssler (1992), Solanki (1993), and Stenflo
(1994).

The classification scheme used in this article is roughly the same as that adopted
by Paper I, except that we removed the Section on the LCR circuit theory; recent
work on this approach can now be found in Section 8. Section 2 covers the acoustic
heating, pulsational heating and heating in early-type stars. Section 3 the magnetoac-
coustic waves, Section 4 Alfvén waves, Section 5 magnetoacoustic surface waves
(sometimes loosely called surface Alfvén waves), Section 6 the heating by magne-
tic field dissipation (DC mechanisms), Section 7 microflare heating, and Section 8
the heating by mass and particle flows as well as by magnetic flux emergence.

Since Paper I our topic has been reviewed in total and in parts by a number of
authors. For example, the acoustic wave heating was reviewed by Ulmschneider
(1990b, 1991, 1993) and Schrijver (1995), general reviews of magnetic heating
processes (both AC and DC mechanisms) see Gomez (1990), Sakurai (1990),
Heyvaerts (1992), Cargill (1992), Priest (1993), and Zirker (1993b). Reviews of

2. Acoustic Wave Heating

The mechanism of acoustic heating has been discussed in detail in Paper I and thus does not need to be reviewed here again. Due to an oversight in the printing the important discussion of acoustic waves in early-type stars was left out of Paper I. Subsection 2.6 is devoted now to this topic. Since the last review in Paper I the acoustic heating picture has greatly gained in precision and it can now be considered as essentially certain, that acoustic waves play an important basic role in the heating of stellar chromospheres of late-type stars (see reviews by Ulmschneider 1990a, b, 1991, 1993). For a very extensive recent review of the basal heating by acoustic waves in cool stellar atmospheres see also Schrijver (1995).

One can summarize (see also Section 2.4 below) the findings as follows. Acoustic shock dissipation is the main heating mechanism for non-magnetic regions of late-type stars. It is the dominant heating mechanism for the entire chromosphere of slowly rotating stars, the so-called basal flux stars (Main-Sequence stars and particularly non-pulsating giants and supergiants), and for F-type stars, where the acoustic heating contribution relative to the rotation-dependent magnetic heating is at its maximum. This picture is consistent with the fact that all late-type stars have surface convection zones, that every turbulent flow field produces acoustic waves, and that acoustic waves invariably develop into shock waves in atmospheres with a significant density decrease. But clearly, acoustic heating does not explain the rotation-dependent chromospheric emission of active chromosphere stars or that of magnetic regions on stars. Here additional magnetic mechanisms, discussed in Sections 3 to 9 are at work. We now discuss the evidence in detail.

2.1. Acoustic Energy Generation

There are two types of analytical treatments which allow to calculate the acoustic energy generation from Kolmogorov-type turbulent convection in the solar convection zone. Goldreich and Kumar (1990) considered the wave generation by turbulent convection in a plane-parallel, stratified atmosphere with gravity. The model consists of two semi-infinite layers, a lower adiabatic and polytropic one to represent the convection zone, and an upper isothermal layer to model the stable photosphere. For the adiabatic layer they assume a convective energy flux given by
the mixing-length theory. If $\nu_A$ is the acoustic cut-off frequency of the isothermal layer, then acoustic waves with $\nu > \nu_A$ freely propagate in this layer, while those with $\nu < \nu_A$ are trapped in the top part of the convection zone ($p$-modes and $f$-modes). Wave generation is concentrated at the top of the convection zone because the turbulent Mach number $M_t \ll 1$ peaks there. The authors find that the dimensionless efficiency for the conversion of the convected energy into wave energy is $\epsilon \sim M_t^{2.5}$, for $p$-modes, $f$-modes, and propagating acoustic waves. For acoustic waves most of the energy is generated in the top scale height of the convection zone and is carried by waves with $\nu > \nu_A$.

Kumar (1994) extended this approach by computing power spectra of acoustic waves in the range of $\nu = 5.2$ to 7.5 mHz, above the acoustic cut-off frequency, and comparing them with observations by Duvall et al. (1991), which show a multi-peaked power spectrum that rapidly falls off towards high frequencies. In this calculation a quadrupole source term is evaluated assuming a Kolmogorov turbulent energy spectrum, taking convective velocities from a mixing-length solar model, and adjusting the depth of the velocity peak to best fit the observed spectral peaks. These latter peaks are interpreted as due to constructive interference from propagating acoustic waves. By concentrating on the above mentioned frequency range, high-frequency contributions to the source terms were neglected, which makes the spectrum unreliable at higher frequency. The acoustic wave source was found to be located $140 \pm 60$ km below the solar surface.

The other type of analytical treatment is by Musielak et al. (1994). Here a mixing-length model of the solar convection zone is split up into a series of isothermal horizontal layers, the acoustic spectral contributions of which were integrated over height. In this work, which follows Stein (1967), the description of turbulence was considerably improved. On basis of theoretical simulations and observations, it was found that the turbulence is best represented by an extended Kolmogorov spatial and a modified Gaussian temporal energy spectrum. In contrast to the previous results, the acoustic wave spectra and wave energy fluxes do not depend very sensitively on the turbulent energy spectrum. The sound emission is almost purely quadrupolar. Typical total acoustic fluxes are of the order $F_A = 1.2 \times 10^8$ erg cm$^{-2}$ s$^{-1}$ ($\alpha = 2.0$) with a peak of the acoustic frequency spectrum near $\nu = 16$ mHz are found, which scale with the mixing-length parameter as $\alpha^{3.8}$. The main contribution of the spectrum is from near the photospheric cut-off frequency of $\nu = 6$ mHz to a frequency roughly ten times higher. Because the theory is linear (no shocks) the computed fluxes most likely constitute lower limits for the generated acoustic energy.

A very different approach to calculate acoustic energy fluxes is based on the time-dependent numerical radiation-hydrodynamical convection zone simulations by Stein and Nordlund (1991) as well as by Steffen et al. (1991) and Steffen (1994). Stein and Nordlund (1991) have calculated 3-D radiation-hydrodynamic models for the solar convection zone and have evaluated the vertically directed ‘acoustic energy flux’, which however also contained contributions due to convective
motions and acoustic modes which are trapped because of their low frequencies. A net flux of $\approx 7 \times 10^9$ erg cm$^{-2}$ s$^{-1}$ was found. The acoustic energy flux in the propagating frequency range was of the order of $\approx 2 \times 10^7$ erg cm$^{-2}$ s$^{-1}$, which is close to the value given by Steffen et al. (1991). More recent models, which include also effects due to time-dependent ionization of hydrogen suggest that this value should only be considered as a lower limit (see, e.g., Rast et al., 1993).

Steffen et al. (1991) and Steffen (1994) have computed time-dependent 2-D solar convection zone models. In these calculations, which due to the two-dimensional geometry failed to reproduce the Kolmogorov turbulence spectrum, acoustic spectra were found, which peak near the cut-off frequency at $\nu = 5.3$ mHz and which decay rapidly towards higher frequencies. The authors find that the peak of the averaged convective velocity occurs at a depth of 150 km and conclude that the convective motions in the subphotospheric layers supply ample acoustic energy to account for the observed chromospheric radiation losses.

A comparison of Stein’s method with the Goldreich and Kumar (1990) approach is not easily accomplished, as noted by the latter authors. Kumar’s (1994) exponentially decreasing spectral peaks agree well with observations, and both the numerical simulations of Steffen et al. (1991) and Stein and Nordlund (1991) show such exponentially decreasing spectral peaks. Yet Musielak et al. (1994) display a spectrum which rises with frequency at similar low frequencies. The difference may be that the latter authors concentrate exclusively on the propagating waves while the former authors also include interference effects and standing wave power. Moreover, the Kumar (1994) approach neglects high-frequency source function contributions, and it is not known in what way in the numerical simulations the selected grid spacing affects the high-frequency power.

Independent evidence exists that short period wave power is probably much larger than indicated by an exponentially decreasing acoustic wave spectrum. The most significant evidence comes from empirical models of the outer solar atmosphere by Vernazza et al. (1981) as well as Avrett (1985), who in order to correctly fit the observed width of many spectral lines use a large amount of microturbulent broadening, with amplitudes approaching the sound speed. In addition, it is known that failure to use such microturbulent broadening in empirical atmosphere models would produce undesirable effects, like, e.g., unobserved emission peaks in the infrared triplet lines of Ca II (Rammacher, 1993, private communication). Some of this short period power may be due to a nonlinear interference at a given line of sight by acoustic shock trains generated at different spatial locations on the solar surface and propagating at various angles in nonvertical direction.

For stars other than the Sun the acoustic flux calculations of Bohn (1981, 1984), based on the Stein approach, are still the best estimates available, although the exponential spatial energy spectrum and exponential frequency factor used by him is now known to be a poor representation of the turbulence. Because in quoting Bohn’s (1984) acoustic fluxes in Equation (2.14) of our Paper I we suffered from a misprint, we repeat here the correct value (erg cm$^{-2}$ s$^{-1}$)
\[ F_M = 7.1 \times 10^{-27} T_{\text{eff}}^{9.75} g^{-0.5} \alpha^{2.8}, \]  

(2.1)

where the maximum temperature \( T_{\text{eff}} \) for the validity of this formula is given by

\[ T_{\text{eff}}^{\text{max}} = 4466 \ g^{0.05}. \]  

(2.2)

Here \( T_{\text{eff}} \) (K) is the effective temperature and \( g \) (cm s\(^{-2}\)) the gravity of the star. \( \alpha \) is the mixing-length parameter.

For most stars his acoustic spectra show a maximum around periods \( P \approx P_A/5 = 1/(5\nu_A) = 4\pi c_s/(5\gamma g) \), where \( \nu_A \) is the acoustic cut-off frequency at the bottom of the photosphere, \( c_s \) the sound speed and \( \gamma = \frac{5}{3} \) the ratio of specific heats. The Bohn acoustic fluxes would be considerably decreased if a Kolmogorov spatial spectrum and a modified Gaussian frequency factor for the turbulence were taken. However, this reduction is offset by the larger values of the mixing-length parameter \( \alpha = 1.7 \) to 2.0 currently favored (Kumar, 1994; Musielak et al., 1994). These high values for \( \alpha \) are also indicated by a comparison of the peak value of the convective velocity in mixing-length models with that in time-dependent numerical convection zone simulations (Steffen, 1993, private communication).

The much larger convective velocities in F-stars as compared to M-stars suggest that the acoustic flux generation peaks near F-stars. This could explain the small variability of the chromospheric emission with rotation found for these stars as here the large acoustic contribution apparently drowns the rotation-related magnetic heating contribution (Walter and Schrijver, 1987, see also Schrijver, 1993). That picture is supported by Simon and Landsman (1991) who analyze measurements of C \( \Pi \) 1335 {\AA} in several hundred low resolution IUE spectra. They find that the CII emission of Main-Sequence stars reach a maximum near F0 and declines towards late A stars. In addition, they observe that the emission for stars earlier than F5 is uncorrelated with rotation. As a rotation correlation would mean a magnetic heating mechanism, their findings are seen as support for the acoustic heating of chromospheres.

For greater \( T_{\text{eff}} \) Main-Sequence stars have higher acoustic fluxes due to the larger convective velocities. Similarly, as compared to Main-Sequence stars, giants have larger acoustic fluxes due to the greater convective velocities in these stars. Yet in subsequent propagation, acoustic waves will suffer greater radiation damping in giant stars (Ulmschneider, 1989).

2.2. **Empirical Chromospheric Emission and Directly Observed Acoustic Flux**

It has long been thought (e.g., Ulmschneider, 1970) that the height-dependence of the chromospheric emission could be a powerful means to identify the nature of the chromospheric heating mechanism. By including losses in the Fe II lines, Anderson and Athay (1989a, b), improving earlier empirical loss determinations
by Vernazza et al. (1981) find that the solar chromospheric cooling flux is for many scale heights proportional to the gas pressure. As shown by Ulmschneider (1990b, Figure 9, 1993), this cooling flux can be balanced theoretically by a monochromatic limiting acoustic shock wave flux with a wave period of around $P = P_A/5 \approx 40$ s. This agreement is remarkable due to the following reason. All monochromatic shock waves eventually reach limiting strength, and for a given gas pressure the limiting flux of such a shock wave becomes independent of the initial wave energy (see Paper I), and depends only on one theoretical parameter, the wave period $P$. The remarkable fact is that there is indeed a specific value of $P$ which gives a perfect fit of the theoretical heating rates and empirical cooling rates over a height range of many scale heights.

The Anderson and Athay (1989a, b) chromospheric cooling fluxes are found to be consistent with the directly observed propagating acoustic wave flux (Endler and Deubner, 1983; Deubner, 1988), see also Figure 9 of Ulmschneider (1990b). Deubner (1988) finds acoustic fluxes of $2.0 \times 10^7, 1.2 \times 10^6, 4.5 \times 10^5$ erg cm$^{-2}$ s$^{-1}$ at the heights 300, 800, 1500 km, respectively. As compared to a large number of earlier acoustic wave observations, cited by the above authors, which usually tend to give much lower flux values, it was stressed that for reliable acoustic wave detection, very stable, high resolution, long duration observations are required and that the effects of seeing and of the modulation transfer function have to be carefully removed (see also Ulmschneider, 1990b, 1993).

Rutten et al. (1991) have discussed the minimum radiative emission flux observed from late-type stellar chromospheres and coronae in the Ca II, Mg II, Si II, C II, Si IV, C IV lines and in X-rays. This extensive survey includes Main-Sequence stars and giants. The two types of stars appear to have the same minimum emission (called basal flux) for given color. The basal fluxes for Ca II vary from a value of $2 \times 10^6$ erg cm$^{-2}$ s$^{-1}$ for F-stars to $1 \times 10^5$ erg cm$^{-2}$ s$^{-1}$ for K-stars, while for Mg II from $5 \times 10^5$ erg cm$^{-2}$ s$^{-1}$ to $1 \times 10^5$ erg cm$^{-2}$ s$^{-1}$ for the same stars. The authors have defined observational basal flux lines and attribute the minimum emission near these lines to heating by a non-magnetic mechanism. The excess heating above the basal flux is attributed to a magnetic mechanism.

Nieuwenhuijzen et al. (1994) derived shock strengths from the observed micro-turbulence in super- and hypergiants. For the least luminous stars the shocks were consistent with what is expected from the weak shock theory. A comparison for cool stars of the shock related energy fluxes with acoustic fluxes by Bois (1981, 1984), allowing for chromospheric radiation damping, they found that the fluxes are compatible with the observations.

2.3. ACOUSTIC WAVE PROPAGATION AND EXCITATION OF CHROMOSPHERIC OSCILLATIONS

The largest body of new work in the acoustic wave heating picture comes from detailed studies of the wave propagation and from calculations of the resonant
response of the chromosphere to the excitation by such waves. We first review
the discussion of the chromospheric Ca II K bright points, then mention recent
improvements in acoustic wave computations and finally results about the atmo-
spheric excitation by detailed acoustic frequency spectra.

Hansteen (1990) investigates the effect of acoustic waves on optically thin
transition layer lines and concludes that if the ionization balance of the emitting
ion is solved consistently with the hydrodynamic equations, then the waves can
produce line shifts as large as that of the wave velocity amplitude.

Kalkofen (1990a, b, 1991) discusses the heating of the bright points and arrives
at the conclusion that large amplitude, long period (2 to 4 min) waves dissipate
enough energy to account for the chromospheric structure.

Rutten and Uitenbroek (1991), Fleck and Schmitz (1991) (who were the first to
show that the 3-min oscillation, instead of being a cavity mode, can be explained as
the basic cut-off frequency (local) resonance of the atmosphere), Deubner (1991)
as well as Kulaczewski (1992) provided extensive reviews of the bright point
phenomenon. That the bright point phenomenon was produced by a resonance of
the chromosphere was also found by Rammacher and Ulmschneider (1992) on
the basis of monochromatic acoustic wave calculations. These latter authors found
that when at the base of the photosphere the atmosphere is excited by acoustic
waves with periods below a critical value of about $P = 30$ s, then chromospheric
pulsations in the 3-min period range are generated, which are periodically driven by
the process of shock overtaking. Shock overtaking means that two or more shocks
merge to form single strong shocks, which greatly influence the atmospheric energy
and momentum balance.

The explanation of the chromospheric bright point phenomenon was also the
aim of Carlsson and Stein. The main contribution of these authors was to pro-
vide for the first time a fully self-consistent radiation-hydrodynamic calculation
of acoustic waves in the solar chromosphere in one-dimensional, plane-parallel
gometry (Carlsson and Stein 1991, 1992, 1993, 1994). In these calculations the
H-ionization, the time-dependent and non-equilibrium (NLTE) population densi-
ties for the multi-level H-atoms as well as the radiation fields are simultaneously
and consistently taken into account. Their computations automatically include
the ionization pumping mechanism originally proposed by Lindsey (1991). By
comparing their result with calculations assuming LTE and neglecting radiation
(or assuming NLTE, but disregarding the time-derivative and advective terms in
the statistical rate equations for hydrogen) Carlsson and Stein conclude that any
of these simplifying assumptions would greatly misrepresent the actual heating
behavior of acoustic waves in the chromosphere. Their monochromatic 180 s wave
calculations reproduce many aspects of the observed Ca II bright point behavior
rather well, which is strong evidence that this feature is indeed attributable to an
acoustic wave phenomenon.

These theoretical simulations also reproduce (Skartlien et al., 1994) a puzzling
observation that the V–V phase shift between velocity fluctuations in the 8542 Å
and 8498 Å Ca II IRT lines is essentially zero, while in model C of Vernazza et al. (1981) these lines are formed 300 km apart. These results were interpreted by a formation of the lines at similar physical heights in the wave permeated atmosphere. Yet considerable unobserved V–V phase shifts were found between the IRT lines and Ca II K.

As pointed out by Deubner (1994) and Fleck et al. (1994) there are still a number of puzzling inconsistencies of this theoretical picture when compared with observations, which hint that we may still miss an important part of the physics of the bright point phenomenon. The observations show that the ratio of the 3-min wave period to the ‘shock jump period’ (the time span in which the line core shifts from a maximum red to a subsequent maximum blue position) is no more than about 3, and that this ratio appears even to decrease towards the upper chromosphere. In addition, the V–V phase differences between the Ca II K and Ca II IRT lines is very different from the observed zero degrees. This suggests it is necessary to carry out more refined three-dimensional wave propagation modelling, and to look more carefully into the behavior not only of the local resonances but also of the chromospheric cavity resonance, taking into account the physics and geometry of the transition layer.

There is evidence by Cook (1992) and Wang et al. (1995), discussed below in Section 9, that the chromospheric bright point phenomenon, different from what has been claimed by Deubner (1991) and Rutten and Uitenbroek (1991), may be a phenomenon associated with magnetic flux tubes.

Jordan (1993) investigated the feasibility of heating the solar chromosphere by spectra of weak acoustic shock waves. Here a series of waves in the short period range of 10–50 s, are allowed to heat independently on basis of the time-independent weak shock theory. He finds that the heating is concentrated near the temperature minimum area and that the middle and higher chromospheric losses cannot be adequately balanced by acoustic heating. He thus argues that acoustic heating is not a viable mechanism for balancing the chromospheric losses. This conclusion has to be taken with utmost caution, as important physical effects, like the nonlinear shock formation process, the presence of the full wave spectrum, the shock overtaking phenomenon which is prevalent for all short period wave calculations as well as the nonlinear, nonequilibrium and time-dependent effects in the radiation have all been neglected. In addition, his findings are in contradiction to the monochromatic weak shock heating results cited above, which compare well with the cooling rates of Anderson and Athay (1989a, b), also used by him.

Cuntz (1990) computed time-dependent acoustic wave models for the outer atmosphere of three late-type giants and supergiants. His primary goal was to estimate the geometrical extent of their chromospheres by evaluating the decrease of the mean electron densities. His models show rather small chromospheric extents (< 0.2 $R_*$), which are in quantitative agreement with other estimates based on detailed semiempirical chromosphere models and are consistent with the analysis of the contribution function of distinct chromospheric lines (Judge, 1990). Other
estimates, which point to very extended chromospheres are now considered as unreliable as these models assume that hydrogen is fully ionized. Cuntz sees his work as further support of the acoustic wave picture.

Buchholz and Ulmschneider (1994) have constructed theoretical acoustic chromosphere models for Main-Sequence stars and simulated spectral line emission in Mg II and Ca II assuming partial redistribution. The theoretical chromosphere models were computed with Bohn's (1984) acoustic energy fluxes on the basis of monochromatic acoustic waves with periods equal to the maximum of Bohn's acoustic spectra (\(P = P_A/5\)). The authors were able to reproduce rather well the observed basal flux line of Main-Sequence stars for both Mg II and Ca II by Rutten et al. (1991) and consider this result as an important confirmation of the acoustic heating theory.

Cuntz et al. (1994) computed theoretical acoustic chromosphere models for three giant stars with \(T_{\text{eff}} = 5012\) K and \(\log g = 3\) and with 1/1, 1/10, and 1/100 solar metal abundances. Subsequently, they simulated spectral line emission in Mg II and Ca II using partial redistribution. The theoretical chromosphere models were again computed on the basis of monochromatic waves with acoustic energy fluxes by Bohn. The periods are those of the maximum of Bohn's acoustic spectra. The authors were able to reproduce rather well the observed basal flux line by Rutten et al. (1991). They also could reproduce the observed insensitivity of the emission flux in Ca II and Mg II against changes in metal abundance. In the case of Mg II, these results agree with observations of Dupree et al. (1990), who obtained Mg II energy losses from 10 metal-deficient giant stars of Population II. The basal flux and the metallicity results both are considered as further confirmation of the significance of the acoustic heating process.

We now turn to response studies of the solar and of stellar chromospheres to the excitation by acoustic waves. Following Fleck and Schmitz (1991, 1993), who evaluated analytical solutions for sinusoidal excitations of an isothermal solar-type atmosphere model, starting suddenly at a time \(t\), Kalkofen et al. (1994) have investigated the continuous and pulse excitation of such an atmosphere by both small and large amplitude acoustic waves. They find that the initial switch-on effect of the calculation produces strong resonances of the 3-min (\(\nu \approx 6\) mHz) type which overlay the incident signal. In the linear case, the authors were able to derive an analytical asymptotic formula for the time behavior of the entire oscillation, which shows that after a given time the resonances decay (with \(\sim t^{-3/2}\)) and only the incident excitation signal survives. For nonlinear excitation, however, due to the process of shock-overtaking, persistent 3-min type shock waves are found. The period of these shock waves increases with atmospheric height.

Sutmann and Ulmschneider (1995a) extended this work to atmosphere models with arbitrary temperature gradients. They confirmed the work by Kalkofen et al. and showed that the resonances decay rapidly in atmospheres with an outward temperature rise, but slowly in an isothermal atmosphere and very slowly in an atmosphere with an inward temperature rise. The decay behavior was found to be
similar, whether the excitation was by a pulse or by continuous wave action. The response to excitation by a stochastic wave spectrum turned out to be a series of uncorrelated transient events, each with its rapid decay of the resonance.

Sutmann and Ulmschneider (1995b) studied the excitation of the solar atmosphere with large amplitude acoustic waves. Both monochromatic waves and acoustic spectra were considered. The waves were computed in the adiabatic approximation. For monochromatic excitation a critical frequency $\nu_{cr} \approx \frac{1}{30}$ Hz similar to that of Rammacher and Ulmschneider (1992) was found, which separates different domains of resonance behavior. Upon excitation with frequencies $\nu < \nu_{cr}$ the atmospheric resonance decays rapidly with time as in the small amplitude wave case, while for $\nu > \nu_{cr}$, persistent resonance oscillations occur, which are caused by shock over takings. Excitation by acoustic spectra always leads to the $\nu > \nu_{cr}$ behavior. Independent of the spectral shape and the energy of the incident wave, acoustic spectra generate oscillations mainly at frequencies $\nu = 6$–7 mHz at the top of the chromosphere. A superposed photospheric 5-min oscillation decays rapidly with height and does not influence the remaining resonance oscillation. A very important result was that the mean shock heating rate by acoustic spectra in the Sun, irrespective of the shape of the acoustic spectrum (Gaussian, exponential, box or random) was found to be roughly similar to that of a monochromatic wave of period $P = P_A/5$, where $P_A = 4\pi c_s/\gamma g$ is the acoustic cut-off period. In addition, irrespective of the initial spectral shape and energy of the excitation and even of the gravity and effective temperature of the star, a universal average shock strength $M_S = 1.5$ was found, which is similar to the shock strength of a $P = 35$ s $\approx P_A/5$ monochromatic wave. This universal strength, moreover, is in surprising agreement with the universal strength value found by Ulmschneider (1990b) on the basis of monochromatic limiting strength acoustic shock waves with period $P = P_A/5$.

2.4. SUMMARY, A CONSISTENT ACOUSTIC HEATING PICTURE

That there is growing evidence for a consistent acoustic heating picture has already been pointed out in the reviews of Ulmschneider (1990a, b, 1991, 1993) and recently in the detailed review of the basal flux phenomenon by Schrijver (1995). Direct wave observations, empirical cooling flux determinations, the inferred microturbulence, and theoretical wave energy generation calculations all point to the fact that a sufficient amount of acoustic wave flux is available in a spectral range extending from the cut-off frequency at $\nu_A = 1/P_A = 5.3$ mHz to a frequency roughly an order of magnitude higher. Theoretical wave propagation calculations, starting with various acoustic wave spectra, all show a similar height-dependent dissipation of the acoustic flux. This dissipation behavior is found to be similar to that by a monochromatic acoustic wave with a period $P \approx P_A/5$, which is in good agreement with the height-dependent empirical chromospheric cooling rate. Wave propagation calculations using monochromatic waves with periods $P = P_A/5$ for Main-Sequence stars permit the computation of theoretical chromosphere models
and allow the evaluation of theoretical basal emission fluxes in the cores of strong chromospheric resonance lines of Ca II and Mg II. These theoretical emission fluxes agree very nicely with the observed basal emission flux limits in Ca II and Mg II. A similar evaluation of theoretical emission fluxes for giants also agrees with the empirical emission flux limits which are the same for Main-Sequence stars and giants. In addition, the theoretical flux determinations in giants are largely independent to changes in the metal abundances at least within a factor 100, consistent with observations. For giants, moreover, the geometrical extents of acoustically heated chromospheres are in good agreement with results from recent semiempirical chromosphere models.

There are still many minor difficulties with the acoustic heating picture (see Ulmschneider, 1990b, 1993). One would like to have more accurate empirical and theoretical acoustic fluxes, understand better the phase relations in the Ca II lines, as well as the nature and magnitude of the microturbulence and its relation to nonvertical wave propagation. One also has to repeat many of the above cited wave propagation and excitation calculations using a fully self-consistent wave code, employing acoustic frequency spectra, and consistently including the physics of the transition layer onset. The relationship between the local and cavity resonances in the chromosphere as well as the nature of the universal shock strength needs better understanding.

2.5. Pulsational Waves

Already in Paper I we have discussed pulsational waves with periods \( P > P_A \) as a likely cause of non-magnetic heating in Cepheids, RR Lyrae stars, \( \delta \) Scuti stars and Mira-type variables. In the case of Mira stars, a landmark paper has been given by Bowen (1988), who studied the propagation of pulsational shocks in an AGB star atmosphere, assuming 1 M\(_\odot\), 240 R\(_\odot\), \( T_{\text{eff}} = 3000 \) K and a period of 320 days. All these parameters are relatively close to those proposed for Mira (\( \rho \) Cet) itself. Bowen found that the pulsational shocks greatly extend the atmosphere and also promote the generation of mass loss. In addition, the energy deposition by the shocks forms a chromosphere, which by the author is referred to as a ‘calorisphere’. The shocks lead to significant line emission, such as in Ca II H and K, Mg II h and k, and H\( \alpha \) (as well as other lines), which closely follows the pulsation cycle of the star. The reason for this behavior is that the photospheric pulsation introduces strong individual shocks, which give rise to radiative energy losses by the gas of the post-shock regions. Luttermoser and Bowen (1990) have studied radiative transfer effects in Mira-type atmospheres including the formation of lines. The change of radiative emission as function of the stellar pulsation cycle is also found in various observational studies. Gillet et al. (1985) have focused on changes in H\( \alpha \) in the case of Mira. In the case of the Mira star S Car, the time-dependent behavior of the Mg II line profiles has been measured by Bookbinder et al. (1989). They found remarkable changes in both the Mg II h and k lines, including the ratios of
these components. Andrievsky and Garunov (1991) discussed the generation of line emission in a δ Sct-type atmosphere. They also found that the chromospheric emission and its variability change with phase, which can be interpreted as a result of a periodic inflow of energy into the star’s outer layers and subsequent shock formation.

2.6. ACOUSTIC SHOCKS IN EARLY-TYPE STARS

It is now generally accepted that the winds of early-type stars are driven by radiation pressure (see, e.g., Cassinelli, 1979). From X-ray observations one finds that for early-type stars there is a correlation between X-ray luminosity and bolometric luminosity (Pallavicini et al., 1981) in contrast to the situation in late-type stars, where there is a correlation between X-ray luminosity and rotation. The X-ray emission in early-type stars shows the presence of coronal structures which are very different from the connected loop-like coronal layers in late-type stars. It has been proposed that these X-ray emitting regions are located in the hot post-shock regions of moving atmospheric gas blobs or trains of shocks either in the stellar wind or in the terminal shock region where the stellar wind hits the interstellar medium. These wind shocks are accelerated by the intense radiation field of the star (e.g., Lucy, 1982a, b). For reviews and discussions of recent time-dependent calculations see Owocki et al. (1988), Rybicki et al. (1990), Castor (1991), as well as Owocki (1990, 1991, 1992).

3. Heating by Fast and Slow Magnetoacoustic Body Waves

In this section we consider slow and fast mode magnetoacoustic waves in homogeneous magnetic fields as well as longitudinal waves in magnetic flux tubes. For older work we refer to Section 3 of Paper I. See also the reviews of Musielak (1992) and Narain and Agarwal (1994).

3.1. MAGNETOHYDRODYNAMIC WAVE GENERATION

As Alfvén waves and magnetoacoustic waves are generated in a similar way, we include the discussion of Alfvén wave generation in the present subsection. The dissipation of Alfvén waves is reviewed in Section 4. For a general review of acoustic and magneto hydrodynamic (MHD) wave generation by turbulence in stellar convection zones see Musielak (1991).

The generation of different types of MHD waves in the solar atmosphere has been studied primarily by using analytical methods based on the theory of sound generation by Lighthill (1952), see also Paper I. Kulsrud (1955) and Osterbrock (1961) extended this theory by including magnetic field effects, and Musielak and Rosner (1987, 1988) improved it by accommodating the presence of stratification and an embedded uniform magnetic field in the wave generation region (see also...
Rosner and Musielak, 1989). For a more detailed review of this work see Paper I. More recently, Collins (1989a, b, 1992) has modified this type of wave generation theory to explore the excitation of MHD waves by periodic velocity fields in diverging magnetic flux tubes.

The common feature of these studies is that they look at the magnetic field in a non-local way to get mean generated wave fluxes. Further advances occurred when a detailed local flux tube field geometry was considered. This led to the work by Musielak et al. (1989) as well as Musielak, Rosner et al. (1995a, b) for longitudinal and transverse tube waves. Finally as an investigation based on the Lighthill theory and concentrating on the local field geometry the work of Yun and Lee (1990) as well as Lee (1993) should be mentioned who studied the generation of MHD waves inside sunspot magnetic fields.

There are also methods of MHD wave generation which are not based on the Lighthill approach. In these methods magnetic flux tube models are perturbed from the outside with velocity fluctuations of an observed magnitude. Such an approach has been taken by Choudhuri et al. (1993a), Choudhuri et al. (1993b), Huang et al. (1995) and Zhugzhda et al. (1995).

The basis for such an approach are recent observations of the proper motions of footpoints of magnetic flux tubes at the photospheric level (Muller, 1989; Muller et al., 1994). These observations clearly show that horizontal velocities as large as 3 km s\(^{-1}\) occur in the solar photosphere. Velocities of this magnitude and larger have also been reported by Title (1994, private communication). These authors have recognized that the interaction between the large velocity motions and the magnetic flux tubes may become an efficient source of magnetic tube waves which propagate along the tubes and carry energy to the chromosphere and corona. A rough estimate of the generated wave energy fluxes by Muller et al. (1994) clearly demonstrates that the amount of wave energy available for heating is sufficient to sustain the mean level of the observed radiative losses from both the solar chromosphere and corona.

Roudier et al. (1994) analyze solar granulation observations and find that the size and brightness of intergranular bright points, which are presumably due to magnetic network elements, increases simultaneously, contrary to previous observations. They interpret this result as a consequence of the arrival of mechanical energy at the magnetic flux tubes through shaking by turbulent granules. This mechanical energy contributes towards the heating of the solar corona.

The observations of large velocity fluctuations affecting foot points of magnetic flux tubes are also found to be in good agreement with recent theoretical advances. Time-dependent numerical simulations of the solar convection zone performed by several different groups (e.g., Nordlund and Dravins, 1990; Nordlund and Stein, 1991; Cattaneo et al., 1991, Nesis et al., 1992; Steffen, 1993) all suggest the presence of motions with horizontal velocities larger than 2 km s\(^{-1}\) near the top of the solar convection zone. In these numerical simulations, the presence of
horizontally propagating shocks near the top of the solar convection zone has been detected (Cattaneo et al., 1991; Steffen, 1993; Steffen et al., 1994).

For the generation of torsional Alfvén waves the picture developed by Schüssler (1984a, b) applies, that the flux tubes are surrounded by cyclonic nonmagnetic downflows (resulting from angular momentum conservation when flows from an extended region converge into a narrow downflow channel), in which fluctuations and friction will excite waves. This type of wave production has to be augmented by torsional wave generation from other wave modes via nonlinear mode-coupling.

We now review the MHD wave generation in greater detail. Musielak et al. (1989) have investigated the interaction between turbulent motions in the solar convection zone and thin magnetic flux tubes. They have considered vertically oriented magnetic flux tubes and restricted their approach to the linear regime. The obtained results indicate that the wave energy flux carried by longitudinal tube waves along a single magnetic flux tube are of the order of $10^7$ erg cm$^{-2}$ s$^{-1}$ or less, which seems to be too little to account for the observed enhanced heating in the chromospheric network. This work was recently modified and extended by Musielak et al. (1995a), who have shown that the flux can be considerably higher if a more refined treatment of the longitudinal tube wave generation is considered. In a similar treatment for transverse tube waves, Musielak et al. (1995b) has shown that the wave energy flux carried by these waves can be of the order of $10^8$ erg cm$^{-2}$ s$^{-1}$.

Solar convective flow generates torsional, kink and sausage magnetic flux tube waves in the presence of magnetic fields (Stein and Nordlund, 1991). The transmitted fast magnetoacoustic waves are severely refracted and may be totally internally reflected because of the increase in Alfvén speed with height. The difficulty in getting energy to the upper chromosphere and corona may be overcome by waves ducted along magnetic flux tubes. Such waves are produced directly by the convective flow and may also be produced by coupling to the acoustic wave flux incident on the chromospheric magnetic canopy. The wave energy that reaches the outer atmosphere is dissipated there by phase-mixing, resonant absorption and nonlinear coupling to compressive magnetoacoustic waves that form shocks. Numerical simulation shows that it is difficult to get a single type of wave that can be copiously produced by the convective flow as well as to transport its energy into the upper chromosphere and corona to heat these layers. Therefore mode-coupling is likely to play a crucial role in the heating.

Collins (1992) compares the flux of Alfvén waves, derived using Lighthill’s method for a localized source in an isothermal homogeneous plasma and constant magnetic field, with his fast and slow MHD wave results (Collins, 1989a, b). Since a small fraction of fast and slow wave flux generated in the solar photosphere is transported to the corona, the ratio of the Alfvén to the total flux sets an upper limit on the efficiency of coronal heating from waves emitted by photospheric motions. For a simple class of spherical sources, adopted as model for oscillating granules
with 5-min period, the Alfvén fluxes are comparable to those required to heat active regions.

Choudhuri et al. (1993a, b) have investigated the generation of magnetic kink waves by rapid footpoint motions of the magnetic flux tube. The linear analytical investigations were carried out on basis of a simple model consisting of two isothermal regions to represent the chromosphere and the corona. They argue that occasional rapid motions can account for the entire energy flux needed to heat the quiet corona. The authors find that pulses are much more efficient than continuous excitation to get wave energy into the corona and that the energy flux from pulses actually increases when there is a transition layer temperature jump in the atmosphere.

Similar work has been carried out by Huang et al. (1995), who numerically investigate the nonlinear time-dependent response to purely transverse shaking of a thin exponentially spreading vertical magnetic flux tube embedded in a solar convection zone and atmosphere model. The shaking was imposed at different heights by using a Kolmogorov-type turbulent energy spectrum scaled to various specified rms velocity amplitudes which are determined by observations. In these nonlinear excitation calculations much larger fluxes of the order of $10^9$ erg cm$^{-2}$ s$^{-1}$ were found. In contrast to previous analytical studies, this seems to indicate that there is enough wave energy to account for the enhanced heating observed in the chromospheric network, and that magnetic tube waves may also play a role in the heating of other regions of the solar atmosphere. The authors attribute the reason for the larger flux to the fact that in the nonlinear case the horizontal tube motion is appreciable compared to the diameter of the tube while in the analytical case it is not.

For the longitudinal tube wave generation using the same nonlinear approach where the shaking was replaced by applying external pressure fluctuations resulting from the turbulent energy spectrum, Ulmschneider and Musielak (1995) likewise found an order of magnitude larger fluxes of a few times $10^8$ erg cm$^{-2}$ s$^{-1}$, relative to the analytical results. In both calculations shocks may increase the generated wave power even further.

This type of investigation has also been carried out by Zhugzhda et al. (1995) who computed the time-dependent formation of kink shock waves in longitudinal-transverse waves propagating along a vertical, exponentially spreading solar magnetic flux tube. The work is an extension of the time-dependent treatment of longitudinal-transverse MHD tube waves by Ulmschneider et al. (1991). The kink shock was found to be always accompanied by a longitudinal shock and, despite of the different propagation speeds of the longitudinal and transverse waves, the joint shock propagated with a common speed which indicates strong mode-coupling. After a purely transverse pulse excitation of one sec duration a large almost exclusively longitudinal wave pulse developed.
3.2. MAGNETOACOUSTIC WAVE PROPAGATION

The smallness of dissipative coefficients in many astrophysical situations requires that large gradients must develop in the dynamical evolution of the system, that is, small spatial scales must form, for efficient dissipation to take place. This applies particularly to the smallest spatial scales which occur in waves, namely shocks, where the heating by viscosity, thermal conductivity and resistivity usually by many orders of magnitude exceeds that which occurs in a wave with a smooth variation over the wavelength. It is thus of great importance to study situations where in the course of the wave propagation small scales form naturally.

In the framework of incompressible MHD and for one-dimensional cases using a slab geometry, where the physical variables depend only on the direction perpendicular to the magnetic field direction, Califano et al. (1990a, b) studied the resistive dissipation of MHD waves in nonuniform media. Using a normal mode approach they show analytically the existence of a new class of solutions which form small scales over the entire inhomogeneous region and not only at some isolated ‘resonant’ regions. Electrical resistivity is the sole damping mechanism considered by them. The results of this linear analysis should remain valid even when nonlinear effects are taken into consideration, they remark.

Mok and Einaudi (1990) discussed the validity of the normal mode approach and described simple extensions of their theory to the compressible case.

Califano et al. (1992) extended their work to the more realistic compressible case. They confirm the formation of small scales over the entire inhomogeneous region previously found in the incompressible case and identify the wave modes in the inhomogeneous medium in terms of those in homogeneous media. In particular, the wave modes were found to be generalizations of the well-known shear Alfvén and slow magnetoacoustic modes of homogeneous plasmas. The fast mode remains essentially ideal and disappears in the resistive regime. The most promising agents seem to be the shear Alfvén waves which behave almost incompressibly.

In a weakly dissipative, incompressible medium Malara et al. (1992) study the two-dimensional propagation of shear Alfvén waves and slow magnetoacoustic waves in the $y - x$ plane where the magnetic field is in the $y$-direction and the Alfvén speed varies in the $x$-direction. The time-evolution of arbitrary initial perturbations is followed numerically using a pseudo-spectral method. They find that the rate of energy deposition is strictly related to the formation of small scales which are formed by nonlinear interactions of the propagating modes and by the nonuniformities of the assumed medium. The authors point out that the damping is different from pure phase-mixing because that occurs with field perturbations in $z$-direction, perpendicular to the $y - x$ plane. Further progress will occur when more realistic compressible cases are considered. Einaudi et al. (1993) readdress these results and put them into a broader perspective.

Chiuderi (1993) in a remark at a conference cautions people against relying too much on results based on normal mode analysis of linearized equations because it
is difficult to recognize the normal modes in the simulated results. Further it seems that the available energy is dissipated before one reaches the asymptotic normal mode behavior.

Mikić (1990) computed the time-dependent evolution of magnetic arcade systems and coronal loops in three dimensions, using a semi-implicit combination of a spectral and finite difference code. The equations include viscosity and resistivity. In his arcade model he shows that current sheets form on an Alfvén time scale and that resistive reconnection liberates magnetic energy which causes the ejection of a plasmoid. Applying a twisting flow to a coronal loop model he finds that it becomes kink unstable.

The propagation of magnetoacoustic waves in inhomogeneous media has also been studied by Ryutova et al. (1991). Their medium is an atmosphere containing an ensemble of tightly packed bundles of magnetic flux tubes. Due to the nonlinearities of the wave as well as the inhomogeneities and statistical properties of the medium both shock waves and solitons form which subsequently dissipate the wave energy. Their computer simulations confirm the enhanced dissipation due to the strong and random inhomogeneities.

Uberei (1990) discusses the Landau damping of acoustic waves in a plasma consisting of an ensemble of magnetic flux tubes. She finds that energy transfer of the acoustic waves to the plasma embedded with the flux tubes depends very critically on the strength of the magnetic field of the flux tubes and the values of the interface parameters.

Woods et al. (1990) model the solar chromosphere-corona transition region assuming the mechanical heating to be the result of the damping of hydromagnetic waves. They suggest that a mixture of fast-mode and slow-mode waves may provide the appropriate heating mechanism in the lower transition region. The authors conclude that only a more detailed model of such a mechanism can determine whether this proposal is viable or not.

The effect of magneto-acoustic waves on spectral line formation and the question of how these waves might be differentiated from sound waves have been discussed by Maltby (1990).

Zhou (1990) solves the MHD equations for Alfvén waves numerically for three electron densities and finds that there are two regions of rapid dissipation when these waves propagate from the transition region to the corona. These regions occur, respectively, in a range of several hundred kilometers above the base of transition region and in the corona at a distance of 1–3 $R_\odot$. It is concluded that wave heating could be more important when the magnetic field divergence becomes stronger.

Ferriz-Mas and Moreno-Insertis (1991) study the propagation and damping of slow-mode MHD shock waves in a cylindrical magnetic flux tube embedded in a field-free homogeneous fluid. The shock dissipation is computed by modifying the Brinkley & Kirkwood theory and allowing the thermodynamic cycle to proceed along the Weymann or Schatzman paths. We feel that the uncertainties in this type of
approach can be avoided by a time-dependent treatment like that of Ulmschneider et al. (1991) and Zhugzhda et al. (1995).

Erdélyi and Markí (1991) balance the empirical chromospheric cooling of the Vernazza et al. (1981, VAL) solar model C against the dissipation of magnetoacoustic waves derived from a weak shock formula (see Paper I), assuming various wave periods. They find that waves with a period of 45 s reproduce best the VAL temperature distribution.

Hasan (1991) considers energy transport in an intense flux tube on the Sun. He finds that overstable oscillations with a characteristic time of order 600 s are set up and concludes that these oscillations are probably not important in heating the chromosphere and corona. Radiative transport is found to dominate the energetics in the photosphere but in the lower layers the main contribution is from enthalpy transport.

The dissipation of ducted, fast magnetoacoustic waves by ion viscosity and electron heat conduction in a radiating, optically thin atmosphere is examined by Edwin and Zhelyazkov (1992). They find that the dissipation length of the wave increases with magnetic field strength and decreases with increasing density and is a few wavelengths for waves of period of several seconds in the active corona. They argue that since the oscillations with such periods have been observed in the corona so the waves could, given the right conditions, be dissipated there. A more thorough rediscussion of this work by Laing and Edwin (1994) showed however, that these waves are not readily dissipated in the solar corona and that it is unlikely that they will play a role in the heating of the solar atmosphere.

Zhukov (1990, 1992) finds that tunnel leakage of fast magnetoacoustic waves leads either to Joule heating of the medium in the vicinity of the Alfvén resonance if the inclination angle of the magnetic field is smaller than a critical angle, or to excitation of Alfvén waves at the Alfvén resonance if the inclination angle is larger than the critical angle. This suggests that nonradiative heating of the corona can be due to solar \( p \)-mode oscillations. The author estimates an energy flux of order \( 10^7 \) erg cm\(^{-2} \) s\(^{-1} \) in the region of the ‘merged’ magnetic field.

The propagation of linear thermal and magnetosonic waves in a general optically thin plasma is studied by Ibáñez and Escalona (1993). In particular a plasma with solar abundances in the temperature range \( 10^4 \text{--} 10^8 \) K is analyzed. Five different kinds of heating mechanisms are considered. Applications to the different regions of the solar atmosphere are outlined.

The possible role of slow and fast magnetoacoustic waves damped by ion viscosity and electron thermal conductivity for the coronal heating has been investigated by Porter et al. (1994a, b) using both a homogeneous field and a slab model. The damping was included selfconsistently in the pure hydrogen plasma model as was optically thin radiation loss. Employing a normal mode analysis, damping rates were computed on basis of a sixth order dispersion relation, for a series of specified coronal loop parameters. The authors conclude that slow mode waves with periods of less than 300 s and fast mode waves with periods of less than 100 s may provide
adequate heating of active regions. They cite a considerable number of observations of nonthermal broadening of EUV and X-ray transition layer lines with velocity amplitudes of up to 60 km s\(^{-1}\) which suggest sufficient slow and fast wave flux, contrary to previous chromospheric and transition layer wave flux determinations. In addition, they point out that the wave flux does not need to originate entirely from the photosphere but may arise also as a consequence of reconnection events below coronal heights.

For a general summary of body wave heating see Section 4.3.

4. Heating by Alfvén Body Waves

In this section we consider Alfvén waves in homogeneous magnetic fields as well as transverse and torsional Alfvén waves in magnetic flux tubes. For the discussion of the Alfvén wave generation see Section 3.1. Older work on Alfvén body wave heating is refereed in Section 4 of Paper I.

4.1. Heating via Shocks and Mode-Coupling

As discussed in Section 3, a promising way to increase the dissipation rate of magnetoacoustic waves is to study the generation of small-scale structures. In the case of Alfvén body waves the same reasoning applies and the study of properties of nonlinear (large amplitude) waves in spatially inhomogeneous structures appears to be very promising, see, e.g., the work by Califano et al. (1992), Malara et al. (1992) discussed in Section 3.2. Even in a geometrically simple situation, like the thin flux tube case, the nonlinear properties of the waves are important, because they lead to mode-coupling and shock formation, the latter being the ultimate of small-scale structure. By mode-coupling, energy from a mode which is difficult to dissipate can be transferred to a more efficiently damped mode.

Ferri\-z-Mas et al. (1989) presented a nonlinear treatment of longitudinal-torsional MHD tube waves, using the thin flux tube approximation and carrying the radial expansion to second order to consistently include azimuthal velocities and twisted magnetic fields. Comparing their results with exact solutions for a uniform flux tube in a non-stratified medium, they find that their method is best for describing surface waves, while body waves are not so well represented. It is highly desirable to extend this work to allow the development of longitudinal and torsional (switch-on) shocks and study mode-coupling.

Hollweg (1992) investigates the nonlinear evolution of a single torsional Alfvénic pulse launched from the solar photosphere on a thin vertical magnetic flux tube which extends into an open coronal region. He finds that the nonlinear dynamics leads to injection of an impulsive flux of energy into the corona. On the basis of this result he suggests that at least some of the sudden events are the consequence of nonlinear wave dynamics, that is, microflares need not be magnetic reconnection driven.
Ulmschneider et al. (1991), employing the method of characteristics to solve the nonlinear MHD equations in the thin flux tube approximation, computed the nonlinear time-dependent development of longitudinal-transverse MHD tube waves in vertical solar magnetic flux tubes. They showed that after purely transverse shaking considerable mode-coupling occurred and longitudinal wave power developed. Following Herbold et al. (1985), Zhugzhda et al. (1995) extended this treatment to allow the computation of longitudinal and transverse shocks. This work is described in Section 3.1.

4.2. Resonant, Turbulent, and Viscous Heating

Alfvén waves are observed in the solar wind propagating outward from the Sun (Hollweg, 1991), they dissipate via MHD turbulence at a rate which is enough to balance the observed heating of the solar wind. For the Alfvén wave heating of the corona the torsional and transverse (kink) Alfvén waves both have the property of propagating energy along the magnetic field and therefore can in principle explain the observed association between enhanced coronal heating and enhanced magnetic field strength. In the case of torsional Alfvén waves, enough energy can reach the corona from the top of the convection zone, despite the reflections which occur due to the strong vertical Alfvén speed gradient, whether the field lines are open (as in coronal holes) provided the flux tubes move about at photospheric heights with velocities of several km s$^{-1}$, or closed (as in coronal loops) provided transmission enhancing loop resonances are excited. The various dissipation mechanisms of Alfvén waves are critically discussed.

Zhugzhda (1991) analyses the problem of propagation and absorption of Alfvén waves in coronal loops. He points out that there are two major problems, namely the strong reflection of Alfvén waves on their way to the corona and the weak dissipation of these waves in the solar corona. The first difficulty is overcome through loop resonance phenomena where Alfvén waves propagate through coronal loops without reflecting at resonant frequencies. He concludes that effective dissipation of Alfvén waves by phase-mixing and resonant absorption together with loop resonance phenomena can lead to the solution of the coronal heating problem. The conclusion that non-compressive Alfvén waves are the best candidates for heating the corona and the solar wind has also been reached by Leer and Hansteen (1990).

Strauss (1991) discusses the difficulties of damping shear Alfvén waves by phase mixing or resonant absorption and concludes that these processes dissipate only after a long propagation time. He points out that the open coronal structures very likely do not have straight field lines but rather a helical field geometry. Different from the straight field case where shear waves will be essentially incompressible, shear waves will lead to $\nabla \cdot \mathbf{v}_\perp$ fluctuations in helical fields. He suggests compressional viscous heating by ion-ion collisions associated with these fluctuations as a new heating mechanism and estimates that this mechanism is by a factor of
100 more efficient than resonant absorption or phase mixing. Different from these latter mechanisms it is not difficult for the new mechanism to produce a uniform heating of the loop.

An examination of the existing theory of Alfvén wave generation in the subphotospheric convection zone and the transmission of these waves up into the coronal hole indicates that it is only the Alfvén waves that can reach coronal heights and that they are not generated sufficiently strongly as to supply more than a fraction of the necessary $5 \times 10^5$ erg cm$^{-2}$ s$^{-1}$ in the coronal hole (Parker, 1991a). Observations suggest an upper limit of $1 \times 10^5$ erg cm$^{-2}$ s$^{-1}$. Parker (1991b) concludes that there is no known mechanism to explain the necessary dissipation of such waves within the first 1–2 $R_\odot$ to account for the principal heat input to the coronal hole. Alfvén waves with a period of 100 s or more may dissipate effectively over distances of 5–20 $R_\odot$ and also may serve to accelerate the solar wind to high velocity (500–800 km s$^{-1}$).

Moore et al. (1991a, b) study the reflection of Alfvén waves in an isothermal hydrostatic coronal hole model with open magnetic field on basis of the Klein–Gordon equation approach (Musielak et al., 1991). They find that reflection occurs if the wavelength is of order of the scale height of the Alfvén velocity. For Alfvén waves with periods of about 5 min and for realistic densities, magnetic field strengths and field spreading in the model, the waves are trapped within the hole if the coronal temperature is slightly less than $10^6$ K. They escape to accelerate the solar wind for higher temperatures. The dependence of trapping on the spreading rate and inclination of the magnetic field suggests that polar coronal holes should be cooler at their edges than in their centers, in agreement with the observation of thermal radio emission. They suggest that network microflares might be the source of Alfvén waves that heat coronal holes. The potential magnetic field is assumed to decrease according to $B \sim R^{-m}$, where $R$ is the radial distance from the Sun center and $m$ is an even integer such that for $m = 2$ the field spreads radially. The cases $m = 4$ and 6 are also investigated. At each height in the coronal hole a critical period, $P_c$, exists at which strong reflection sets in. The results are exhibited for a base radius of $R = 1.15 R_\odot$, a field strength of 10 G and an electron density of $n_e = 3 \times 10^7$ cm$^{-3}$. Their analysis suggests that the temperature of the coronal hole is regulated by heating via trapped Alfvén waves.

The above results have been extended and modified by Krogulec et al. (1994) who find that Alfvén wave reflection is strongly model-dependent and that strongest reflection can be expected in models with base temperatures higher than $10^6$ K and densities lower than $7 \times 10^7$ cm$^{-3}$. Alfvén waves with periods as short as several minutes have negligible reflection, while reflection is significant for waves with periods of the order of one hour.

A mechanism, called ‘intermittent magnetic levitation’ for the dissipation of reflected Alfvén waves has been proposed by Moore et al. (1992) in which Alfvén waves propagate from their site of generation to the coronal hole flux tube. The waves with periods greater than the critical period get reflected by the density
gradient and they push up against the coronal plasma. As a result of this, a quasi-hydrostatic vertical lifting of plasma takes place in such a way that there is more plasma at each height than in the absence of the reflecting waves. Because events like granular flows, microflares and spicules that generate Alfvén waves are episodic and intermittent, it is expected that the coronal plasma in the flux tube at any instant would be rising or falling. During the rise the plasma gains gravitational potential energy which during the fall gets converted into thermal energy. More detailed recent investigations by Musielak and Moore (1995, private communication) show that it is really the waves with periods in the minute range and shorter which are responsible for the main part of the Alfvén wave levitation.

Assuming a steady state Lin and Zhang (1992) compute the propagation of Alfvén waves along a one-dimensional cool coronal loop model. The waves are dissipated by particle collisions. The loop material consists of pure hydrogen and is allowed to radiate assuming the thin plasma approximation. From their results the authors conclude that Alfvén waves constitute an important heating mechanism for cool (~ 10^4 K) loops.

A study of coronal loop heating by discrete Alfvén waves has been made by De Azevedo and De Assis (1990) as well as De Azevedo et al. (1991a, b) and Shueoka et al. (1992). Discrete Alfvén waves (DAW) are a new class of Alfvén waves which can be described by a two fluid model with finite ion cyclotron frequency or by an MHD model with plasma current along the magnetic field. They are the eigenmodes of the coronal loop structure and do not suffer reflections due to density variations. The coronal loop is modeled as a semi-toroidal plasma. The global chromospheric oscillations which shake the feet of the coronal loops should be responsible for the excitation of these waves. As is desirable, these waves can transport energy along the magnetic field lines. The authors conclude that the absorption of the DAW by the coronal plasma through viscosity can account for at least 20–30% of the heating rate of 10^{-9} \text{ erg cm}^{-3} \text{ s}^{-1} to keep the coronal temperature constant.

Goossens (1991b) outlines the basic properties of MHD waves in nonuniform plasmas and focuses on those aspects of these waves which are important for understanding wave heating theories of the solar corona. In particular, he discusses MHD waves in an infinite and uniform plasma, in one-dimensional nonuniform plasmas, quasimodes, discrete Alfvén waves (DAW), heating by resonant absorption and the continuous spectrum of two-dimensional equilibrium states. It is found that stronger density stratification in a loop enhances the heating efficiency. Slowly varying current profiles lead to uniform heating along the radius while sharp current profiles give rise to localized heating in the outer portions of the loop. Steady-state computations show that the DAW is unimportant for the heating. The two dimensional treatment shows the existence of toroidicity induced Alfvén waves (TAW). He concludes that resonant absorption and phase-mixing of such waves seems quite promising.

Sudan (1991) studies the propagation and dissipation of Alfvén waves in stochastic magnetic fields and the generation of microstructure by random footnote
motion. Microstructure, that is irregular and stochastic fine scale field geometry, greatly enhances Joule heating and shortens the resistive decay time by orders of magnitude. The two-dimensional time-dependent algorithms presently available are unable to provide quantitative answers, and the development of new methods which allow to resolve features of very small scale is urgently needed.

In a two-dimensional model of both slab and axi-symmetric geometry Similon and Zargham (1992) determine numerically the transmission and reflection coefficients of shear Alfvén waves for an open magnetic structure on the Sun. They find that the transmission coefficients from the photosphere to 4 $R_\odot$ for wave periods between 1 and 5-min range from 5 to 30%. This means that Alfvén waves contribute significantly towards the heating of the corona.

For the sheared coronal arcade model of Heyvaerts and Priest (1992, discussed below in Section 6.4) Inverarity and Priest (1993b) computed the turbulent and viscous heating by Alfvén waves. The individual arcade loops act as a resonant cavity. The heating flux from the first few harmonics was estimated to be of the order of $10^6$ erg cm$^{-2}$ s$^{-1}$, sufficient for active region energy requirements. For high-frequency waves there is little difference between the heating obtained with resonant and non resonant waves. The authors find that whenever a resonance occurs the Alfvén waves heat as much as is dissipated by DC heating (see their work in Section 6.4).

Khabibrahmanov and Mullan (1994) discuss a new type of heating mechanism of shear Alfvén waves propagating along a vertical magnetic field in the presence of gravity in cases where the collision frequency is much smaller than the cyclotron frequency. Because particles circling the horizontal magnetic field component $B_\perp$ of the wave gain energy on the downward swing in the gravitational field, their Larmor radius becomes bigger than that on the upward swing, resulting in a horizontal drift, the well-known gravitational field drift. The acceleration due to the establishment of this drift uses up wave energy which constitutes a heating mechanism. The authors find that this mechanism heats heavier ions more efficiently than protons, which is consistent with the proportionality between the ion temperature and the particle mass, observed in the solar wind.

Kleva and Drake (1992) investigate the nonlinear coupling of parent Alfvén waves to generate Alfvén waves which have a shorter wavelength than the parent waves. This short wavelength Alfvén wave can in turn nonlinearly couple with the parent wave to generate yet shorter wavelength Alfvén waves. Thus, if the foot point motion of the magnetic field lines in the photosphere produces Alfvén waves with a wavelength $\lambda$, the nonlinear interaction of these waves will generate Alfvén waves with a much shorter wavelength. These nonlinearly generated short-wavelength modes dissipate energy through Coulomb collisions much more rapidly than do the longer wavelength parent waves since the resistive damping rate of a mode is $\sim \lambda^{-2}$. In order to find the effect of a background spectrum of Alfvén waves on the rate of dissipation of a test shear Alfvén wave, numerical calculations have been performed by the authors. For typical coronal conditions they find that the
damping rate of an Alfvén wave in the presence of a background wave spectrum is several orders of magnitude larger than the classical collisional damping rate.

Callebaut and Tsintsadze (1994) investigate Alfvén waves where dissipation and dispersion (Hall effect) are included.

4.3. **Summary, the heating picture by MHD body waves**

From the above work we draw the following conclusion. At the present time it appears clear that Alfvén waves are responsible for the heating of the solar wind, but that it is not possible to say anything definite about their exact role and that of magnetoacoustic waves in chromospheric and coronal heating. Parker (1991a, b) concluded that the generation and heating by Alfvén body waves appears to be insufficient to balance the principal chromospheric and coronal energy losses within the first 1–2 \( R_\odot \). But new observational and theoretical developments indicate a much greater MHD body wave flux. In addition, new work on the formation of small-scale structures including shocks, as well as on pulsational resonances produced by wave reflection indicates a much greater heating rate by MHD body waves. This indicates that the heating by MHD body waves is more important than previously thought.

5. **Heating by Slow and Fast Magnetoacoustic Surface Waves**

In this section we discuss MHD waves which are restricted to the regions of strong field changes and propagate along these interface regions. These waves are also called surface Alfvén waves, although, as was pointed out, e.g., in the excellent review by Roberts (1991), they should better be called slow and fast magnetoacoustic surface waves as they are compressive. We follow this usage here. A very intensely investigated and very natural heating mechanism for such waves is resonant absorption which takes place in the narrow region near the field line where the Alfvén speed is equal to the surface wave speed. Note that resonant absorption is different from resonant heating. The latter works by reflection in a loop of finite length and thus in a fundamental way depends on the loop length; here various mechanisms do the actual dissipation and this heating mechanism is not restricted to surface waves. Resonant absorption, on the other hand, is a local phenomenon and does not depend on the loop length. However, in many studies one combines both mechanisms and lets the reflected waves be damped by resonant absorption. The generation of surface Alfvén waves is similar to that of other MHD waves, it is therefore included in Section 3.1. For older work on this wave type see Section 5 of Paper I.
5.1. Surface Wave Propagation

Although coronal plasmas are considered collisional, it is widely assumed that particle collisions do not provide effective energy dissipation. De Assis and Tsui (1991a, b) as well as De Assis et al. (1991) show that the wave-particle resonant interactions can dissipate energy fast enough to sustain the coronal temperature. They study a magnetoacoustic surface wave and a kinetic Alfvén wave (KAW). The coronal loop is modeled as a cylindrical resonant cavity. The surface wave dissipates via phase-mixing. The KAW is found to be more suitable.

An elegant description of the resonance absorption theory in relation to coronal heating has been given by Davila (1991). He considers shear viscosity, resistive dissipation, compressive viscosity, nonlinear wave-wave interactions (e.g., Kelvin–Helmholtz instability) and dissipation in a turbulent magnetic field. The compressive viscosity for coronal parameters is estimated to be about 10 orders of magnitude larger than the shear viscosity or resistivity. Kelvin–Helmholtz instability has a threshold and could lead to episodic or quasi-periodic heating. He concludes that resonant absorption is a viable mechanism for heating the corona of the Sun and other late-type stars. Observations of the power spectrum at the base of the corona are needed as input for the theory. These observations should be carried out in ions which are present at or above the transition region.

The resonant absorption process for the heating of active region coronal loops through numerical simulation of the MHD equations for a fully compressible, low-β, resistive plasma has been studied by Steinolfson and Davila (1993). In particular, they take into account the variation of the Alfvén speed with radial distance, the temporal variations of the velocity components with time, the global mode frequency variation with density, the spatial distribution of the velocity and magnetic field within a driven mode and the ohmic heating as function of distance, time, and frequency. The simulations show that the dissipation occurs primarily in a thin resonance layer, in agreement with previous studies. Also it is confirmed that the period-averaged ohmic heating in the stationary state is independent of the dissipation magnitude and the dissipation layer thickness scales with resistive dissipation as $S^{-1/3}$, where $S = \tau_R/\tau_A$ is the Lundquist number, the ratio of resistive decay time to the Alfvén transit time. They conclude that this study definitely provides support for resonant absorption mechanism but it remains to be demonstrated that the necessary heating can be accomplished with velocity oscillations similar to those that are observed.

Chitre and Davila (1991) as well as Davila and Chitre (1991) study the effect of acoustic waves impinging on the almost horizontally directed magnetic arches of the low-lying chromospheric canopy. They demonstrate that the acoustic energy upon entering the magnetic arches can be resonantly absorbed in a narrow resonance layer. The authors derive simple expressions for the heating and, using reasonable estimates for the physical situation, derive heating rates of $1 \times 10^5$ erg cm$^{-2}$ s$^{-1}$. 

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at a canopy height of 1000 km which is sufficient to balance the observed upper chromospheric radiation losses.

Poedts et al. (1990a, b) study the efficiency of resonant absorption of Alfvén waves as a solar coronal heating mechanism in the framework of linearized, compressive and resistive magnetohydrodynamics. The loops are approximated by straight, cylindrical, axisymmetric plasma columns with equilibrium quantities varying only in the radial direction. The waves incident on coronal loops excite them. The efficiency of the heating mechanism and the energy deposition profile in the stationary state depend strongly on the characteristics of the external driver (source) and the equilibrium state. It is found that for typical coronal loops a considerable part of the energy supplied by the external source is dissipated ohmically to thermal energy and that the mechanism of resonant absorption is very efficient. In addition the heating rate is found to be proportional to the square of the magnitude of the background magnetic field which is a desirable feature. Since the spectrum of waves which is incident on a coronal loop is not known, only the efficiency of the heating mechanism could be estimated. It is concluded that resonant absorption appears to be a viable heating mechanism for coronal loops.

Goossens (1991a) investigates resonant absorption in coronal loops numerically. In order to interpret these results the analytical theory is also presented. A wave is specified by the azimuthal wave number \( m \) and the longitudinal wave number \( k \). Another wave number \( n \) is defined such that \( k = n\pi/l \), where \( l \) is the length of the loop. In order to conform to a real situation he considers a number of waves in spite of the fact that the spectrum of incoming waves is not known. He finds that the heating efficiency depends on the wave numbers, being highest for \( m = 1 \) and \( n = 1 \). It is non-negligible for \( |m| \) up to 5 and \( n \) up to 10. For \( m = 1 \) and \( n = 1 \) the fractional absorption is almost uniform over the total radius of the loops. For higher values of \( n \) (but \( m = 1 \)) the fractional absorption becomes more and more concentrated in the outer layers of the loop. Also a stronger density stratification in the loop enhances the heating efficiency.

A great advance constitutes the success by Sakurai et al. (1991) and more recently by Goossens et al. (1995) (see also Goossens, 1994) to be able to treat the resonant absorption problem analytically. This advance allows to study in a more illuminating way the basic physics of the resonant absorption process. It also provides a fundamental conservation law for resonant Alfvén waves as well as jump conditions that make it possible to compute these waves without having to solve the dissipative MHD equations numerically.

The heating efficiency and time scales of solar coronal loop heating by resonant absorption has also been studied by Poedts (1991), in the framework of compressible and resistive MHD. As usual, the loops are approximated by straight, cylindrical axisymmetric plasma columns excited by an incident wave (external source) at the plasma surface. The source is taken to be periodic, multiperiodic or stochastic. In the periodic excitation case, phase-mixing occurs in the initial phase and the plasma energy accumulates in a thin layer around a resonance point where
the excitation frequency equals the local Alfvén frequency. Due to finite electrical conductivity the system attains a stationary state where the power supplied by the external source is exactly balanced by the ohmic dissipation rate in the layer. The time scale of the dissipation is much smaller than the typical life time of coronal loops.

Ruderman (1991a, b, 1992) considers MHD surface wave propagation on a single magnetic interface in a cold plasma taking ion viscosity into account. Using a reductive perturbation method he obtains the equations governing the behavior of small amplitude surface Alfvén waves which he studies numerically. The wave steepening leads to strong acceleration of wave damping. The dependence of the wave flux on distance is calculated. He shows that in the case of small viscosity the wave-damping distance predicted by the nonlinear theory is an order of magnitude smaller than that predicted by the linear theory.

Ruderman and Goossens (1993) extending the work of Ruderman (1991a, b, 1992) study the resonant absorption of small amplitude surface Alfvén waves in nonlinear incompressible MHD for a viscous and resistive plasma. They use a reductive perturbation method to obtain the equations governing the spatial and temporal behavior of small amplitude nonlinear surface Alfvén waves. Numerical solutions of these equations are obtained under the initial condition that at time $t = 0$ the spatial variation is purely sinusoidal. Their numerical results show that nonlinearity accelerates the wave damping due to resonant absorption. They conclude that resonant absorption is a more efficient wave damping mechanism than can be anticipated on the basis of linear theory.

The effect of line-tying of solar coronal loops on both stability and heating by means of resonant absorption of Alfvén waves is investigated numerically by Halberstadt et al. (1991). The Alfvén waves are excited by an external source imposing a helical perturbation at the boundary of of a line-tied plasma cylinder. In a line-tied case the heating is radially much more distributed than in a loop without line-tying. The modes localize in the middle of the loop hence energy is preferentially deposited in the upper layers of the corona.

Halberstadt and Goedbloed (1993) discuss the incorporation of the photospheric boundary conditions in the theory of resonant heating of coronal loops. They find that the Alfvén continuum is responsible for the resonant heating of coronal loops which are line-tied to and bounded by the photosphere and excited at their footpoints by photospheric motion. In case of a multiperiodic source, several resonances are excited simultaneously so that several plasma layers are heated simultaneously. In resistive MHD the coronal loops are heated inhomogeneously even when the power spectrum of the external source is uniform. When the loop is excited at random the coronal plasma is heated primarily in a plasma layer around the magnetic surface where the local Alfvén frequency equals the frequency of the quasi-mode produced. This theory is extended in Goedbloed and Halberstadt (1994).

Poedts et al. (1994) investigate the heating of inhomogeneous coronal loops by resonant absorption in the framework of linear resistive MHD. They define
the quality $Q$ of a resonance as the ratio of the total energy contained in the loop plasma system to the dissipation per driving cycle of the external source (incident waves). They find that the resonances in coronal loops have bad quality, i.e., the dissipation per cycle is appreciable and yields a lot of ohmic heating per driving cycle. As a result of the generation of small length scales the time scales of the heating processes are shorter than the lifetimes of the coronal loops. They conclude that resonant absorption is a viable heating mechanism for coronal loops.

Poedts and Goedbloed (1994) discuss the heating of coronal loops by resonant absorption and phase-mixing by incident wave energy in the framework of 3D nonlinear MHD using numerical simulations.

5.2. **Heating by Locally Produced Surface Waves**

So far the surface wave heating phenomenon was discussed in a coronal loop scenario, where waves are introduced at the photosphere by rapid foot point motions into a vertical magnetic field with a strong horizontal variation. Martens et al. (1992) study a different situation where a coronal arch system loops in the $x-z$ plane over a neutral line extending in $y$-direction. Introducing slow shearing motions in $y$-direction they follow the temporal development of the arch system from the initial equilibrium state using a time-dependent numerical code. After some critical time, equilibrium is lost and in situ wave generation at coronal height takes place. The wave energy is assumed to be damped by resonant absorption. This in situ wave generation removes the need for large wave energy fluxes at the photospheric footpoints.

It can be debated whether this mechanism should be classified as DC heating (because the energy input is by slow footpoint motions) or AC heating (because of the dissipation mechanism). For similar work on the dissipation via waves generated by loss of equilibrium see that of Mikić et al. (1990), Longcope and Sudan (1992), and Sudan and Longcope (1992) reviewed below in Section 6.2.

5.3. **Summary Surface Wave Heating**

Magnetooacoustic surface waves are effectively damped by resonant absorption. A great advance constitutes the fact that the resonant absorption problem can now be treated analytically. This allows the basic physics of the resonant absorption process to be studied in much greater detail. However, with the tendency of these waves to dissipate in narrow surface layers, the problem is how easily they can generate a uniform heating of the loops. The surface waves are not only generated in the convection zone and introduced into the coronal loops there by fast footpoint motions, but also in the corona, by loss of equilibrium due to the buildup of stresses by slow footpoint motions. An interesting way to enhance the heating of the network is also the idea by Chitre and Davila that acoustic wave energy gets intercepted by the large collecting area of the magnetic canopy and converted into MHD waves which travel down into the narrow network tubes.
6. Heating by Current (or Magnetic Field) Dissipation

This section reviews the DC heating mechanisms which heat by magnetic field dissipation. These mechanisms operate by transporting mechanical (magnetic field) energy into the chromosphere and corona via slow photospheric foot point motions, with time scales \( \tau > l_\parallel / c_A \), and dissipate this energy by various processes. Here \( l_\parallel \) is the loop length and \( c_A \) the Alfvén speed. The DC mechanisms are different from the AC mechanisms which transport energy via waves excited by fast foot point motions with time scales \( \tau < l_\parallel / c_A \). The AC mechanisms have been discussed in Sections 3 to 5. For older work on DC mechanisms and additional details see Section 6 of Paper I.

6.1. Reviews of DC Mechanisms

Heyvaerts (1990) critically compares the similarities and differences of the proposed DC heating theories. He also outlines the generation of MHD turbulence and the role of field line stochasticity in coronal structures.

Priest (1990a, b) reviews the basic theory and recent numerical simulations of magnetic reconnection on the Sun based on the ideas of Sweet et al.. He describes several solar phenomena where reconnection is believed to operate.

Gomez (1990) reviews the magnetic heating theories of the solar corona as well as the thermal stability of coronal loops. He briefly mentions wave heating theories, then discusses DC mechanisms like reconnection, anomalous Joule heating, dissipation by turbulence and Taylor’s relaxation. For a prescribed heating the scaling laws for loops are summarized and the stability of various coronal loop models is discussed.

Van Ballegooijen (1990) reviews the magnetic heating mechanisms of stellar coronae with emphasis on the heating by field-aligned electric currents. Large current densities are needed if the heating is by classical or anomalous Joule dissipation. He argues that gradual foot point motions will result in spatially continuous magnetic fields which contradicts Parker’s envisioned spontaneous formation of current sheets.

Berger (1991b), see also Berger (1991c), reviews the different types of heating of closed coronal flux tubes via current dissipation. The coronal magnetic energy of the tubes is supposed to be generated by random photospheric motions. He describes the heating in the single flux tube Sturrock–Uchida model, in the Parker model with random braiding of many flux tubes and in the Heyvaerts–Priest model which assumes Taylor relaxation. It is found that the dissipation of coronal magnetic fields must predominantly occur in a small fraction of volume near current sheets.

Aly (1992) critically reviews the theory of magnetic reconnection in the solar corona as well as the heating of the coronal plasma. He discusses large-scale reconnection and the formation of plasmoids in arcade systems, the spontaneous
formation and dissipation of current sheets as well as turbulent reconnection and Taylor’s relaxation.

Parker (1990a–c, 1992) reviews the coronal magnetic field on the Sun. He emphasizes that the X-ray corona is not primarily a wave phenomenon and the dissipation of waves plays a minor role in it. The random continuous shuffling and mixing of the footpoints of the bipolar magnetic fields, in which the X-ray corona resides, leads to the spontaneous appearance of current sheets which dissipate as transient bursts of magnetic reconnection (nanoflares). These current sheets arise in any magnetic field subject to continuous deformation. Thus the main heat source close to the Sun (1–5 $R_\odot$) is provided by the spontaneous current sheets (tangential discontinuities) which are driven by the continuous motions of the footpoints of the field in the photospheric convection. Waves generated by the photospheric convection dissipate only at 5 $R_\odot$ and beyond.

6.2. STUDIES OF ENERGY BUILDUP AND OF INSTABILITY GROWTH

Berger (1991a) studies the buildup of twist due to helical and translational photospheric flows acting on the footpoints of a Sturrock–Uchida model (a single coronal flux tube anchored with both ends in the photosphere). Although a considerable increase of the magnetic energy is found ($\approx 7 \times 10^5$ erg cm$^{-2}$ s$^{-1}$) the energy buildup rate is an order of magnitude below the energy requirement for active regions. In addition it appears difficult to dissipate this energy except when singular current sheets are involved. In addition to the buildup of energy by twists Berger (1994) studies the formation of different types of braids, the energetics and heating in braided magnetic fields, and the influence of fragmentation of the tubes as a consequence of reconnection.

Zirker (1993a) uses the results of a 3D solar numerical convection simulation to estimate the power input into coronal loops from the twisting of the photospheric magnetic field in intergranular vortices. He finds that field line twisting could supply a substantial amount if not all of the required energy for the quiet corona.

Liewer and Pyne (1990) investigate the growth rates of linear tearing modes in forced current sheet equilibria in the solar corona and compare them with growth rates of such modes in diffuse current sheets. Their numerical studies confirmed earlier analytical work and showed that the growth rate in forced equilibria is larger by a factor of $\approx 10^4$. Actual growth times for various coronal regions with typical widths of the sheared field area $a$, shear field strengths $B_\perp$ and gas densities $n$ are shown in Table I. These growth rates are found to be comparable to the reconnection time scales which are required for the heating of the corona by magnetic field dissipation.

Lou (1992) investigates numerically the onset of mechanical, tearing, thermal, and resistive instabilities in a cylindrical electric current sheath embedded in a coaxial magnetic flux rope. The background model for the sheath assumes a helical field and an energy balance between cross-field thermal conduction, Joule heating
and optically thin radiation. The onset of instability is studied by deriving and solving perturbation equations on basis of this background model.

Mikić et al. (1990) study the three-dimensional time-dependent evolution of twisted magnetic flux tubes numerically, by solving the ideal MHD equations. Their coronal loop is represented by a cylinder with the field lines initially in axial direction, in which the top and bottom surfaces represent the two photospheric foot points where the field is line tied. Applying radially dependent oppositely directed helical twisting velocities $v_\theta(r)$ at the two footpoints, the temporal evolution is computed and the equilibrium properties and linear stability behavior are discussed. The authors find that when the twist exceeds $4.8\pi$ on the axis, the loop becomes kink unstable.

Sudan and Longcope (1992) using a 3D spectral code compute the time-dependent twisting and untwisting of solar magnetic loops by random photospheric motions. Motions which are faster than the Alfvén transit time result in shear Alfvén waves and are rapidly damped in the stochastic field of the loop. Slow motions twist and untwist the field lines quasistatically. When the buildup of stresses becomes large enough the quasistatic equilibrium is lost and Alfvén waves are generated which are rapidly damped. These impulsive events are identified as nanoflares. In a long numerical run covering $10^6$ Alfvén times, 1093 such impulsive events were found. The heating of coronal loops is a direct consequence of the photospheric motions.

Longcope and Sudan (1992), on basis of the reduced MHD equations for long and slender coronal flux tubes, develop a procedure to compute the current system in the entire tube and its associated plasma motions from a given current distribution at one foot point. Inverting this formalism gives the current system as function of the foot point motions. The authors find that there are situations where a sudden loss of equilibrium occurs and high-frequency Alfvén waves appear.

6.3. **SPONTANEOUS FORMATION OF CURRENT SHEETS, TAYLOR RELAXATION**

By theoretical reasoning and detailed numerical simulation Karpen et al. (1990, 1991; see also Antiochos, 1990) find that a sheared, initially potential magnetic field without preexisting nulls cannot produce coronal current sheets, and therefore
cannot be invoked to explain coronal heating. In topologies with null points the current discontinuities do readily occur. On the Sun, preexisting null points should occur in the complex topology and mixed polarities of active regions, where current-sheet formation and dissipation may take place.

Dahlburg et al. (1991) with a time-dependent incompressible and dissipative numerical code investigate the evolution to a current free state of a coronal magnetic arcade system which has been stressed by helical motions at its foot points. The authors did not find instabilities or any other significant manifestation of solar activity.

Low (1990, 1991) shows that if a finite threshold in the size of the boundary displacements is exceeded a current sheet should form which contradicts the conclusion of Karpen et al. (1990, 1991) as well as the objection to the spontaneous formation of current sheets by gradual foot point motions voiced by van Balleghoijen (1985, 1990) (see also van Ballegooijen 1994). He attributes the null result of Karpen et al. to an insufficiently resolved photospheric boundary region and a poor simulation of the plasma motions which have a very high magnetic Reynolds number. Low (1992), by taking into account the effects of pressure gradients and gravity, shows that the quasi-static evolution of the atmosphere, in response to a continuously changing distribution of the plasma pressure at the atmospheric base, can bring an initially smooth state to one in which an electric current sheet forms.

Bhattacharjee and Wang (1991) show that, despite of contrary claims, current sheets do indeed form in Parker’s model (where two flux tubes of the same helicity are wrapped around each other) due to the conservation of helicity (Taylor relaxation), if a small but finite amount of plasma resistivity is present. Smooth helical flows at the foot of Parker’s model will generate current sheets where the efficient Sweet-Parker reconnection takes place which heats the model. This work is extended by Wang and Bhattacharjee (1992) by including the time-dependence. They found that current sheets develop on the Alfvénic time scale with an amplitude which increases linearly with time. Due to the finite resistivity these current sheets lead to reconnection and substantial heating.

Vekstein et al. (1991a) and Vekstein and Priest (1991) study the dynamic formation of current sheets generated by the quasistatic evolution of the magnetic fields. Initially the 2D magnetic field is taken to be potential with the ends of each line of force rooted in the photosphere. This magnetic field is then deformed by slowly shearing its footpoints. As opposed to Karpen et al. (1990) they find that a realistic treatment of the photospheric boundary region leads to current sheets along the separatrices. They suggest that these current sheets could produce coronal heating but may also trigger flares. Vekstein and Priest (1992, 1993) continue this study and extend it by considering the evolution of an X-point into a current sheet with cusp points due to foot point shearing.

Priest (1991) reviews formation of current sheets and finds that in a 2D magnetic field, current sheets may form near X-type neutral points or separatrices in response
to footpoint motions (converging or shearing) or to magnetic instability. In 3D cases the sheets may develop near separators or separatrix surfaces, but also more generally near any potential singular line in a sheared field. He presents a self-consistent theory for heating by MHD turbulence in which heating depends only on the boundary motions and geometry. He suggests that closed magnetic regions of stellar chromospheres and coronae are filled with many small current sheets which are continually forming, filamenting and dissipating.

Vekstein et al. (1991) discuss the solar coronal heating produced by slow random helical motions, which are introduced at the photosphere. They consider the case of a magnetic arcade system and that of an array of closely packed flux tubes with the same helicity. The relaxation by magnetic reconnection to a minimum magnetic energy state is affected through the Taylor–Heyvaerts–Priest relaxation process using a phenomenological relaxation equation with a characteristic time-scale $\tau_F$. Successive magnetic reconnection configurations are calculated analytically.

Vekstein et al. (1993) (see also Vekstein, 1994) further develop the theory of heating the solar corona by the Taylor–Heyvaerts–Priest relaxation process. They consider an arcade system where the heating is produced by small helical footpoint motions introduced at the photosphere. They permit incomplete relaxation to the linear force-free minimum magnetic energy state. When the relaxation time ($\tau_F$) is much smaller than the time scale for footpoint motions ($\omega^{-1}$) then complete relaxation is achieved, otherwise intermediate relaxation is reached, which in practice is still sufficient to provide the observed coronal heating. They show that infinities in the theory are avoided because the excess helicity can be ejected into the solar wind.

The work of Vekstein et al. (1993) was further extended into the nonlinear regime by Wolfson et al. (1994). Assuming initially a potential field configuration of a bipolar region with adjacent open field, that features a current sheet and a Y-type neutral point, they evolve the torsion coefficient $\alpha$ in the force-free field equation iteratively, using a finite difference numerical scheme, until the final relaxed state is found. For different heights of the neutral point, different helicities and values $\alpha$ are found. The computations show that there is a limiting helicity above which presumably catastrophic mass ejection starts.

6.4. EXPLICIT VISCOUS, TURBULENT, RESISTIVE, OR RECONNECTION HEATING

Heyvaerts and Priest (1992) calculate the solar coronal heating rate produced by slow random motions, which are introduced at the photosphere in a sheared arcade system. They simplify the problem by using a slab model. The photospheric motions are introduced by shaking the top and bottom slab surfaces in a fixed transverse direction. Assuming turbulent viscosity and resistivity, a stationary state heating rate is analytically evaluated. For a quiet region coronal loop of length $10000 \text{ km}$, electron density $n_e \sim 2 \times 10^{10} \text{ cm}^{-3}$, magnetic field strength 30 G, the DC heating flux turns out to be about $2.4 \times 10^5 \text{ erg cm}^{-2} \text{ s}^{-1}$, whereas for
an active region loop it is about $2 \times 10^6$ erg cm$^{-2}$ s$^{-1}$. The turbulent velocities range from 20 to 30 km s$^{-1}$. Similar analytical work by Inverarity and Priest (1993a) considered the case of a twisted straight flux tube. Inverarity et al. (1995) have recently extended the sheared arcade system case by numerically solving the equations for the turbulent viscosity, magnetic diffusivity, velocity and energy flux density.

Dahlburg and Picone (1989) and Picone and Dahlburg (1991), with a 2D compressible MHD code with viscosity and resistivity, compute the time-dependent evolution of a given simple vortex system for various subsonic and supersonic flows and different Lundquist numbers. They show that MHD turbulence develops with very narrow regions of high current density and shocks.

Roumeliotis (1991) supposes that most of the Sun’s lower transition layer, consisting of plasma in the temperature range ($3 \times 10^4$ to $3 \times 10^5$ K), is confined in thin sheared magnetic boundary layers separating adjacent flux loops, where it is heated by Joule dissipation of intense, field-aligned currents. With field strengths of $B = 250$ G, shear angles $\delta \theta = \pi/2$ and a thickness of the boundary layer $\delta x = 1$–10 km he estimates current densities of $J = 4$–40 A m$^{-2}$. For a simple circular loop model and various specified current densities he finds the equilibrium state by solving the time-dependent hydrodynamic equations. There is a critical current density which separates hot and cool loop models.

Spicer (1991) reviews the coronal heating by Joule dissipation of DC magnetic field aligned currents. He differentiates voltage driven and current driven field aligned currents. There are two heating scenarios. The ‘filament model’ assumes that the coronal emission originates from a large number of unresolved current channels with a very small filling factor. In the ‘volume heating model’ the radiation comes from the large volume between an extensive system of very thin hot current filaments. He estimates that classical Joule heating may work in the filament model where about $10^3$ to $10^4$ current filaments, each of 0.1–1.0 km thickness are needed to produce the required emission from a coronal loop. For the volume heating model he needs from $10^3$ to $10^5$ even thinner filaments with temperatures from $10^9$ to $10^7$ K, respectively, requiring anomalous Joule heating. The structuring mechanisms which lead to the filamented current system need further study.

Boozer (1992) derives evolution equations for an ideal magnetic field embedded in an arbitrary chaotic flow. He finds that the plasma flows in the solar photosphere which are driven by solar convection may amplify the currents sufficiently to rapidly thermalize that energy through Joule heating.

Hirayama (1992) investigates the Joule heating in the boundary layer of slender vertical magnetic flux tubes in the solar photosphere. The tubes sit in the intergranular lanes where the surrounding gas, which has been brought up elsewhere by the granulation, converges and flows back down into the Sun. The current, produced by the interaction of the radially converging gas motion with the vertical magnetic field, flows in azimuthal direction around the tube and is dissipated by Joule heating. For a tube field of 1500 G, maximum horizontal velocities of 2 km s$^{-1}$ and
densities from solar models he finds heating fluxes of \(1 \times 10^9\) erg cm\(^{-2}\) s\(^{-1}\) in the photosphere below the temperature minimum. This heating appears to fit well with observational requirements.

Sakai and Koide (1992) have classified the different kinds of magnetic reconnection situations which occur between two adjacent magnetic flux tubes. As the field in the two tubes can be either parallel or antiparallel, and the current in each tube can run either parallel or antiparallel to the field, there are 9 different reconnection situations which indicates different kinds of coronal loop heating and flares.

Beaufumé et al. (1992) use new high resolution NIXT and vector magnetograph observations to derive a theoretical model of DC coronal heating processes taking place in coronal loops. Their model is characterized by a cyclic series of events. (a) The plasma loop initially has a high electrical conductivity, and the electric field generated through photospheric motions induces a skin current in the outer layers of the loop. (b) This current sheath is subsequently disrupted by the excitation of instabilities involving magnetic reconnection, whereby the magnetic energy is converted into particle acceleration and plasma heating. Due to the low transverse thermal conductivity this leads to very hot \(T \sim 10^7\) K filaments with a very small filling factor in agreement with observation. (c) The thermal energy transport across the magnetic field lines is locally enhanced by the disruption of the current sheath and heat diffuses across and along the loop. Due to the high thermal conduction along the magnetic field, this input of energy leads to an evaporation process of the chromospheric material at the foot points of the loop which produces a density increase in the loop and induces brightness variability, since the emission varies with the square of the electron density. The resulting turbulence subsides and the cycle starts again with step (a).

Litwin and Rosner (1993) discuss principal constraints on mechanisms for heating of stellar coronae, such as the requirements of heat dispersal, the observed variations in the coronal emission and the observed perpendicular structure of the magnetic fields. The authors argue that anomalous Joule heating or mechanisms working with magnetic field line stochasticity are much too slow to account for the observed rapid heating of coronal loops. They suggest that reconnection between adjacent coronal flux bundles is the most plausible mechanism and point out that hyperresistivity could provide an appropriate mechanism that leads to fast heating, as well as to a large transverse dimension of the heated region.

Gomez and Ferro Fontan (1992) compute the generation of MHD turbulence in solar coronal loops due to interweaving of magnetic field lines driven by the turbulent velocity field of the convection zone. They obtain an estimate of the typical dissipation length at which the magnetic field heats the plasma by Joule dissipation and also discuss the impact of numerical convection zone simulations on the coronal heating problem.

Gomez et al. (1993b) present a model for the heating of the solar corona where the turbulent photospheric foot point motions drive an MHD turbulent regime in the corona. For the spatial fluctuations in the corona they derive a theoretical power
spectrum which varies with the wave number like $k^{-3}$, which is in agreement with their high resolution NIXT observations (Gomez et al., 1993a).

Gomez et al. (1995) discuss the formation of small-scale current-carrying structures in coronal loops driven by foot point motions as a way to enhance the magnetic energy dissipation. They show that the structures develop due to a nonlinear instability in a time scale which is the geometric mean of the Alfvén time and the turnover time of the footpoint motions.

Sakai et al. (1994) studied the heating of a current carrying coronal loop by means of a 3-dimensional electromagnetic plasma particle code. They showed that both the electrons and ions in the loop can be heated in the direction perpendicular to the loop and that a large temperature anisotropy develops where the perpendicular temperature is 10 times larger than the parallel temperature. Starting with pinching oscillations, large amplitude kinetic kink waves are excited which accelerate the ions.

6.5. Driven 3-Dimensional Reconnection

While the above subsections are mainly concerned with the heating of coronal holes and coronal loops this subsection is concerned with the third distinct X-ray feature discovered with Skylab, the coronal bright points. Priest (1994) as well as Priest et al. (1994a) and Priest et al. (1994b) propose a Converging Flux Model for heating X-ray bright points. There are three phases in this model, namely, a preinteraction phase in which two initially unconnected opposite polarity photospheric magnetic fragments approach each other, an interaction phase when the fragments reconnect in the corona and create a bright point and a cancellation phase when reconnection in the photosphere produces the cancelling magnetic feature.

Three-dimensional numerical modelling of particular bright points observed with the Normal Incidence X-ray Telescope (NIXT) by Parnell et al. (1994a) shows good agreement with the proposed model. The Converging Flux Model was extended successfully by Parnell et al. (1994b) to opposite magnetic fragments of differing strength. These computations show that the energy requirements of bright points are satisfied.

6.6. Summary of DC Mechanisms

The field of DC mechanisms has been greatly advanced by the study of energy buildup, braiding and instability growth. The circumstances of current sheet formation and energy dissipation are now much better understood. The large amount of work devoted to DC heating clearly shows that these mechanisms are considered to be very important and may well be the most important heating mechanisms. Here especially Parker's idea of micro and nanoflares, driven by small-scale reconnection, appears to be most promising. Although a DC heating mechanism, microflares are reviewed separately in Section 7.
7. Heating by Microflares/Transients

It appears to be well established that microflares and nanoflares are caused by small-scale reconnection of twisted magnetic fields. For this reason literature on microflares can also be found in Section 6 in the discussions of reconnection. The events following the sudden injection of energy are similar to those in the spicule phenomenon which is another important heating source for the corona. Spicule phenomena, which do not necessarily require the presence of flares because they are usually explained by (shock) wave processes, are discussed in Section 8. For earlier work on heating by microflares see Section 8 of Paper I. We also want to point out again that in our present work we do not review the abundant flare literature but only those processes which provide a steady heat input into the chromosphere and corona. For a discussion of observations of the equivalent of microflares in the more active dMe stars see Cheng and Pallavicini (1992).

Mandrini et al. (1990) study the characteristics of energy release in microflares. They find that the properties of the more common microflares are similar to those of the larger flares, which points to the same kind of basic physical process. They state that the interaction of magnetic loops belonging to two or more bipoles within an active region is an essential ingredient for the triggering of flares. Their X-ray studies suggest that the bulk of the energy is not released at the interface between the interacting bipolar magnetic loops, but instead close to the internal reservoir in each loop.

Sturrock et al. (1990) propose a spicule type model in which the solar corona is heated episodically by short bursts with subsequent long periods of radiative cooling. Analyzing observational data for the quiet solar atmosphere they were able with their model to offer an explanation both of the negative slope part of the differential emission measure (DEM) curve below temperatures of $10^5$ K and the positive slope part of the DEM curve above $10^5$ K. They find that the required injection rate is close to that of spicules.

Raymond (1990) shows that the microflare model of Sturrock et al. agrees fairly well with the EUV and X-ray spectra computed by him, although the model, in the high transition layer, predicts a factor of two more emission than observed. To assume time-dependent ionization instead of instant ionization improves the agreement with observation for the He II 304 line.

Parker (1991c, see also his work in Section 6) points out that almost all strong continuous deformations of an initially uniform magnetic field in an infinitely conducting fluid cause the field to develop internal tangential discontinuities (current sheets) as the field relaxes to equilibrium. He suggests that the dissipation at the current sheets in the bipolar magnetic fields of active regions is the principal heat source for the solar X-ray corona. The dissipation is episodic so that the magnetic energy is converted to thermal energy in small transient bursts, called nanoflares, in which about $10^{23-25}$ erg of energy is released. The X-ray corona may be understood as a cloud of nanoflares. Taking $10^{23}$ erg per nanoflare as an average,
the mean output of $10^7$ erg cm$^{-2}$ s$^{-1}$ requires about one new nanoflare per second in each area $10^3 \times 10^3$ km$^2$. Parker (1993) extended this nanoflare heating theory to include the formation of current sheets by discontinuous fluid motions and finds that in this type of current sheets, reconnection is not efficient enough. Only when current sheets are formed by the field topology (topological dissipation) is the reconnection rapid enough to satisfy the coronal heating requirements.

Moore et al. (1991a, b) discuss the possible role of network microflares in the heating of the quiet corona. Microflares occur at a rate of $10^3$ s$^{-1}$ over the surface of the Sun. In each microflare about $10^{26}$ erg of energy is released. The total power output is thus about $10^{29}$ erg s$^{-1}$, which is equivalent to a flux of $10^6$ erg cm$^{-2}$ s$^{-1}$, sufficient to heat the quiet corona. The authors picture the miroflares as very compact, occurring in a region of no more than 1000 km diameter, inside a magnetic bipole of this size, from where the energy is transferred out into the adjacent legs of large coronal loops, where it is distributed along the loop by conduction and mass flows (see their Figure 1). For their discussion of the wave phenomena associated with these events see Section 4.

Numerical calculations on a 20000 km quiet-Sun coronal loop with an initial apex temperature of $8 \times 10^5$ K and a pressure of 0.2 dyn cm$^{-2}$ at the base of the transition region have been performed by Sterling and Mariska (1991) as well as Sterling et al. (1991), see also Sterling et al. (1993, 1994). The radiative losses (thin plasma approximation) and classical thermal conductivity are balanced by a background heating of $4.3 \times 10^{-4}$ erg cm$^{-3}$ s$^{-1}$. The microflare is assumed to deposit thermal energy in a sinusoidal pulse of duration 30 s. Three types of microflares of low ($10^{24}$), medium ($10^{25}$), and high energy ($10^{26}$ erg) were considered. The gas flows and ejections which develop do not resemble spicules, but are rather similar to the $v = 100$ km s$^{-1}$ features seen in the UV lines. Weaker microflares do not produce prolonged significant heating and density increases. Nanoflare type heating within the chromosphere-corona transition layer has also been investigated by Glencross (1994).

Mullan (1991) predicts that nanoflare heating of the solar corona will produce magnetic plasmoids with initial dimensions of order 300 km moving with Alfvén speed. Fluxes of mass and momentum are comparable to solar wind values. Plasmoids may lead to acceleration of the solar wind plasma and may already have been detected in radio occultation studies.

A magnetic loop with a length of about 20000 km and foot points rooted in the chromosphere has been considered by Kopp and Poletto (1990, 1991). A sudden release of energy ($\sim 10^{25}$ erg) by a nanoflare raises the loop temperature to about $10^8$ K without immediately changing its density. The enhanced temperature causes enhanced downward thermal conduction leading to an evaporative filling of the loop from the chromosphere. This process continues until the back pressure of the loop plasma becomes sufficient to impede further evaporation so that the filling stops and the plasma radiatively cools and at the same time begins to fall back
into the chromosphere. The time behavior of the emission measure for this single nanoflare event is computed.

Dere et al. (1991) study explosive events and magnetic reconnection using the Naval Research Laboratory’s HRTS instrument. They find that the explosive events, which typically show themselves by CIV line profile broadenings of up to 300 km s⁻¹, tend to occur on the edges of high photospheric magnetic field regions and identify them with reconnection during the cancellation of photospheric magnetic flux. An extensive discussion of these explosive events is also given by Cook and Brueckner (1991).

Hudson (1991), on basis of the observed solar flare rate, discusses the nanoflare coronal heating theory. He extrapolates the observed rate of flares of given energy down to nanoflare energies and shows that this gives too little total energy for the coronal heating requirements. In order for the nanoflare picture to work, a much higher rate of nanoflare events must occur with a distribution distinctly different from that of the observed flare rate. This could mean a different physics for the nanoflares. There also must be a high energy cut-off of the flare distribution.

Zirker and Cleveland (1993a, b) support Hudson’s (1991) conclusions and suggest that there are two types of flares, twisting and braiding flares. Large-scale photospheric motions involving entire flux tubes lead to braiding of neighboring loops, which through reconnection occasionally produces the observed large flares. Smaller scale vortical motions lead to the twisting of individual loops, producing instabilities and small-scale reconnection relaxations (nanoflares), which would account for the bulk of the power for the heating of active regions. Because Berger’s (1991a) braiding criterion generates too much flare energy, the authors see their model in better agreement with observation.

Cargill (1993) discusses the heating of the solar corona by nanoflares. He shows that even if these nanoflares are impulsive events and occupy a small heating volume the corona may still appear to be homogeneous. In particular, the filling factor of the coronal volume radiating in EUV and X-rays to the total volume could be of the order of unity.

Lee et al. (1993) express a relationship between the total energy released into the corona and the X-ray observations. They do not find any direct evidence for the proposal that corona may be heated by numerous small flares.

Einaudi and Velli (1994) discuss the energy storage and dissipation scenario of flares and find that there must be at a large number \( N = 10^{13} \) locations in the solar corona and at all times current sheets of size \( \Delta V \approx \lambda l \), where \( \lambda \approx 30 \) m is the Taylor microscale, \( l \approx 20 \) cm the dissipative length scale in the turbulent cascade and \( L = 10^{10} \) cm the coronal loop length.

Benz (1995) reviews flare heating of the solar and stellar coronae on basis of radio and X-ray evidence. He finds that regular flares do not have enough power to be the dominant heating mechanism even for quiet regions. This situation may be better for active, rapidly rotating stars. Micro bursts point to microflares, yet these flares are not well understood.
The analysis of high-resolution HST transition layer lines of AU Mic (dMe) by Linsky and Wood (1994) and Capella (G8III+G1III) by Linsky et al. (1995) shows evidence that two types of magnetic heating mechanisms are at work: AU Mic and the hot binary of Capella show very broad components in the C IV and Si IV lines which have widths of about 200 km s\(^{-1}\), which these authors attribute to microflare heating, and a less broad component of about 30–50 km s\(^{-1}\) width (shown by all three stars), which may be attributed to MHD wave heating. Similar results were obtained for Procyon (F5 IV–V) and HR 1099 (dG9) (Linsky, 1995, private communication).

8. LCR Circuit Approach, Heating by Mass/Particle Flows and Flux Emergence

In this section we discuss mechanisms which inject energy into the chromosphere and corona by mass and particle flows as well as by magnetic flux emergence. For earlier work on these processes see Section 9 of Paper I. We also discuss intermittent processes and the attempt to collect all the diverse magnetic heating mechanisms in a unified LCR theory, the latter is more extensively discussed in Section 7 of Paper I.

8.1. Unified Resonant LCR Circuit Heating

By attempting to reproduce the observed nonlinear relationship between the coronal X-ray emission and the chromospheric Ca II K line flux with help of the unified LCR circuit heating approach, Cram (1991) tries to obtain greater insight in the coronal heating mechanism. He proposes that this nonlinearity may be explained by the fact that the (optically thin) coronal emission is proportional to the volume and the (optically thick) chromospheric emission to an area.

Narain and Kumar (1993a) use the circuit approach to study the heating of solar coronal holes of various temperature and magnetic field strength through the reflection and trapping of Alfvén waves with different periods. A detailed review of the electric circuit approach of coronal heating has been given by Narain and Kumar (1993b). In particular, the basic circuit (LCR) relations, the unification schemes and some stellar applications such as the enhanced X-ray flux in early M-dwarfs, the bimodal behavior of black hole accreting X-ray sources, the magnetic energy in a sheared arcade as well as the circuit representation of Alfvén waves are briefly described.

Narain and Kumar (1995) study the electrodynamic heating efficiency for AC and DC mechanisms and point out that for certain coronal loop lengths and dissipation times the efficiency exceeds unity and loses its physical meaning.
8.2. Heating by Magnetic Flux Emergence

Brueckner (1990a, b) investigates the magnetic energy conversion in the solar transition zone. UV spectra from Spacelab-2 point to a firm connection between newly emerging flux and non-thermal broadening of transition layer lines. This establishes newly emerging magnetic flux as an important energy source for the solar corona and the solar wind. He sees the transition layer as the location where the magnetic energy conversion takes place, facilitated by the observed large non-thermal motions and the unique conditions in that layer. The spatial filling factor is small (<0.01) which indicates a highly filamented plasma. Large fluctuations in density, magnetic field strength and currents appear to facilitate fast reconnection. He sees the areas of strong plasma oscillations in the transition layer as the origin of coronal heating and particle acceleration.

8.3. Heating by Spiculaire Flows, Particle Flows and Intermittent Processes

Cheng et al. (1991) as well as Cheng (1992a–c) numerically simulate a solar spicule with a single acoustic pulse which is generated in the photosphere and propagates upward along a flux tube. They take into account radiation, ionization, thermal conduction and mechanical heating. The authors were able to simulate fairly well the observed properties of spicules including the persistent downflows seen in the C_{IV} line at a temperature of $10^5$ K in the chromosphere-corona transition layer. As their numerical code conserved mass and did not produce a massive solar wind the systematic downflow phenomenon was found to be not due to a net downward mass flow. The authors find that spicules can play a significant role in the heating of the upper solar atmosphere.

A similar time-dependent wave calculation by Wikstøl and Hansteen (1994) which in addition solve the time-dependent rate equation for the hydrogen ionization shows a persistent blue-shift of the C_{IV} transition layer emission line 1548 Å instead of a net red-shift as observed and as obtained by Cheng (1992a). They conclude that the rebound shock model does not explain the observed downflows in the transition layer lines.

Scudder (1992a, b) postulates that at the foot of the transition layer, where the gas becomes optically thin and fully ionized, the Maxwell distribution function of the particles somehow (waves, shocks?) obtains a strong suprathermal tail. Because the Coulomb cross-section gets rapidly smaller the higher the particle energy, the particles of this tail rise essentially without collisions against the gravitational potential to coronal heights, such that only the hottest survive. As the broad suprathermal distribution at the base leads to a broad distribution in the corona the temperature increases with height to values in excess of $10^6$ K, without depositing wave or magnetic field energy into the gas above the base of the transition region. This heating theory is applied to open and closed flux tubes, to the Sun and stars.
The temperature inversion process is slightly more efficient in closed than in open tubes. The solar wind ions are ‘heated’ through this mechanism in proportion to their masses. For a quantitative application a kappa-distribution function is introduced where $\kappa$ describes the degree of departure from the Maxwell distribution function. This $\kappa$ is adjusted to fit various observations. Scudder (1994) applies this heating theory to coronal holes where for a chosen range of $\kappa$ he obtains the right mass loss and nonthermal line widths. Scudders interesting theory critically depends on the existence of a suprathermal velocity distribution of the particles which in turn needs another so far unknown heating mechanism that generates a significant number of hot particles at the foot of the transition region. It further remains to be seen whether the collisions are indeed unimportant enough to preserve the hot tail on its way up into the corona. Here solar wind observations may provide additional evidence.

Hammer et al. (1993) critically examine the heating mechanism in coronal holes. A heating mechanism not only provides a total heating flux but also specifies the spatial distribution of this heating so that observations can be used to identify the heating mechanism. On basis of current semi-empirical models they find that most of the heat input in the solar coronal holes must be deposited over a characteristic length of about half a solar radius which is too long for spicules to be the main heating mechanism and too short for propagating monochromatic Alfvén waves. In the light of this constraint they examine the possibility that coronal heating is an inherently intermittent process, with quiet and active phases. In active phases, the inner part of the corona expands as a result of magnetic field reconfigurations or of the wave pressure associated with MHD wave packets. During quiet phases, the plasma sinks back and releases its potential energy, which is ultimately converted into thermal energy. They show that the required velocities are consistent with observations and conclude that the proposed process could be an important heating mechanism in the magnetically open corona.

9. Some Relevant Observational Evidences

In this section we review a few observational papers which we feel have bearing on the question of the heating mechanisms. Reviews of additional observational papers can be found in Sections 2 to 8. For recent observational reviews of solar photospheric magnetic flux tubes see Solanki (1993) and Stenflo (1994).

9.1. Evidence for Acoustic Heating Mechanisms

Judge and Stencel (1991) study the global thermodynamic properties of the outer atmospheres of giant stars. They find that the mass loss rates are not strongly dependent on the actual physical processes driving the winds and suggest that nonlinear processes act to regulate wind energy fluxes. Chromospheric heating
and mass loss are two responses of outer atmospheres of cool giants to an input of mechanical energy generated in deeper layers. There are two sources of mechanical energy, viz., subphotospheric convective motions and global pulsations of the star. They picture that short period acoustic waves are responsible for chromospheric heating and long period waves lead to mass loss.

Brown et al. (1992) describe a time series of Doppler measurements of the solar photosphere with moderate spatial resolution, covering a portion of the solar disk surrounding a small sunspot group. They find that in the frequency range $5.5 \leq \nu \leq 7.5$ mHz, a small fraction of the surface area emits a disproportionate amount of acoustic energy. The regions with excess emission are characterized by a patchy structure at spatial scales of a few arc seconds and by association with regions having substantial magnetic field strength. These observations bear on the conjecture that most of the acoustic energy driving solar p-modes is created in localized regions occupying a small fraction of the solar surface area.

Mg II h and k emission line observations with the IUE satellite for 69 K and 88 M dwarfs have been analyzed by Mathioudakis and Doyle (1992) and Doyle and Mathioudakis (1993). They find that there is an empirical lower limit in the Mg II emission flux, which is consistent with acoustic heating as given by Bohn (1981, 1984). Doyle et al. (1994) detected M dwarfs with even lower chromospheric activity, having Mg II and Ca II emission fluxes of $4 \times 10^3$ and $1 \times 10^3$ erg cm$^{-2}$ s$^{-1}$, respectively. Similar very low emission fluxes for M-dwarfs below the published basal flux lines have been found by Byrne (1993, 1994).

Mullan and Cheng (1994a) in theoretical modeling a low chromospheric emission M-dwarf suggested that the corona of this star might be heated acoustically. They find that acoustic heating may maintain a cool corona with a temperature of the order $0.7\times 10^6$ K.

A similar conclusion was reached from EUVE observations of the dK4 star HD 4628 by Mathioudakis et al. (1995). They interpret detection by EUVE and non-detection by ROSAT of this star as evidence for a cool corona with coronal temperatures of less than $10^6$ K, which is possibly heated acoustically.

Mullan and Cheng (1994b) have modeled the chromosphere and corona of the F5IV–V star α CMi by using a time-dependent radiation hydrodynamic code and employing acoustic waves. They compared their model with reported fluxes of Mg II and Lα lines as well as with X-ray fluxes and with differential emission measure distributions. They found that the models and observations were consistent and that the required acoustic fluxes were easily within range of the theoretically predicted fluxes.

9.2. EVIDENCE FOR MAGNETIC HEATING MECHANISMS

Von Uexküll et al. (1989) on basis of a time sequence of disk center Hα spectrograms confirm that in the interior of the chromospheric network cells the oscillatory behavior dominates whereas at the boundaries one generally finds random motions
having an rms velocity of the order 4 km s\(^{-1}\). Since the gas pressure is orders of magnitude lower than the magnetic pressure, the gas motions are controlled by the magnetic field. Using Parker’s formula for the magnetic energy flux for slow foot point motion they find that the random motions provide enough heating to balance the radiation losses of the high chromosphere.

A comparison of Kitt Peak magnetograms and high resolution rocket observations at 1600 Å by Cook and Ewing (1990) enables them to derive a quantitative linear relationship between the brightness temperature of the observed fine structure elements and the magnetic field strength in the range 10–150 G.

Dere (1990) analyses UV spectra of the solar transition region obtained by the Naval Research Laboratory’s HRTS instrument which measures Doppler shifts to an accuracy of 1–2 km s\(^{-1}\). In contrast to the classical picture where the transition zone is a thin region maintained by the heat conducted down from the corona, high resolution spectroheliograms show that the transition zone consists of elongated structures which must be heated locally because the field-aligned temperature gradients are insufficient to support the necessary conductive flux. An analysis of the observationally determined emission measures and pressures of transition region structures reveals that they consist of fine scale subresolution structures with dimensions less than 100 km. Further evidence for subresolution structure comes from the line widths which are much broader than expected from nonthermal broadening. He argues that length scales of about 10 km or less occur which, would greatly facilitate viscous and Joule heating.

Smartt and Zhang (1990, 1993) as well as Li et al. (1994) observe transient enhancements at the projected intersections of coronal loops. They find that the brightness of such enhancements gradually increases to a marked maximum and then fades with a typical lifetime of about 30 min. The observed phenomenon is interpreted as localized loop coalescence and magnetic reconnection.

Damé et al. (1991) discuss interferometric techniques which could allow to achieve 0.01 arc sec angular resolution. Such high resolution would be very important for the unraveling of the different magnetic heating mechanisms.

Böhm-Vitense and Werth (1991) study UV emission lines and continuum fluxes from late-type giants. For cooler giants of spectral type F and later they find a correlation between large C IV/ C II ratios and an excess continuum flux at 1950 Å. As large C IV/ C II ratios indicates magnetic heating mechanisms, the energetically expensive excess continuum emission must mean strong heating, possibly by Alfvén waves.

Solar Maximum Mission satellite observed coronal X-ray line broadening in a non-flaring active region. Line ratio analysis by Saba and Strong (1991) yields an electron temperature of order 3 \(\times\) 10^6 K with a 10% variation. They interpret the observed coronal line broadening as a signature of the mechanisms responsible for coronal heating in active regions. They found excess non-thermal line broadening corresponding to velocities in the range 45 to 60 km s\(^{-1}\) for the Mg xi ion. This broadening may come from flows, from turbulent motions, or from oscillations.
of the magnetic field. They have not been able to discriminate among various mechanisms of heating.

Turner et al. (1991) compare linear combinations of observed spectra of the Hα and Ca II lines from the chromospheres of a quiet and an active M dwarf star with that of an intermediate activity star. They conclude that the spectrum of the intermediate star cannot be explained by simply adding the extreme chromospheric states with appropriate filling factors, but that the radial structure of the chromospheric heating is important.

Cerruti-Sola et al. (1992) make a detailed comparison of Mg II h and k lines in late-type stars of different activity levels and spatially resolved solar regions of different degrees of magnetic activity. They conclude that there is good qualitative and quantitative agreement with the picture that different levels of chromospheric activity in stars is produced by different fractions of the stellar surface covered by magnetic fields.

Jordan (1992) reviews the methods of semi-empirical modeling of solar coronal loops. The review concentrates on what is known about the observable parameters and on what can be deduced from them about the physical state of the loops and its energy balance.

Dravins et al. (1993) analyze the data on the very old (9–10 Gyr) star β Hyi (G2IV) collected from IUE and EXOSAT. They find a much weaker coronal X-ray flux than for the Sun. Comparing the Sun, α Cen, β Hyi, and Arcturus they suggest that β Hyi is an intermediate step to losing the observable corona altogether. Stars of this type and older ones provide a unique opportunity to study the coronal heating mechanisms and the rotational braking history.

Harmon et al. (1993) study the structures above pores and small sunspots using the high resolution X-ray observations of the solar corona obtained from the NIXT Instrument. On basis of these data they argue that coronal heating requires magnetic field stressing from imposed motions internal to coronal flux bundles, rather than from displacement of the flux bundles.

Fleming et al. (1993) present X-ray data, both detections and upper limits, from the ROSAT all-sky survey for most known M dwarfs later than M5, as well as from selected ROSAT pointed observations of some of these stars. They compare these data with similar data for early M-dwarfs in an attempt to probe the nature of the magnetic dynamo and the coronal heating mechanism for the late M dwarfs. It is found that late M dwarfs can have coronae which are just as active as those of early M dwarfs and that the coronal heating efficiency for stars at the saturation boundary, that is stars where presumably the stellar surface is entirely covered by magnetic fields, does not drop at spectral type M6.

Zirker (1993b) discusses observational tests for coronal heating theories. None so far could give unambiguous answers.

Houdebine and Doyle (1994a, b) modeled the hydrogen spectrum of active dMe stars and from the line flux ratios of Hα/Lα conclude that a very high transition layer pressure and a thin transition region are simultaneously required.
Gomez et al. (1993a) have analyzed very high resolution X-ray images from the solar corona obtained with the NIXT instrument. They find that the spectral power of the spatial intensity fluctuations varies with the wave number as $k^{-3}$. That is, the coronal structures show no preferred length scale down to the resolution limit of 0.75''.

Moses and Cook (1994) as well as Kjeldseth-Moe and Cheng (1994), (but see also Simnett 1994) discuss explosive transition region events which appear to remove plasma and magnetic fields from regions undergoing magnetic reconnection.

Wang et al. (1995) provided unprecedented observations for direct measurements of solar intranetwork magnetic fields. More than 2500 individual intranetwork elements and 500 network elements were identified and the magnetic flux was measured in Big Bear deep magnetograms of a quiet region of $300 \times 235$ arc sec size. The authors found that more than 20% of the total flux is in the form of intranetwork elements at any given time. That many chromospheric bright points are in fact magnetic in nature has also been found by Cook (1992).

10. Conclusions

We arrive at the following conclusions:

1. There is now a fairly consistent picture of the acoustic heating of stellar chromospheres. The acoustic wave generation calculations, the solar wave observations, the empirical chromospheric losses and theoretical heating simulations appear all to be consistent. In addition, simulations of stellar chromospheric line emission on basis of acoustically heated chromosphere models agree well with the observations (basal flux line). This implies that nonmagnetic chromospheric regions of late-type stars are heated by shock dissipation of acoustic waves which are generated in the stellar surface convection zones. Towards late-type giants the acoustic heating is augmented by shock heating from pulsational waves. For slowly rotating stars, which have few or no magnetic fields, the acoustic and pulsational waves are the dominant chromospheric heating mechanisms.

2. The coronal structures of early-type stars are very different from those in late-type stars. The X-ray emission there apparently arises in the hot post-shock regions of gas blobs which are accelerated in the stellar wind by the intense radiation field of these stars.

3. Except for F-stars, where acoustic heating has its maximum contribution, the chromospheric heating of rapidly rotating late-type stars is dominated by magnetic heating, either in form of MHD wave dissipation (AC mechanisms) or magnetic field dissipation (DC mechanisms). The MHD wave and magnetic field energy derives from turbulent and convective motions in the stellar surface convection zones which displace the magnetic field lines. MHD waves are also generated in the corona from loss of equilibrium in overstressed fields (caused by slow footpoint
motion) or by reconnection. The spectrum of the field line movements determines whether AC or DC heating occurs. The high-frequency part of the motion spectrum leads to AC heating, the low frequency part to DC heating.

(4) The coronae are probably almost exclusively heated by magnetic mechanisms, although a few dK, dM star cases have been found where cool acoustically heated coronae may be present. However, these cases need confirmation.

(5) Despite the known fact that Alfvén waves are responsible for the heating of the solar wind it is widely concluded (e.g., Parker, 1991a, b) that the generation and heating by Alfvén body waves is insufficient to balance the principal chromospheric and inner coronal energy losses. The same applies to magnetoacoustic waves. However, new observational and theoretical developments indicate a much greater MHD wave flux and new work on the formation of small-scale structures including shocks, as well as on pulsational resonances produced by wave reflection, indicates a much greater heating rate by MHD body waves. Here higher resolution observations of the speed of footpoint motions as well as more detailed studies of the formation of small-scale structures and of the dissipation in filaments are needed.

(6) The workers on the heating by magnetoacoustic surface waves all agree that resonance absorption, which works well in terrestrial plasmas, must certainly be important in chromospheres and coronae. This mechanism is also a logical choice for interaction at the boundaries between magnetic flux tubes and nonmagnetic areas (e.g., Chitre and Davila, 1991).

(7) The large amount of work on the heating by dissipation in current sheets (DC heating) clearly shows that this mechanism is very important and may well be the most important magnetic heating mechanism. Here particularly Parker’s idea of micro and nanoflares, driven by small-scale reconnection, appears to be most promising. This also agrees well with the increasing importance of flares when going to later type stars like dMe stars which are more magnetically active than the Sun.

(8) The emergence of magnetic flux will certainly enhance local heating as does the mass injection by spicules. The interesting idea of Scudder (1992a, b, 1994) to ‘heat’ the corona by the diffusion of a hot tail of a non-Maxwellian distribution function shifts the heating problem down into the upper chromosphere, where this hot tail must be produced by another heating mechanism. In addition, this theory needs a more detailed study of the influence of particle collisions in the high chromosphere and lower transition layer. The LCR approach tries to unify very different mechanisms and may be premature at the present time.

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References

At the end of each reference we specify the Section(s), where the respective work is discussed.


