1. INTRODUCTION

It is known that the intensities of many emission lines produced in gaseous nebulae are extremely sensitive to the electron temperature $T_e$ (see, for example, Czyzak et al. 1986). Mean electron temperatures of planetary nebulae have traditionally been derived using the intensity ratios of the $3P_{1/2} - 1D$ and $1D - 1S$ forbidden transitions within the $2s^22p^2$ ground term of $O^+$ and $N^+$, namely, $R_{O^+} = I(9599 + 5007 \, \AA)/I(4363 \, \AA)$ and $R_{N^+} = I(6548 + 6584 \, \AA)/I(5754 \, \AA)$ as shown by, for example, Seaton (1960), Osterbrock (1974), Kaler et al. (1976), and Aller (1984).

In Kaler (1986), mean electron temperatures for $[N \, II]$ and $[O \, III]$ were derived in a sample of planetary nebulae, and then compared. Here we present new calculations of the $[N \, II]$ and $[O \, III]$ electron temperature diagnostics derived using improved $R$-matrix atomic data on electron impact excitation rates, and use these to reanalyze the Kaler (1986) planetary nebula dataset.

2. ATOMIC DATA AND THEORETICAL RATIOS

The model ion for $N^+$ consisted of the six energetically lowest fine-structure levels, namely, $2s^22p^2$ and $1D - 1S$ forbidden transitions within the $2s^22p^2$ ground term of $O^+$ and $N^+$, namely, $R_{O^+} \approx 4959 + 5007 \, \AA/4363 \, \AA$ and $R_{N^+} \approx 6548 + 6584 \, \AA/5754 \, \AA$ as shown by, for example, Seaton (1960), Osterbrock (1974), Kaler et al. (1976), and Aller (1984).

In Kaler (1986), mean electron temperatures for $[N \, II]$ and $[O \, III]$ were derived in a sample of planetary nebulae, and then compared. Here we present new calculations of the $[N \, II]$ and $[O \, III]$ electron temperature diagnostics derived using improved $R$-matrix atomic data on electron impact excitation rates, and use these to reanalyze the Kaler (1986) planetary nebula dataset.
In calculating \( R_{[\text{NII}]} \) and \( R_{[\text{OIII}]} \) an error of 10\% in the electron impact rates of \( O^{+2} \) and \( N^+ \) was assumed (Aggarwal 1983; Stafford et al. 1994), as well as an error of 10\% in the Einstein A coefficients (Keenan and Aggarwal 1989; Mendoza 1983). This leads to an overall error of 15\% in the line ratios.

Over the last several decades, standard formulas have been employed for the derivation of electron temperatures from the \( R_{[\text{NII}]} \) and \( R_{[\text{OIII}]} \) line ratios, the coefficients of which have slowly evolved as the atomic parameters have improved. Previously, the formulas used in obtaining \( T_e \) were derived from simplifications of the \( p^2 \) equilibrium solutions (see, for example, Seaton 1960; Osterbrock 1974; Kaler et al. 1976; Aller 1984). However, using the atomic data discussed above, in conjunction with the statistical equilibrium code of Dufton (1977), we have derived relative \([\text{NII}] \) and \([\text{OIII}] \) level populations for a range of electron temperatures \( T_e = 5000-20,000 \) K and densities \( n_e = 10^4-10^6 \text{ cm}^{-3} \) applicable to planetary nebulae, and hence explicitly calculated the \( R_{[\text{NII}]} \) and \( R_{[\text{OIII}]} \) line ratios as a function of electron temperature and density.

The results were fitted to expressions of the form found in Kaler (1986), using MATHEMATICA, a commercial Computer Algebra package. Although it does not have a nonlinear curve-fitting procedure built in, the function NonlinearFit[] is an external package that is distributed with the program which allows very general models to be fitted to empirical data, obtaining least-squares best fits for the parameters. As warned in the package’s documentation, it was necessary to have a fairly good idea of the probable values of the parameters in order to prevent divergence of the algorithm used. However with suitable starting values given, NonlinearFit[] returned results within a few minutes on a PC. This allowed us to obtain the following formulas:

\[
T_e[N\text{II}] = \frac{10697.5}{\log R_{[\text{NII}]} - 0.888527 + \log(1 + 0.310077x)} \quad (1)
\]

and

\[
T_e[O\text{III}] = \frac{14516.0}{\log R_{[O\text{III}]} - 0.871411 + \log(1 + 0.046874x)} \quad (2)
\]

In Eqs. (1) and (2), \( x = 10^{-2}N_e/\sqrt{T_e} \), where \( N_e \) is the electron density (\text{cm}^{-3}) of the nebula. These formulas should be more accurate than those derived previously, due to (a) the adoption of improved atomic data, especially for electron impact excitation rates, and (b) the fact that we have explicitly calculated the line ratios using detailed model ions rather than using approximate standard formulas. The latter is of particular importance at higher densities, where the effects of higher-lying levels must be taken into account.

The mean electron temperatures found using these formulas are dependent on \( R_{[\text{NII}]} \) and \( R_{[\text{OIII}]} \), and therefore dependent on their inherent errors. Consequently, from the expected error of 15\% in \( R_{[\text{NII}]} \) and \( R_{[\text{OIII}]} \) it is found that there is an estimated probable error of approximately 6\% and 7.5\% in \( T_e \) \([\text{NII}] \) as \( x_{[\text{NII}]} \) approaches 0 and 1, respectively, while for \( T_e \) \([\text{OIII}] \), there is an estimated probable error of 4.5\% for all \( x_{[\text{OIII}]} \).

### 3. RESULTS AND DISCUSSION

Mean electron temperatures calculated for the planetary-nebula sample of Kaler (1986), using Eqs. (1) and (2), are presented in Table 1 (see Kaler 1986 for details of the relevant observations). In deriving these electron temperatures...
the values of $x_{\text{O III}}$ and $x_{\text{N II}}$ were taken, where possible, from Stanghellini and Kaler (1989). The mean electron density from the [Cl III] $I(\lambda 5517)/I(\lambda 5537)$ ratio was used to find $x_{\text{O III}}$, while the electron densities from the [O II] $I(\lambda 3727)/I(\lambda 3729)$ ratio and [S II] $I(\lambda 6717)/I(\lambda 6730)$ ratio were used to determine $x_{\text{N II}}$. Otherwise, the mean of the available data in Kaler (1986) was employed. The data come from such a variety of sources that it is difficult to evaluate errors, which, moreover, are hidden by real variations within the nebulae. Typical and reasonable errors of 15% in the line ratios lead to errors in either electron temperature (for typical $T_e$) of about 700 K. Examination of the scatter in the comparison diagrams presented by Kaler (1986) (in which the temperatures from one source are compared with those of another) suggests the “errors” to be applied to the electron temperatures for an individual source to be about 1000 and 1500 K for [O III] and [N II], respectively, much of which, however, is the result of real differences and therefore quite consistent with a 15% observational error.

We corrected all the data for interstellar reddening before
the temperatures were calculated using the Cardelli et al. (1989) extinction function, expressed in the common form where the corrected line intensity equals that observed times $10^{c/h\beta}$, where $c$ is the logarithmic extinction at H$\beta$, i.e.,

$$I_{\text{corrected}} = I_{\text{observed}} \times 10^{c/h\beta}.$$  

For the purpose of correcting the line ratios, we used $\Delta f[O III] = 0.16$ and $\Delta f[N II] = 0.145$ (see Kaler 1986). We took values of $c$ from Cahn et al. (1992).

It must be noted that it is not completely obvious that the extinction function derived from observations of hot stars near the galactic plane within 1 kpc or so of the Sun applies to the dust along the light paths to the planetary nebulae in this data set, nor to the dust which may be within those nebulae, nor that the effects of scattering by that dust are correctly taken into account by using the extinction function derived from stars.

Mean derived electron temperatures are presented in

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Fig. 4—Same as Fig. 3 but for the [O III] electron temperatures.

Fig. 5—Plot of the [N II] electron temperatures against those derived from [O III], using the present results given in Table 1. The line is the 45° slope.

Fig. 6—Same as in Fig. 5 but using the electron temperatures of Kaler (1986).
Table 1. These results differ considerably from those originally presented by Kaler (1986). Figures 1–6 display comparisons between the electron temperatures derived here and those found by Kaler. In Fig. 1, both sets of $T_e$ [N II] are plotted against each other, while a similar diagram is presented in Fig. 2 for $T_e$ [O III]. The $45^\circ$ slope drawn on both graphs shows that for both N II and O III, the electron temperatures derived here are systematically lower than those of Kaler, with the mean ratio of $T_e$(present) to $T_e$(Kaler) =0.93 and 0.91 for [N II] and [O III], respectively.

The discrepancies between the mean electron temperatures derived by Kaler (1986) and those in the present paper become more pronounced with increasing electron density. To emphasize this dependency on $N_e$, $\Delta T_e = |T_e$(Kaler) – $T_e$(present)| was averaged over log $N_e$ bins of approximately 0.6 dex, and this average, denoted $\Delta T_e$, is plotted against log $N_e$ in Figs. 3 and 4 for [N II] and [O III], respectively. Note that in Fig. 3, 3 out of the 80 values for $\Delta T_e$ [N II] were omitted, those for NGC 3242, NGC 6537, and M 1-41, because $\Delta T_e$ in each case was over a factor of 10 greater than the other values in that density range, and their inclusion changed the form of the graph. It is unclear as to why this anomaly exists.

The newly derived values of mean electron temperature lead to slightly smaller discrepancies between $T_e$ [N II] and $T_e$ [O III] than were found in Kaler (1986). Figures 5 and 6 show $T_e$ [N II] plotted against $T_e$ [O III] derived using the new results and those of Kaler (1986), respectively. Also plotted on the figures is the $45^\circ$ slope for $T_e$ [N II] = $T_e$ [O III]. It is clear from inspection of the figures that the points are generally clustered more densely in the region surrounding the $45^\circ$ line in Fig. 5 than in Fig. 6. This is supported by the average discrepancy between $T_e$ [N II] and $T_e$ [O III] (2070 and 2210 K for the present results and those of Kaler, respectively). It is difficult to evaluate how meaningful this difference is, but it seems reasonable that the probable errors in the average $T_e$ [N II] and $T_e$ [O III] would not cause these values to overlap. Thus, it is interesting to note that the new atomic data lead to more agreement between the mean electron temperatures of [N II] and [O III].

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