EFFECTIVE TEMPERATURE-COLOUR AND SURFACE BRIGHTNESS-COLOUR RELATIONSHIPS FOR THE LATEST-TYPE STARS

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ABSTRACT. New calibrations of six broad-band optical and near infrared colours \((U - B, B - V, V - R, V - I, R - I\) and \(V - K\)) against accurate effective temperatures \((T_{\text{eff}})\) derived from empirical and semi-empirical (infrared flux) methods for G, K and M giants are presented. Polynomial coefficients for these relationships are given. We also present independent relationships between the visual surface brightness parameter \(F_V\) and the colours \((V - R), (V - I), (R - I)\) and \((V - K)\). We find that \((U - B)\) and \((B - V)\) are unsuitable as \(T_{\text{eff}}\) indicators. The tightest relationship in each case found for the \((V - K)\) index. Our mean relationships agree reasonably well with similar relationships derived by other authors, where such are available. These relationships are suitable for use in the determination of spot areas and temperatures in active late-type stars.

1. INTRODUCTION

Photometric variability in active late-type stars is widely interpreted as due to the rotational modulation of the visibility of very large starspots (see e.g. Byrne 1992a, 1992b). Areas and temperatures of these spots are derived from light and colour curves via models which depend critically on the calibration of surface brightness and effective temperature versus colour relationships. In calculating a model light curve of a rotating spotted star for a given spot distribution, one of the difficulties lies in evaluating the distribution of the light coming from the spot. The character of the spot fluxes are not yet certain. There are three different approaches.

- Assume the spot light is the same as that of a star of the same \(T_{\text{eff}}\) and use spectrophotometric fluxes (Dorren 1987).
- to use the Barnes-Evans surface brightness–colour relationships (Byrne et al. 1995), or
- to use blackbody fluxes (La Fausi & Rodonò 1983).

According to Dorren (1987), there is no certainty that any of these accurately represents the true spot flux distribution. Nevertheless, it seems that representing spot flux distributions by a combination of the Barnes-Evans relationships and stellar flux distributions offers the most physically reasonable compromise (see e.g. Poe & Eaton 1985). In general, since magnetic pressure will partially evacuate the umbrae of spots, this effect can be mimicked by using the flux distribution of a star of luminosity class one higher than the parent spotted star.

In this paper we undertake the calibration of both monochromatic surface brightness and broadband colour index against effective temperature over a range of effective temperatures appropriate to the study of spots on active late-type stars.

2. DATA

2.1. Temperatures

We have searched the literature for published effective temperatures derived from a totally empirical method where possible, i.e. from measurements of stellar angular diameters and bolometric fluxes. Here we make use of data from Ridgway et al. (1980), Di Benedetto & Rabbia (1987) and White & Kreidl (1984). These data are all derived via the above empirical method, the only differences being in the way in which the stellar diameters were obtained, i.e., lunar occultation or Michelson interferometry.

To extend the sample towards the coolest M giants, we had to make use of effective temperatures coming from semi-empirical infrared-flux method (IFM) (Blackwell & Shullis 1977). This method computes the ratio \(R\) of the total stellar flux to a monochromatic infrared flux for model atmospheres covering a range in temperature and gravity. Comparing the observed values of \(R\) with those obtained from the model atmospheres, \(T_{\text{eff}}\) can be determined, the effect of gravity being rather small. Temperatures derived via the IFM are taken from Blackwell & Shullis (1977), Blackwell et al. (1980), Blackwell et al. (1986), Tsuji (1981) and Leggett et al. (1986).

A number of the above temperature sources required small adjustments due either to incorrect values of interstellar absorption or subsequent development of the method. For these corrections we refer to McWilliam (1990).

All the stars in this investigation are of luminosity class III. We have rejected a few stars from the original lists where their classification as giants disagreed with their classification in the SIMBAD database.

2.2. Photometry

Johnson \(U BV RI K\) photometry was taken from the J P11 measurements held at the SIMBAD database. Those stars with
Fig. 1. Relationships between $T_{\text{eff}}$ and the colour indices (a) $(U - B)$, (b) $(B - V)$, (c) $(V - R)$, (d) $(V - I)$, (e) $(R - I)$ and (f) $(V - K)$ for giant stars. The solid lines represent the fits given by the cubic polynomials in this work. We also plot on the upper two panels the data for dwarfs and their fits (heavy dotted line). The rest of the curves on the four lower panels are relationships found by other authors (see text).
Table 1. Coefficients of polynomial fits to \( T_{\text{eff}} \)-colour relationships

<table>
<thead>
<tr>
<th>Colour</th>
<th>( c_0 )</th>
<th>( c_1 )</th>
<th>( c_2 )</th>
<th>( c_3 )</th>
<th>rms</th>
</tr>
</thead>
<tbody>
<tr>
<td>( U - B^b )</td>
<td>1.389</td>
<td>-5.270 ( \times 10^{-5} )</td>
<td></td>
<td></td>
<td>0.1178</td>
</tr>
<tr>
<td>( B - V^b )</td>
<td>3.800</td>
<td>-6.127 ( \times 10^{-4} )</td>
<td></td>
<td></td>
<td>0.0587</td>
</tr>
<tr>
<td>( U - B^c )</td>
<td>-96.70</td>
<td>0.07037</td>
<td>-1.648 ( \times 10^{-5} )</td>
<td>1.261 ( \times 10^{-9} )</td>
<td>0.1387</td>
</tr>
<tr>
<td>( B - V^c )</td>
<td>-32.62</td>
<td>0.02535</td>
<td>-6.131 ( \times 10^{-6} )</td>
<td>4.806 ( \times 10^{-10} )</td>
<td>0.0556</td>
</tr>
<tr>
<td>( V - R^c )</td>
<td>40.66</td>
<td>-0.02476</td>
<td>5.178 ( \times 10^{-6} )</td>
<td>-3.65 ( \times 10^{-10} )</td>
<td>0.1082</td>
</tr>
<tr>
<td>( V - I^c )</td>
<td>83.23</td>
<td>-0.05068</td>
<td>1.053 ( \times 10^{-5} )</td>
<td>-7.36 ( \times 10^{-10} )</td>
<td>0.2145</td>
</tr>
<tr>
<td>( V - K^c )</td>
<td>85.08</td>
<td>-0.04812</td>
<td>9.397 ( \times 10^{-6} )</td>
<td>-6.18 ( \times 10^{-10} )</td>
<td>0.2930</td>
</tr>
<tr>
<td>( R - I^c )</td>
<td>39.04</td>
<td>-0.02331</td>
<td>4.721 ( \times 10^{-6} )</td>
<td>-3.20 ( \times 10^{-10} )</td>
<td>0.1117</td>
</tr>
</tbody>
</table>

\(^a\)Colours are computed from temperatures using \( Y = \sum C_i T_{\text{eff}}^i \), where \( Y \) is the colour and \( C_i \) are the coefficients. \(^b\)Dwarfs. \(^c\)Giants.

measured \( T_{\text{eff}} \) that were lacking all or any of these magnitudes in that catalogue were rejected from the calibration even if those magnitudes could have been taken from other sources.

This was done to maintain consistency in the way we obtained the photometric measurements.

As Barnes et al. (1978) pointed out, stars which have their angular diameters measured by lunar occultation or interferometry should be near enough to need no correction for interstellar absorption. Furthermore, Warner (1972) showed that the reddening line is almost parallel to the intrinsic distribution of \( F_V \) versus colour index. Therefore, the uncertainties in the reddening correction do not impede the determination of the intrinsic distribution. Nevertheless, some of the data needed to be corrected for interstellar extinction. In some cases, they were even over-corrected, i.e., the magnitude of the interstellar correction was overestimated. This was rectified in the paper by McWilliam (1990).

2.3. Angular diameters

The visual surface brightness parameter \( F_V \) was defined by Barnes & Evans (1976) as;

\[
F_V = 4.2207 - 0.1V_0 - 0.5 \log \phi'
\]

in which \( V_0 \) is the unreddened apparent magnitude in the Johnson system and \( \phi' \) is the stellar angular diameter expressed in milli-arcseconds.

The angular diameters for the calibration of this relationship were again taken from the literature (Ridgway et al. 1980; Di Benedetto & Rabbia 1987; Blackwell & Shallis 1977; Leggett et al. 1986; Blackwell et al. 1980; Richichi et al. 1992; Tsuji 1981).

The work by Ridgway et al. (1980) relied upon lunar occultation measurements of the stellar angular diameter and has a limiting accuracy of \( \sim 1 \) mas. Their resulting temperature uncertainties range from 110 K to 450 K for the K giants. For the present calibration we have replaced their value for the diameter of \( \delta \) Crys at 4.75 \( \pm 1.13 \) mas by the more recent measurement by Richichi et al. (1992) from a lunar occultation of 3.91 \( \pm 0.17 \) mas.

A more accurate empirical measurement of \( T_{\text{eff}} \) comes from the work by Di Benedetto & Rabbia (1987) using Michelson interferometry. They claim an average internal accuracy of \( \pm 61 \) K. However, the number of stars analyzed in this work is small and it does not cover a large range in temperature.

The remaining angular diameters come from the IFM. In Tsuji (1981), these diameters are not given explicitly, therefore, we had to calculate them using the expression;

\[
\phi' = 4.125 \times 10^8 \left( \frac{F_{\text{bol}}}{\sigma T_{\text{eff}}^4} \right)^{\frac{1}{2}}
\]

where \( \phi' \) is in milli-arcseconds, \( \sigma \) is the Stefan–Boltzmann constant in erg cm\(^{-2}\) s\(^{-1}\) K\(^{-4}\) and \( F_{\text{bol}} \) is in erg cm\(^{-2}\) s\(^{-1}\) K\(^{-4}\). The errors for these diameters were not given by Tsuji (1981). However, we can say that the accuracy of the IFM is limited by the accuracy with which the total to monochromatic stellar flux ratio \( R \) is measured. This, in turn, depends on the accuracy of absolute flux measurements in the ultraviolet, visible and infrared regions. A full discussion of these errors is beyond the scope of the present work.

The uncertainties for \( F_V \) take into account only the contribution due to the uncertainties in \( \phi' \) in Equation 1. This is so because the uncertainties in the measurement of the angular diameters dominate over those in the photometry.

All the angular diameters determined from lunar occultation and Michelson interferometry took limb-darkening into account. Usually, a polynomial function is used to correct for the centre-to-limb variations of the intensity profile. This function is expressed as follows;

\[
I(\mu) = I(1) \left[ 1 - \sum_{i=1}^{n} a_i (1 - \mu)^i \right]
\]

where \( \mu \) is the cosine of the azimuth of the emissive point on the stellar surface. The coefficients \( a \) are provided by a least-square fit to the limb-darkening, computed by model atmospheres. Di Benedetto & Rabbia (1987) chose a quadratic fit to the limb-darkening models by Manduca (1979) as a first approximation. Ridgway et al. (1980) computed limb-darkening correction factors from the model intensities of Johnson (1974), Manduca et al. (1977) and Manduca (1979).

The IFM, on the other hand, produces true angular diameters, so that no limb-darkening correction needs to be applied.
Table 2. Coefficients of the polynomial fits to $F_v$-colour relationships$^a$

<table>
<thead>
<tr>
<th>Colour</th>
<th>$c_0$</th>
<th>$c_1$</th>
<th>$c_2$</th>
<th>$c_3$</th>
<th>range</th>
<th>rms</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V - R$</td>
<td>3.9329</td>
<td>-0.4017</td>
<td>0.0067</td>
<td>-0.0085</td>
<td>$0.60 &lt; (V - R) &lt; 2.55$</td>
<td>0.0111</td>
</tr>
<tr>
<td>$V - I$</td>
<td>3.9855</td>
<td>-0.3387</td>
<td>0.0571</td>
<td>-0.0035</td>
<td>$1.00 &lt; (V - I) &lt; 4.80$</td>
<td>0.0104</td>
</tr>
<tr>
<td>$R - I$</td>
<td>3.9819</td>
<td>-0.8695</td>
<td>0.4323</td>
<td>-0.0993</td>
<td>$0.40 &lt; (R - I) &lt; 2.40$</td>
<td>0.0136</td>
</tr>
<tr>
<td>$V - K$</td>
<td>4.0172</td>
<td>-0.1907</td>
<td>0.0151</td>
<td>-0.0009</td>
<td>$1.95 &lt; (V - K) &lt; 7.05$</td>
<td>0.0081</td>
</tr>
</tbody>
</table>

$^aF_v = \sum c_i X^i$, where $c_i$ are the coefficients and $X$ the colour.

Fig. 2. Relationships between the surface brightness parameter $F_V$ and the colour indices $(V - R)$, $(V - I)$, $(V - K)$ and $(R - I)$ (upper left, right, lower left and right panels respectively). The solid lines represent the fits given by the cubic polynomial functions in this work. On the $(F_V, (V - R))$ plot we also show the relationships found by Barnes & Evans (1976) (dotted line), Barnes et al. (1978) (dashed line) and Eaton & Poe (1984) (dashed-dotted line) and on the $(F_V, (V - K))$ plot the relationship by Di Benedetto (1993) (dashed line).
3. RESULTS

3.1. Effective temperature-Colour relationships

Figure 1 shows the data for each colour against \( T_{\text{eff}} \) from Table 1. In the upper two panels, we also show the trend in \((U - B)\) and \((B - V)\) followed by M dwarfs, whose colours were taken from Leggett (1992). The temperatures for these M dwarfs were obtained using our \((T_{\text{eff}}, V - K)\) relationship. On these figures, the solid and dotted lines represent, respectively, the best third order polynomial fits for the giants and linear fits for the dwarfs. We give the coefficients of these fits in Table 1 along with the rms of the residuals of the data from the fit.

In the case of the M dwarfs, the \((V - K)\) colour was taken as a temperature indicator, since no direct determination was available for them. The \((V - K)\) colour was used because of its longer baseline, bracketing the region of the flux maximum in these stars.

All the fits are valid for a range of \( T_{\text{eff}} = 3200 - 5200 \) K, except for the \((U - B)\) and \((B - V)\) fits in which the valid ranges are from 3200 to 5000 K, for giants, and from 3300 to 4200 K for dwarfs. We see in these last two colours that the colour-\( T_{\text{eff}} \) relationship is not single valued for giants over the ranges 1.00 < \((U - B)\) < 2.00 and 1.45 < \((B - V)\) < 1.60. For the dwarfs, we see that the \((T_{\text{eff}}, U - B)\) relationship is almost flat but with a very large dispersion, while the \((T_{\text{eff}}, B - V)\) relationship is almost linear but again with a very large scatter.

For the colours \((V - R)\), \((V - I)\), \((R - I)\) and \((V - K)\) we over-plot (dashed line) the relationships given by McWilliam (1990) in his work, the agreement being quite reasonable. For the \((V - K)\) index, we present also the fitting curves found by Ridgway et al. (1980) and Di Benedetto (1993). As can be seen, they agree reasonably well and the differences are, generally, less than \( \pm 6 \) %.

The errors in the photometry should be of an accuracy of \( \sim 0.01 \), yielding error bars of a size much smaller than the size of the symbols used in the plots. Thus, they are completely dominated by errors in \( T_{\text{eff}} \) for individual stars. However, since we give mean relationships with \( T_{\text{eff}} \) as independent variable, we express rms errors in terms of colour. Intercomparison of the errors, therefore, needs to be weighted by the range of colour in each index.

Taking the weighted rms values by colour range, we see that the tightest relationship is obtained for the \((V - K)\) index.

3.2. Surface brightness-Colour relationships

As described in Section 2.3 above, we have obtained from the literature 73 angular diameters for giant stars of spectral type from G to M. Making use of Equation 1 we calculated the surface brightness parameter which, along with the colour indices, permitted us to derive new calibrations for this parameter.

In Table 2 we give coefficients for the cubic polynomial fits to our data for each of the colours. We also plot these functions in Figure 2. This is done for all the colours except for \((U - B)\) and \((B - V)\) since, as we discussed above, they do not yield unique temperatures for very cool stars. We also over-plot in Figure 2 the \((F_{\nu}, (V - R))\) relationships derived by Barnes et al. (1976, 1978) and Eaton & Poe (1984) for comparison.

We adopted a cubic polynomial fit to the \((V - R)\) colour relation since three different linear functions were required to fit the data throughout the colour interval. A polynomial seemed a simpler representation and there is no physical reason for this. We followed the same reasoning to obtain the fits for the remaining relationships even though two of them – \((V - I)\) and \((V - K)\) – could have been fitted by a pair of linear functions.

Referring to the rms values for the scatter from the mean relationships we can see that the most accurate relationship is obtained for \((V - K)\).

4. DISCUSSION

The relationships given here are aimed at providing basic calibrations of \( T_{\text{eff}} \) and visual surface brightness parameter, \( F_{\nu} \), against directly observable quantities, i.e., optical, near-infrared and infrared colours valid over a range suitable for the investigation of the light curves of late-type spotted stars. Comparison of our new calibrations shows good agreement with previously derived results where these are available.

Our results, as can be seen from Figure 1, show clearly that \((U - B)\) and \((B - V)\) are not good indicators of \( T_{\text{eff}} \) for either giant atmospheres (because of non-uniqueness of the derived \( T_{\text{eff}} \)'s at low temperatures characteristic of spot umbrae) or dwarfs (because of the very large scatter at these lower temperatures and a total lack of temperature sensitivity in the case of \((U - B)\)).

Poe & Eaton (1985) also pointed out that the wavelength dependence of the coefficients of linear limb darkening affect \((U - B)\) and \((B - V)\) especially strongly, hiding, in some cases, the actual variations due to spots. The effects of limb darkening decrease towards longer wavelengths, however, making redder colours more suitable for the analysis of spot temperatures.

Use of near and far infrared colours also opens possibilities to search for evidence of more than one temperature (cf. Byrne et al. 1985), arising either from different umbral temperatures from spot-to-spot, or umbral/penumbral structure in individual spots.

5. CONCLUSIONS

The \((T_{\text{eff}}, V - K)\) relationship for G and K giants is found to be in good agreement with previous relationships published in the literature (Di Benedetto 1993, McWilliam 1990, Ridgway 1980), as are those found for \((V - R)\), \((V - I)\) and \((R - I)\) (McWilliam 1990).

The \((F_{\nu}, (V - R))\) relationship is found to be closer to that by Eaton & Poe (1984) than to that one previously derived by Barnes et al. (1978) but different from either.

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