THE EVOLUTION OF MAGNETIC STRUCTURES DUE TO “MAGNETOSONIC STREAMING”

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ABSTRACT

The Faraday effect in gasdynamics called acoustic streaming and its accompanying nonlinear phenomena have analogies in plasma magnetohydrodynamics. A natural place where these effects may occur is the solar atmosphere with its strongly inhomogeneous magnetic fields concentrated in random magnetic flux tubes. Unlike acoustic streaming in the usual gasdynamics, nonlinear phenomena consisting in the generation of plasma flows by an oscillating magnetic flux tube, “magnetosonic streaming” (Ryutova 1986), is accompanied by a current drive and results in a specific evolution of magnetic structures: depending on the physical parameters of the medium a single magnetic flux tube may be either split into thinner flux tubes or dissolved diffusively into the ambient plasma. The effect of the “magneto sonic streaming,” on one hand, is an obvious candidate for the generation of mass flows at magnetic flux tube sites, and on the other hand, it plays an essential role in the evolution of magnetic structures and ultimately may determine their lifetime. The theory of magneto sonic streaming is general and can be applied to other astrophysical objects that maintain oscillatory motions and contain structured magnetic fields or magnetic domains. We review analytical results and describe the origin of the magneto sonic streaming in magnetic flux tubes due to their interaction with acoustic waves. We study numerically the regime of the “magneto sonic streaming” corresponding to splitting of a magnetic flux tube. Our computer simulation supports and extends the analytical result.

Subject headings: MHD — Sun: magnetic fields

1. INTRODUCTION

The effect of the acoustic streaming was first observed by Faraday in one of his simple trials while assisting the lectures of Professor Devi. Much later Lord Rayleigh was involved in the problem of a musical organ seemingly out of tune without an evident reason. He found that for each tube there must be a threshold with respect to the intensity of sound above which some physical effects turn on and lead to discord in the organ. Soon he found that these physical effects must be related to viscosity of an air in the vicinity of solid obstacles and gave an analytical description (Rayleigh 1884) considering the stationary sound waves in a space between two plane-parallel walls. He found that gas flows must appear due to the viscosity of medium in a thin boundary layer along the tube walls. He wrote in this paper:

"Experiments in acoustics have discovered more than one set of phenomena, apparently depending for their explanation upon existence of regular currents of air resulting from vibratory motion ... such currents, involving as they do circulation of the fluid, could not arise in the absence of friction ..."

"... The more important of the problems relates to generated over a vibrating plate ... This was traced by Faraday (1831) to the action of currents of air, rising from the plate at the place of maximum vibration, and falling back to it at the nodes."

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The acoustic streaming is connected with the viscosity of the medium and with the presence of a solid obstacle or solid boundary walls in a field of sound wave (or other kind of oscillations): near the obstacle sound waves result in the tangential motion and in additional stresses that give rise to the absorption of the energy of oscillations in a thin "acoustic" boundary layer; this energy is transformed into the energy of stationary mass flows. The effect is nonlinear and appears in the second order in the wave amplitude. Although the effect is provided by viscosity, the velocity of established stationary flows actually does not depend on the viscosity coefficient. However, the time required to set up the steady flows is inversely proportional to coefficients of viscosity.

After the studies by Lord Rayleigh, almost half a century passed before the renewal of the problem of the acoustic streaming. Rayleigh himself tried to perform different experiments to further explore these effects. He found strong air streaming as a result of the oscillation of a fork at the mouth of Helmholtz resonator, but with some contradiction in his explanation. Acoustic streaming effects were rediscovered in the experiments on piezoelectric generators and named "Quartz wind": a strong mass flow was observed in front of an oscillating surface of quartz crystal in liquid (Meissner 1926). The flows of liquid generated by an ultrasonic source turned out to be so strong that they were enough to disturb the crystal face. Soon a new manifestation of acoustic streaming was found. Andrade (1931) experimentally studied flows generated by induced standing sound waves about a circular cylinder and found four stationary vortices visible by injection of smoke. Similar results were obtained by Schlichting (1932), who performed
also the calculation of the boundary layer on the cylinder oscillating in viscous fluid. In Figure I calculating streamlines of a secondary flow and an experiment by Schlichting (1932) visualizing the four vortices generated by the oscillating (solid) cylinder are shown.

However, to understand that Andrade's and Schlichting's experiments demonstrate the same effect as quartz wind was reached only when a complete theory of acoustic streaming was built up by Eckart (1948). Since then acoustic streaming has been known also as “Eckart flows.” These studies invoked an activity exploring the acoustic streaming in a wide range of media including the cell membrane in biology (see, e.g., Nyborg 1965).

Especially rich effects like these have been found in magnetohydrodynamics. They were studied first by Ryutova (1986), who considered a nonlinear stage of the interaction of solar magnetic flux tubes with acoustic waves and found that at the site of an oscillating magnetic cylinder there appear qualitatively new effects that are absent in usual hydrodynamics:

1. First of all, while usual Reynolds stresses appear near a solid boundary, the additional magnetic stresses appear both outside and inside of a “magnetic fluid cylinder” leading to generation of plasma flow inside as well as outside magnetic structure. Of these two the inner flow plays a crucial role in the evolution of a flux tube and its dynamics:

a. The magnetic field and plasma density are leveled along the field lines of the induced flow. Depending on the ratio of the duration of wave train (e.g., p-modes in solar photosphere) to the time to establish viscous flow, the magnetic flux tube may be either split into smaller flux tubes (the case of “coherent” wave train) or dissolved diffusively into the ambient plasma (the case of convective motions or “incoherent” wave train). If in the splitting regime the same conditions are fulfilled for newborn flux tubes, each of newborn flux tubes experiences further splitting. This process of filamentation of magnetic structure proceeds until fragmented magnetic tubes meet conditions corresponding to diffusive dissolution. Outer fragments are dissolved first. Note that higher order nonlinear effects can also stop the fragmentation process.

b. The process of the filamentation determines the evolution and probably the lifetime of the magnetic structure.

2. Unlike the acoustic streaming, the “magnetosonic streaming” is accompanied by the generation of electric current: if the absorption of oscillation energy is provided mostly by one of the plasma species (electron or ion, the specific situation depends on the damping mechanism), current drive occurs along the plasma flow. This current leads to distortion and redistribution of the initial magnetic field, and thus, to changing of the equilibrium. Geometry of the current corresponds to that of the generated flow.

3. The most remarkable feature of the effect in a case of magnetic flux tube is that there are two different mechanisms leading to “magnetosonic streaming,” while for the acoustic streaming only the absorption of oscillation energy in a thin boundary layer is responsible:

a. The “magnetosonic streaming” is generated due to the action of the ponderomotive force on plasma. The effect occurs only if the force contains a nonzero vortex part. This mechanism is not connected with the direct absorption of oscillations, i.e., dissipative process, but requires radial inhomogeneity of magnetic field and plasma density across the magnetic flux tube. In hydrodynamics, this requirement would be analogous to the requirement of inhomogeneous compressibility of medium.

b. The other mechanism is similar to those in gas-dynamics and is connected with the absorption of the momentum and angular momentum of flux tube oscillations. Absorption of the momentum leads to the generation of upward and downward mass flows. Absorption of angular momentum causes the rotational mass flows across the flux tube axis.

4. In solar magnetic flux tubes there are two major non-dissipative mechanisms of the absorption of flux tube oscillations. One is the anomalous damping in the resonance point, where the phase velocity of oscillations becomes equal to local meaning of Alfvén velocity (Ryutova 1977). The second is the radiative damping of flux tube oscillations.

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Fig. 1—(a) Pattern of streamlines of the steady secondary motion in the neighborhood of an oscillating circular cylinder. (b) Secondary flow in the neighborhood of an oscillating cylinder. The camera moves with the cylinder. The metallic particles serve to render the flow visible, after Schlichting (1932). (c) Long-wavelength kink oscillations of flux tube.

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due to the radiation of secondary acoustic or MHD waves (Ryutov & Ryutova 1976). This means that even in the absence of dissipative effects, the flux tube itself is able to provide and sustain the generation of plasma streaming and current drive inside and outside it.

The paper is organized as follows. In the next two sections we describe the analytical approach to the problem based on results by Ryutova (1986). Section 2 contains the theory of the magnetosonic streaming generated by the action of ponderomotive force. In this case the major effect is the streaming across the magnetic field, which ends up either in the filamentation or/and in the diffusive dissolving of magnetic field. The lifetime of a magnetic flux tube in terms of the physical parameters of the medium is evaluated. In § 3 we describe the generation of upward mass flow and transverse convection provided by the absorption of the energy of flux tube oscillations. This mechanism can be responsible for a wide range of effects that, in principle, can be observed and can serve as a diagnostic tool. In § 4 we present results of a computer simulation that supports our analytical results. In this paper we restrict ourselves to the simulation of magnetosonic streaming effect by the action of ponderomotive force. The computational studies of the action of other mechanisms, the absorption of oscillations via different processes, are open and will be presented elsewhere. In § 5 we outline some observational evidence that may be related to the effects described above. We emphasize that, although the primary goal of these studies was to understand the dynamics of solar magnetic flux tubes interacting with the convective motions, the theory of magnetosonic streaming is general and can be applied to other astrophysical objects that maintain oscillatory motions and contain structured magnetic fields or magnetic domains.

2. ORIGIN OF MAGNETOSONIC STREAMING AND CURRENT DRIVE IN MAGNETIC FLUX TUBES

The interaction of magnetic flux tubes with convective motions and acoustic waves results in the excitation of a different kind of oscillations propagating along the flux tubes (Ryutov & Ryutova 1976; see also Ryutov & Priest 1993a, b). Among these oscillations the most important role is played by kink oscillations corresponding to the dipole mode of azimuthal wavenumber

\[ m = \pm 1. \]  

These are long-wavelength oscillations with the wavelength \( \lambda = (1/k) \) much larger than the flux tube radius \( R \):

\[ kR \ll 1. \]  

To visualize the nature of magnetosonic streaming, we restrict ourselves to the consideration of this particular mode. We use the cylindrical coordinate system with z-axis directed along the magnetic field.

Long-wavelength kink oscillations (Fig. 1c) are analogous to periodic vibration of a solid cylinder in Schlichting’s experiment shown in Figures 1a and 1b. The analogy is especially complete with the experiment of Andrade who placed a circular cylinder in a field of acoustic waves. We will show that there are conditions under which an oscillating magnetic flux tube generates four vortices similar to those in the experiments of Andrade and Schlichting. Furthermore, the force responsible for the generation of vortices outside the flux tube affects the plasma inside the flux tube and leads to generation of inner plasma flow and current. Plasma density and the magnetic field are leveled along the streamlines of the induced motions and initially smooth magnetic flux tube either splits into four independent flux tubes, or diffusively dissolves in the ambient plasma. Which of these two scenarios is realized depends on the physical parameters of the medium and magnetic flux tube. We discuss first the action of ponderomotive force and then give a qualitative picture of the absorption mechanisms.

2.1. Magnetosonic Streaming due to the Action of Ponderomotive Force

The linearized system of MHD equations describing kink oscillations of a flux tube with radial dependence of background parameters has a form

\[ \rho \frac{\partial \vec{v}}{\partial t} = -\nabla \rho + \frac{1}{4\pi} \left[ \nabla \times \vec{B} \right] \times \vec{B} \]  

\[ + \frac{1}{4\pi} \left[ \nabla \times \vec{B} \right] \times \vec{B}, \]  

\[ \frac{\partial \vec{B}}{\partial t} = \nabla \times (\vec{v} \times \vec{B}). \]  

Tildes here stand for perturbed quantities. For long-wavelength oscillations under the condition (2) incompressibility is maintained with good accuracy:

\[ \nabla \cdot \vec{v} = 0. \]  

Since in the linear regime the z-component of velocity of kink oscillations is negligibly small (flux tube performs a periodic transverse motion), velocity perturbations \( \vec{v}(\vec{r}, \vec{r}, \phi, 0) \) can be expressed through the stream function \( \psi \):

\[ \vec{v}_r = -\frac{1}{r} \frac{\partial \psi}{\partial \phi}, \quad \vec{v}_\phi = \frac{\partial \psi}{\partial r}. \]  

Obviously, the ponderomotive force appearing in the second order has only components in the plane perpendicular to the z-axis, and the problem becomes two-dimensional. In two dimensions the equation of motion for plasma in the second order has a form

\[ \rho \frac{\partial V}{\partial t} = -\nabla \left[ \rho \left( \nabla \frac{B^2(r)}{8\pi} \right) \right] + f + v\Delta V. \]  

All quantities here have a standard meaning; \( v \) is the kinematic viscosity, and \( f \) is the ponderomotive force acting on a unit volume of plasma:

\[ f = -\left( \rho \frac{d\vec{v}}{dt} \right) + \frac{1}{c} \left< \langle \vec{j} \times \vec{B} \rangle \right>. \]  

The averaging is performed over the wave period. Under the condition that the inertial term (the first term in left-hand side of eq. [7]) is small, the equation (7) becomes stationary, describing a velocity field of steady flow:

\[ \rho v\Delta V = -\nabla \left[ \rho \left( \nabla \frac{B^2(r)}{8\pi} \right) \right] + f. \]  

This equation determines the existence or nonexistence of magnetosonic streaming: the streaming can be generated by...
the oscillating flux tube only if the force is nonpotential. In other words, for the generation of secondary flow it necessary that

$$\mathbf{v} \times f \neq 0.$$  \hspace{1cm} (10)

Otherwise, the ponderomotive force leads to an insignificant redistribution of the plasma and magnetic field. We will show below that the nonzero vortex part in the ponderomotive force is provided by the inhomogeneity of magnetic field and plasma density across the flux tube. The magnitude of the force is, therefore, determined by the gradient of magnetic field and plasma density.

The magnitude of $f$ becomes large in the resonance layer where the phase velocity of kink oscillations becomes close to the local Alfvén velocity (Ryutova 1977, see also below). Although the generated flow is of the second-order effect, a strong plasma streaming can arise in a thin resonance layer.

Let us introduce the angular velocity $\Omega = \nabla \times \mathbf{v}$; equation (9) then becomes as follows:

$$\rho \nabla \times \mathbf{f} = \mathbf{v} \times \mathbf{f}.$$  \hspace{1cm} (11)

Thus, equation (11) uniquely describes the velocity field of generated steady flow.

To carry out a quantitative analysis, we solve linear equations for kink oscillations and through linear solution find the ponderomotive force.

We assume that all perturbed quantities are proportional to $\exp (i\omega t - ikz)$ and their dependence on $\phi$ has a form

$$\psi(r, \phi) = \chi(r) \cos \phi$$  \hspace{1cm} (12)

(linearly polarized wave). With equations (6) and (12) the linearized system of MHD for the kink mode is reduced to a single equation for $\chi$ (Ryutova 1977):

$$\frac{\partial}{\partial r} \left[ \frac{\rho(r) - k^2 B_0^2(r) \omega^2}{4\pi \rho} \frac{\partial \chi}{\partial r} \right] - \frac{\partial}{\partial r} \left[ \frac{\rho(r) - k^2 B_0^2(r) \omega^2}{4\pi \rho} \chi(r) \right] = 0.$$  \hspace{1cm} (13)

The important feature of equation (13) is that it has the classical form of the Rayleigh equation with a singularity: the coefficient of higher derivative at some point across the tube becomes zero. In our case this is a point where the phase velocity of kink oscillations $(\omega/k)$ becomes equal to the local Alfvén velocity. As is shown by Ryutova (1977) strong absorption of oscillations takes place at this point. The analytical solution of equation (13) and corresponding damping rate are given in the Appendix.

The solution of equation (13) defines the velocities

$$\mathbf{v}_r = \frac{\chi(r)}{r} \sin \phi, \quad \mathbf{v}_\phi = \frac{\partial \chi}{\partial r} \cos \phi,$$  \hspace{1cm} (14)

as well as linear perturbations of a magnetic field in kink oscillations:

$$\mathbf{B}_r = -\frac{k B_0 \chi(r)}{\omega} \sin \phi, \quad \mathbf{B}_\phi = -\frac{k B_0 \chi(r)}{\omega} \cos \phi.$$  \hspace{1cm} (15)

Having the solution of equation (13) with the help of equations (14) and (15), one can find the expression for ponderomotive force, equation (8). The averaged force has only stationary terms and can be written as follows:

$$f = -\langle \mathbf{E} \cdot \nabla (\rho \mathbf{v}) + \rho(\partial \mathbf{v}) \partial \rangle + \frac{1}{4\pi} \langle \mathbf{v} \times \mathbf{B} \times \mathbf{B} \rangle.$$  \hspace{1cm} (16)

After some algebra, we get the following expressions for components of the ponderomotive force:

$$f_r = \frac{\rho}{2} \cos 2\phi \left[ \chi^2 \left( \frac{x^2}{r^2} - \frac{\chi^2}{r^2} \right) - \frac{\chi^2}{r^2} \frac{\partial \rho}{\partial r} - \frac{k^2 B_0^2}{\omega^2} \right] \times \left[ \chi \left( \frac{1}{r} \frac{\partial}{\partial r} \chi \right) + \chi \frac{\partial \ln B}{\partial r} \right],$$  \hspace{1cm} (17)

and

$$f_\phi = \frac{\rho}{2} \sin 2\phi \left[ \chi \left( \frac{\chi}{r^2} + \frac{\chi}{r^2} \right) - \frac{\chi \chi}{r^2} \frac{\partial \rho}{\partial r} - \frac{k^2 B_0^2}{\omega^2} \right] \times \left[ \chi \left( \frac{1}{r} \frac{\partial}{\partial r} \chi \right) + \chi \frac{\partial \ln B}{\partial r} \right],$$  \hspace{1cm} (18)

where $\chi(r)$ is the solution of equation (13) (see Appendix) and the prime stands for derivative over $r$.

The integration of equation (11) with the expression (17) and (18) gives the velocity field of generated flow. We emphasize that the flow is generated inside the flux tube as well as outside it.

We consider only the angular dependence of ponderomotive force for simplicity:

$$f_r = G(r) \cos 2\phi, \quad f_\phi = H(r) \sin 2\phi,$$  \hspace{1cm} (19)

which immediately shows that streamlines of the field of ponderomotive force are closed in each quarter of the circle at a given radius of the magnetic flux tube. Plasma particles and frozen-in magnetic fields follow the streamlines and ultimately end up with the same geometry as the generated flows (Fig. 2). It is important to compare equation (11) with the classical equation describing the streaming generated by sound waves in a liquid (eq. [25] of Eckart 1948). There is a principal difference between the amplitude of generated flow in the case of acoustic streaming and that in the magnetosonic streaming. In fluid dynamics, where the generation of flow is provided by the viscous stresses and thus by the absorption of oscillation energy, the generating force is proportional to viscous coefficients. Both sides of

\hspace{1cm} (Fig. 2—Field of ponderomotive forces. The dashed line is an effective radius of flux tube.)

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equation (11) (which formally remains of the same form in fluid dynamics) the velocity amplitude of secondary flow turns out to be independent of the magnitude of viscous coefficients unless the bulk viscosity is nonzero. In other words, the amplitude of the steady streaming is proportional to the coefficient

\[ b = \frac{4}{3} + \frac{v'}{v}, \]

(20)

where \( v \) is the ordinary coefficient of shear viscosity measured in cm\(^2\) s\(^{-1}\) and \( v' \) is the bulk viscosity. The time required to set up the steady state depends on the fluid resistance and is inversely proportional to the coefficient of viscosity.

We will see below that a similar conclusion is valid for the magnetosonic streaming that is caused by the absorption of flux tube oscillations (but not by the action of the ponderomotive force!). In this case the amplitude of plasma flow is proportional to the ratio of the damping rate to the coefficient of plasma viscosity. The magnitude of the damping rate is determined by the specific mechanism of absorption, which can be provided by the resonance absorption, by radiation of secondary acoustic waves, or by the usual dissipative mechanisms of absorption such as viscosity, thermocconductivity, and Ohmic losses. This is discussed in § 3.

The situation is different in the case of plasma streaming generated by ponderomotive force: this force is not dissipative, and the amplitude of generated flow is no longer independent of the viscous coefficient but is inversely proportional to it. This means that, at the applicability limit, flow generated by the action of ponderomotive force can reach a significant value.

Below we discuss a qualitative picture of the effect of ponderomotive force, and thus, of the induced flow on the evolution of magnetic flux tubes with the initially smooth radial profile of background parameters.

2.2. Process of Filamentation and Diffusive Vanishing of Magnetic Flux Tubes

The evolution of a flux tube due to the generated plasma flow strongly depends on the physical parameters of the flux tube and surrounding plasma. In particular, it does on the initial size (radius) of the tube, and on the relationship between the time of establishing of viscous flows and the duration of acoustic wave trains interacting with the flux tube. One can see from equation (7) that for large enough flux tubes the viscous term is small and its evolution is determined by the inertial term \( \rho (\partial V/\partial t) \). The quantitative analysis of nonstationary magnetostrictive streaming requires a separate consideration and will be presented elsewhere.

In the present paper we give qualitative analysis of the case when the viscous term is essential and secondary streaming can be described by the averaged stationary equation (9). In the end of this section we compare these results with a rough estimate of the lifetime of a flux tube for which viscous effects are negligible.

We start from qualitative analysis of the field of forces defined by equations (17) and (18).

Denote the amplitude of flux tube displacement in kink oscillations by \( \xi = (\xi^2 + \zeta_0^2)^{1/2} (\xi = \partial \xi/\partial t) \). From equations (3) and (4) we can estimate the magnitude of magnetic field perturbation \( B \):

\[ B \sim \frac{\xi \omega}{v_A}. \]

(21)

The magnitude of ponderomotive force is of the order of \( (B^2/R) \), i.e.,

\[ f \sim \frac{B^2}{8\pi} \frac{\omega^2 \xi^2}{v_A^2} R. \]

(22)

The velocity amplitude of generated flow is estimated from equation (11):

\[ \rho v V = \frac{1}{R} \sim f \]

(23)

or

\[ V \sim \frac{f R^2}{\rho v}. \]

(24)

As discussed above, the amplitude of plasma streaming under the action of ponderomotive force unlike usual acoustic streaming depends on the coefficient of viscosity and is inversely proportional to it. This assertion is valid until the last term in equation (17) is larger than the inertial term.

The quantity \( (R^2/\nu) \) entering estimate (24) is proportional to the time of establishing of viscous flow:

\[ \tau_\nu \sim \frac{R^2}{v}, \]

(25)

and determines the time required for magnetic structure to reach a new state provided by generated plasma flows. The field of forces and, therefore, the character of generated flows, depends on the relation between \( \tau_\nu \) and the duration \( T \) of the acoustic wave train interacting with the flux tube.

We discuss here two cases, those of "coherent" and "incoherent" wave trains.

In the case of a long "coherent" wave train, when

\[ T > \tau_\nu \]

(26)

the estimate (24) becomes as follows:

\[ V \sim \frac{f \tau_\nu}{\rho}. \]

(27)

The plasma density and magnetic field are gradually equilibrated along the streamlines of induced flow and obtain the same symmetry as those of ponderomotive force. This process leads to splitting of magnetic flux into four independent flux tubes. This symmetry is a result of the specific of kink oscillations whose azimuthal number, equation (1) provides the corresponding symmetry of the force:

\[ f = i G(r) \cos 2\phi + H(r) \sin 2\phi. \]

(28)

Farther evolution of the system depends on the behavior of newborn flux tubes. Analytical description of the next stage is difficult, because newborn flux tubes form an ensemble of closely spaced structures with complicated flows inside them. They can interact with each other or/and be influenced by acoustic waves as their predecessors. If newborn flux tubes (or some of them) are influenced mostly by coherent acoustic waves, the filamentation process goes further, and those flux tubes experience further splitting. The behavior in this regime is confirmed by our computer simulation. In this case the lifetime of the magnetic structure is determined by the length of the time required to complete the
filamentation process and by the subsequent vanishing of newborn flux tubes.

In the opposite case of short “incoherent” wave train or convective motions, when

$$T < \tau_v,$$  \hspace{1cm} (29)

The generated flows have a character of stochastic motions. These motions result in a diffusive broadening of a flux tube: the plasma density and magnetic field are smoothed out gradually until the flux tube disappears.

In Figure 3 the schematic change in time of the magnetic field squared and gas kinetic pressure is shown: the decrease of magnetic field and the leveling of gas kinetic pressures inside and outside flux tube lead ultimately to complete “dissolving” of a flux tube.

The “diffusion coefficient,” defining a speed of this process, is of the order of

$$D \sim \frac{\Delta x^2}{T},$$  \hspace{1cm} (30)

where $\Delta x$ is the displacement of an element of a tube estimated as

$$\Delta x \sim VT.$$  \hspace{1cm} (31)

The velocity amplitude of generated flow in this case is of the order of

$$V \sim \frac{f}{\rho} T,$$  \hspace{1cm} (32)

and for the diffusion coefficient we have

$$D \sim \frac{\xi^4 \omega^4}{R^2} T^3.$$  \hspace{1cm} (33)

The lifetime of a magnetic flux tube can be estimated in terms of the diffusion coefficient:

$$t_D \sim \frac{R^4}{\xi^4 \omega^4 T^3}. $$  \hspace{1cm} (34)

This estimate is valid for flux tubes that interact with the acoustic waves with the period $T$ less than $\tau_v$, or with the convective motions with turnout time $T < \tau_v$. At the same time the radius of flux tubes should be small enough to satisfy the condition when the inertial term is small compared with the viscous term:

$$\rho \left| \frac{\partial V}{\partial t} \right| < |\nu \Delta V|.$$  \hspace{1cm} (35)

As mentioned above for thick flux tubes, the term $|\nu \Delta V|$ is negligibly small and their evolution is determined by the inertial term. In this case the generated flow is estimated as

$$V \sim \frac{f t_v}{\rho},$$  \hspace{1cm} (36)

where $t_v$ is the time of the establishing of the generated flow. If this time is less or comparable with the duration of acoustic wave trains, the flux tube experiences the splitting into thinner tubes. The time of splitting is of the order of $t_v$:

$$t_{\text{split}} \sim t_v \sim \frac{R}{V} \sim \frac{\rho R}{f t_{\text{split}}} $$  \hspace{1cm} (37)

or

$$t_{\text{split}} \sim \frac{\rho R}{f} \sim \frac{R}{w_x}.$$  \hspace{1cm} (38)

Transition from the case (34) to (38) occurs when these timescales become comparable, i.e., for a tube radius exceeding the estimate

$$R \sim \xi \omega T.$$  \hspace{1cm} (39)

As mentioned earlier, long-wavelength oscillations of a flux tube can be excited either by shaking its footpoint by convective motions or due to the absorption of acoustic waves (Landau resonance). Magnetosonic streaming, its strength, and its backward effect on flux tube dynamics depends on physical parameters of the flux tube and surrounding medium. As an example we give here order-of-magnitude estimate of generated flows and corresponding lifetimes for photospheric magnetic flux tubes interacting with the convective motions and acoustic wave trains.

For quantitative estimate we adopt the following typical values: the period of acoustic waves $T_p = 300$–180 s, duration of acoustic wave-train $T_{\text{wtr}} = 25$ minutes, lifetime of convective granule $T_{\text{conv}} = 8$ minutes. For the effective viscosity at magnetic flux site we use the observed limit on turbulent velocities, $\bar{v}$, and characteristic length scale of the turbulence, $\bar{l}$, consistent with observations (Title et al. 1992; Yi & Engvold 1993; Beckers 1976): $\bar{v} = 0.4$ km s$^{-1}$ and $\bar{l} = 20$ km. This gives for the effective viscosity $v = 8$ km$^{-2}$ s$^{-1}$. Let us consider flux tubes with $R = 100$ km and less.

For the chosen parameters flux tube with $R = 100$ km is subject of splitting due to the interaction with the acoustic wave-train with the characteristic time $T_{\text{wtr}} = 25$ minutes: for such tubes $\tau_v = R^2/v \approx 21$ minutes, and the necessary condition (26) is fulfilled. Obviously, the duration of the “splitting” process is by the order of the time of the establishing of viscous flows, e.g., $\tau_v$: so that in time $t_{\text{split}} \approx 21$ minutes flux tube with radius $R = 100$ km splits into four newborn flux tubes each with radius roughly of $R = 50$ km.

To estimate the amplitude of corresponding flows generated across the flux tube, we use equation (24), which with equation (22) becomes as follows:

$$V \approx \frac{1}{2} \frac{\xi^2 \omega^2}{\nu} R.$$  \hspace{1cm} (40)

We assume that transverse displacement of a flux tube $\xi$ is not less than $R/2\pi$. This assumption may be justified from the condition (39) where, for periodic motions replacing $\omega T$ by $2\pi$ we come to above estimate. For the velocity of transverse displacement of a flux tube, $\bar{v} = \omega \xi$, this estimate gives

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Fig. 3.—Schematic change in time of squared magnetic field and gas kinetic pressure: the decreasing of magnetic field and the leveling of gas kinetic pressures inside and outside flux tube ends up by diffusive dissolving of flux tube.
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\[ \dot{v} = \omega \xi = R/T_r. \]

And for tubes of radius \( R = 100 \) km we have \( \dot{v} = 0.33 \) \( \text{km s}^{-1} \), which is consistent with the observed values. From equation (40) we find the amplitude of generated flows: \( V = 0.7 \) \( \text{km s}^{-1} \). Newborn flux tubes having radius 50 km may experience further splitting; this time either again due to the interaction with the acoustic wave trains or, more likely, due to the action of convective motions: for these flux tubes \( \tau_c = 5.21 \) minutes \( < T_{\text{conv}}. \)

Duration of this process is of the order of 5.21 minutes. The amplitude of generated flows is \( V \approx 0.1 \) \( \text{km s}^{-1} \). In the third generation, flux tubes whose radius is now about 25 km are too thin to experience farther splitting and should vanish diffusively in time \( \approx 1.3 \) minutes. The total lifetime of a flux tube with \( R = 100 \) km is thus roughly 27 minutes. It is important to note, that the condition (39) determines the lower limit of flux tube size (its radius) below which flux tubes do not experience fragmentation and vanish diffusively. To estimate the “critical” radius given by equation (39) note that “diffusive” regime requires the condition (29).

This means that at given a timescale \( T \), flux tubes with radius \( R \leq (v T)^{1/2} \) are subject to diffusive vanishing with \( \tau_p \approx T \) (with \( T \leq \tau_c \)). If we assume that the minimum “effective” timescale is that of 3 minute oscillations, the minimum “critical” radius is then \( R_{\text{min}} = (v T)^{1/2} = 38 \) km. Of course, this estimate is the order-of-magnitude one, and the value of critical radius may be smaller or larger than 40 km. For example, convective motions also can terminate the fragmentation process in flux tubes with radius about 60–70 km. In this case the flux tube vanishes diffusively in time of \( \approx 7–9 \) minutes.

The estimated timescales, the range of velocities and the dynamics of photospheric flux tubes are in a good agreement with the observed regularities found in subarcsecond measurements (Berger et al. 1994). However, we should bear in mind that the fragmentation process even in the case of photospheric magnetic flux tubes can end up with quite vigorous nonlinear dynamics. That is, diffusive vanishing is only one possibility of the final stage of flux tube dynamics. Given that generated flows are accompanied by current drive with quite complicated geometry. (see, e.g., Fig. 7 below), strong distortion of the topology of magnetic field takes place, and the situation may appear that is favorable for local reconnections. This process can manifest itself as local brightening of small scale flux tubes with signature similar to bright points. Note that according to the observational data (Berger et al. 1994) bright points represent only 50% of magnetic flux. These effects require separate consideration and will be presented elsewhere.

It is important to note that plasma flows are also generated outside the flux tube. In a case of kink oscillations under the conditions close to equation (26) outside a flux tube, four stationary vortices are generated. These vortices, shown in Figure 2 by dotted lines, are similar to those generated in the experiments of Andrade and Schlichting (Fig. 1). But the mechanism of generation of mass flows described above is connected with the action of ponderomotive force, which is obviously absent in the experiment by Holtzmark et al. (1954). In this experiment, as well as in any manifestation of acoustic streaming, the only mechanism leading to the generation of flows is the absorption of the energy of oscillation in a thin boundary layer. As discussed above, along the action of ponderomotive force, in the case of oscillating magnetic flux the mechanism of generation of mass flows connected with absorption of oscillation energy also exists, and in some aspects has no analogy in hydrodynamics. We discuss qualitative features of this mechanism in the next section.

3. GENERATION OF MASS FLOWS DUE TO THE ABSORPTION MECHANISMS

The origin of acoustic streaming in usual hydrodynamics is completely determined by the existence of a thin boundary layer near the solid obstacles where the energy of sound waves (or other kind of oscillations) is absorbed due to the usual viscosity. The absorbed energy is converted into the secondary mass flow. The same mechanism works in the case of an oscillating magnetic cylinder with richer additional effects for the MHD case. Along with the usual dissipative effects that can cause magnetosonic streaming, the feature of magnetic flux tube embedded in moving solar atmosphere is such that the energy of flux tube oscillations is damped out due to the specific nondissipative damping mechanisms. These are the following:

1. Radiative damping when an oscillating flux tube gives off its energy through emission of secondary acoustic or MHD waves (Ryutov & Ryutova 1976; Ryutova & Priest 1993a, b); and

2. Anomalous damping in the resonance layer, where the phase velocity of flux tube oscillations becomes close to the local meaning of Alfvén velocity (Ryutova 1977).

We concentrate here on the second mechanism of anomalous damping of kink oscillations in the resonant layer. We will show that in this case the effect of magnetosonic streaming is especially strong. Physically, the nature of the absorption is the pumping of oscillation energy into the resonance layer where the dissipation occurs. The whole momentum or angular momentum of flux tube oscillations is transferred to the plasma in a thin layer, causing a strong mass flow.

Formally, the expression for the force inducing the secondary streaming has the same form as equation (8), but now the terms that are directly connected with the absorption are taken into account.

Absorption of the momentum of oscillations leads to the generation of upward or downward mass flows. Absorption of angular momentum of oscillations causes the rotational mass flows in a plane perpendicular to tube axis. Nonzero angular momentum can be transmitted, for example, by circularly polarized kink oscillations.

We estimate a force responsible for plasma streaming due to the absorption of the momentum of kink oscillations in the resonance layer.

In the Appendix we describe the solution of equation (13) obtained by Ryutova (1977) for a simplified dependence of plasma density and magnetic field on the tube radius. The corresponding dispersion relation has a form

\[ \omega \approx \frac{kB}{\sqrt{4\pi(\rho + \rho_e)}} \left( 1 + \frac{i\pi}{4} \frac{\rho}{\rho_e} \frac{1}{R} \right). \] (41)

One can see that it contains a large imaginary part corresponding to strong absorption of kink oscillations in the resonance layer. Within the applicability limit, when \( l \) is of the order of radius \( R \) (smooth radial profile of a flux tube),
damping rate becomes comparable with eigenfrequency 
\( \gamma = \text{Im} \omega \sim \text{Re} \omega \).

From equation (41) we have for the spatial damping rate 
\( \alpha = \text{Im} k \):

\[
\alpha = \frac{\pi \omega}{4} \frac{\rho}{v_A} \sqrt{\frac{1}{\rho + \rho_e}} R^2. \tag{42}
\]

The energy of kink oscillations per unit length of a tube is 
\( W = \rho \tilde{v}^2 \pi R^2 \); \( \tilde{v} \) is the magnitude of linear velocity perturbation (see eq. [6]). The momentum of oscillation is

\[
\mathcal{P} = \frac{k_z}{\omega} W, \tag{43}
\]

where \( k_z \) is \( z \)-component of wavevector. The force \( F_z \) appearing due to the absorption of momentum \( \mathcal{P} \) is proportional to

\[
F_z \sim \alpha \frac{\omega}{k_z} \mathcal{P}. \tag{44}
\]

The force acting on a unit volume of a tube is then

\[
f_z = \frac{F_z}{2\pi RL}. \tag{45}
\]

With \( \alpha \) given as equation (42) from equations (44)–(45) we have

\[
f_z \approx \frac{\pi \omega}{8} \frac{\rho}{v_A} \sqrt{\frac{1}{\rho + \rho_e}} \rho \tilde{v}^2 . \tag{46}
\]

As we see, the “model” parameter \( (l/R) \) does not enter in the final expression for the force responsible for the generation of mass flows. The magnitude of generated flows that is proportional to this force is completely determined by the basic physical parameters.

Below we estimate roughly the velocities of flows generated along magnetic flux tubes for two sets of physical parameters, corresponding to (1) chromosphere and (2) low corona. For these estimates it is convenient to rewrite the expression for the force (eq. [46]) through plasma beta. The pressure equilibrium condition gives that \( \rho/\rho_e = \beta/(1 + \beta) \), and equation (46) becomes as

\[
f_z \approx \frac{\pi \omega}{8} \frac{\rho}{v_A} \sqrt{\frac{1}{1 + 2\beta}} \rho \tilde{v}^2 . \tag{47}
\]

The amplitude of generated flows, equation (24), is then

\[
V_z \approx \frac{\pi \omega}{8} \frac{\rho}{v_A} \sqrt{\frac{1}{1 + 2\beta}} \tilde{v}^2 R^2/v. \tag{48}
\]

In chromosphere and corona in the magnetic dominant case \( (\beta \ll 1) \) viscosity is estimated by Braginsky’s kinetic coefficients. Since we are considering the flows generated along magnetic field the viscosity in equation (48) is \( \nu = \eta_0/\rho \), where \( \eta_0 = 0.96n k T_i \tau_i \), or (see Spitzer’s coefficient for kinematic viscosity),

\[
\nu = 2.21 \times 10^{-15} \frac{T^{3/2}}{\ln \Lambda} \text{ g cm}^{-1} \text{s}^{-1}. \tag{49}
\]

For quantitative estimate we use following parameters. For chromosphere: temperature \( T = 10^4 \text{ K} \), Coulomb logarithm \( \ln \Lambda = \ln 10 = 2.3 \), density \( n = 10^{11} \text{ cm}^{-3} \) \( (\rho = 1.67 \times 10^{-13} \text{ g cm}^{-3}) \), magnetic field \( B = 50 \text{ G} \), Alfven velocity is then \( v_A = 345 \text{ km s}^{-1} \), characteristic radius of magnetic structure \( R = 100 \text{ km} \), and wave period \( T_p = 180 \text{ s} \).

For low corona: temperature \( T = 10^5 \text{ K} \), \( \Lambda = 20 \), density \( n = 3 \times 10^9 \text{ cm}^{-3} \) \( (\rho = 5 \times 10^{-15} \text{ g cm}^{-3}) \), weaker magnetic field of \( B = 10 \text{ G} \), Alfven velocity \( v_A = 399 \text{ km s}^{-1} \), and wider structure of radius \( R = 200 \text{ km} \); for the wave period we accept a typical period of Alfven waves \( T_p = 100 \text{ s} \). We assume that the velocity of transverse displacement of a flux tube in chromosphere and corona is proportional to the radius of the structure and wave frequency. This gives in chromosphere \( \tilde{v} \approx 3 \text{ km s}^{-1} \), and in corona \( \tilde{v} \approx 12 \text{ km s}^{-1} \). These values are quite consistent with the estimate obtained, for example, for higher corona energetic consideration (see Parker 1991).

For chosen set of parameters equation (48) gives finally the following estimates for the amplitude of mass flows generated along the flux tube: in chromosphere \( V_z \approx 23 \text{ km s}^{-1} \) and in corona \( V_z \approx 2.5 \text{ km s}^{-1} \). These values seem quite reasonable: velocity of flows generated in chromosphere are close to those observed in spicules, and in the application to chromospheric magnetic structures “magneto sonic streaming” can be considered as a promising candidate for more detailed studies. Much less amplitude of flows generated in coronal structures is also consistent with observations showing that high-velocity and explosive events observed in the transition zone are not usually seen in the overlying corona (see, e.g., Dere 1994). Note, that high-velocity and explosive events in the transition zone require consideration of nonstationary effects in the presence of dissipative instabilities and cannot be described in the frame of the present theory (see Ryutova 1988, and also the Introduction in Ryutova & Habbal 1995).

In the same way as above we can estimate the magnitude of the force appearing due to the absorption of angular momentum of oscillations, which is responsible for the generation of azimuthal flows and currents. The angular momentum directed along the tube axis is estimated as

\[
M_z \approx \frac{W}{\omega}. \tag{50}
\]

The angular momentum absorbed per unit length of a tube is proportional to \( (ak_z/\omega)M_z \). An estimate for force acting on plasma in the azimuthal direction may be obtained from the following relation:

\[
2\pi Rl f_\phi \approx \frac{ak_z}{\omega} M_z \omega, \tag{51}
\]

or

\[
f_\phi \approx \frac{ak_z}{\omega} \frac{W}{2\pi RL}, \tag{52}
\]

and we obtain the same order of magnitude of force as in the case of the absorption of the momentum of oscillation. As mentioned above, if the oscillation energy is mostly absorbed by one of the plasma components (electron or ion), mass flows are accompanied by current drive. In this case the imaginary part of equation (41) contains additional
terms connected with the specific damping mechanism. If the absorption is provided mostly in the electron component of plasma the damping may be a result of (1) electron-ion collision with collisional damping rate
\[ \delta_e \sim \frac{v_{ei} v_A^2}{k_e v_{Te}^2}, \]  
(53)

and (2) Landau damping with
\[ \delta_L \sim \frac{\omega}{k_e v_{Te}} \]  
(54)

\((v_{Te})\) is the electron thermal speed). For example, in a case of Landau damping for spatial damping rate \(\alpha_i\) we have
\[ \alpha_i \sim \frac{\pi r_i^2 T_e}{l^2 T_i v_{Te}} \]  
(55)

Here \(r_i\) is the ion Larmor radius and \(T_e\) and \(T_i\) are the electron and ion temperatures, respectively.

Let us estimate, for example, the magnitude of the azimuthal current generated by the absorption of angular momentum. The force (eq. [52]) causes the motion of electrons with the velocity
\[ u_{\phi} \sim \frac{f_\phi}{n_e m_e}, \]  
(56)

and thus, leads to the appearance of azimuthal current \(j_\phi = e n_e u_{\phi}\) of the order of
\[ j_\phi \sim \frac{\omega}{v_{ei}} \frac{\rho e^2 R}{2 l}. \]  
(57)

In the same way we can estimate the currents generated along the magnetic field due to the action of the \(z\)-component of the force (eq. [46]).

Although the generated currents are of the second-order effect, they play an essential role in the dynamics of the magnetic flux tube leading to the distortion of the initial magnetic field and changing its equilibrium conditions.

4. NUMERICAL SIMULATION

In the present paper we restrict ourselves to the regime corresponding to the filamentation process in our numerical simulation of magnetosonic streaming and current drive.

4.1. Basic Equations and Numerical Methods

We made the following assumptions for our numerical simulations: (1) the medium is an ideal gas, (2) the gas is a polytropic of index \(\gamma\), (3) the magnetic field is frozen in the gas and is vertical, (4) gravity is neglected. Cartesian coordinates \((x, y, z)\) are adopted so that the \(z\)-direction is parallel to the magnetic field. It is assumed that the evolution is two-dimensional with \(\partial/\partial z = 0\). Thus, the basic equations are as follows:
\[ \frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x} (\rho V_x) + \frac{\partial}{\partial y} (\rho V_y) = 0, \]  
(58)

\[ \frac{\partial}{\partial t} (\rho V_x) + \frac{\partial}{\partial y} (\rho V_y V_x) + \frac{\partial}{\partial x} \left( \rho V_x^2 + p + \frac{B_x^2}{8\pi} \right) = \delta_x, \]  
(59)

\[ \frac{\partial}{\partial t} (\rho V_y) + \frac{\partial}{\partial x} (\rho V_x V_y) + \frac{\partial}{\partial y} (\rho V_y^2 + p + \frac{B_y^2}{8\pi}) = \delta_y, \]  
(60)

\[ \frac{\partial B_x}{\partial t} + \frac{\partial}{\partial x} (V_x B_x) + \frac{\partial}{\partial y} (V_y B_x) = 0, \]  
(61)

\[ \frac{\partial}{\partial t} \left[ \frac{p}{\gamma - 1} + \frac{1}{2} \rho (V_x^2 + V_y^2) + \frac{B_x^2}{8\pi} \right] \]
\[ + \frac{\partial}{\partial x} \left[ \frac{\gamma}{\gamma - 1} p V_x + \frac{1}{2} \rho V_x (V_x^2 + V_y^2) + \frac{B_x}{4\pi} V_x \right] \]
\[ + \frac{\partial}{\partial y} \left[ \frac{\gamma}{\gamma - 1} p V_y + \frac{1}{2} \rho V_y (V_x^2 + V_y^2) + \frac{B_y}{4\pi} V_y \right] \]
\[ = \delta_x V_x + \delta_y V_y, \]  
(62)

where \(\delta_x\) and \(\delta_y\) describe the interaction of a flux tube with acoustic waves. All other symbols have their standard meaning.

We consider a plasma with temperature \(T\) and assume the gas to be a polytrope of index \(\gamma = 1.5\). We assume that the magnetic field is parallel to the \(z\)-axis \([B = (0, 0, B_z(x, y))]\). The distribution of the initial magnetic field strength \(B_z(x, y)\) is given by
\[ B_z(x, y) = \left[ 8\pi p(x, y)/\beta(x, y) \right]^{1/2}, \]  
(63)

where
\[ \beta(x, y) = \beta_0 f(x, y), \]  
(64)

\[ f(x, y) = \frac{1}{2} \left[ -\tanh \left( \frac{r - r_0}{w_0} \right) + 1 \right], \]  
(65)

\[ r = [(x - x_0)^2 + (y - y_0)^2]^{1/2}, \]  
(66)

and where \(\beta_0\) is the plasma beta at the center of the flux tube, \(x_0\) and \(y_0\) are the coordinates of the center of the flux tube, \(r_0\) is its radius, and \(w_0\) is the width of the boundary layer between the flux tube and the unmagnetized plasma. In our calculations we used \(\beta_0 = 0.2\) and \(w_0 = 1.0H\), where \(H\) is the characteristic length scale.

The initial density and pressure distributions are calculated by using equations (63)–(66), the equation of the state, and the equation of magnetostatic equilibrium
\[ p(x, y) + \frac{B_z^2}{8\pi} = \text{constant}. \]  
(67)

An incompressible velocity field is initially imposed on the magnetic flux tube. We assume a velocity potential of the form
\[ \Phi(x, y) = A \sin \pi x \sin \pi y, \]  
(68)

and consider the resulting velocity field
\[ V_x = -f(x, y) \frac{\partial \Phi}{\partial y}, \quad V_y = -f(x, y) \frac{\partial \Phi}{\partial x}, \]  
(69)

where \(f(x, y)\) is given by equation (65).

Time-dependent acoustic perturbations are imposed on the magnetic flux tube via interaction terms in the equations
of motion (eqs. [59] and [60]) of the following form:

$$
\delta_x = Af(x, y) \cdot \sin \omega t, \quad \delta_y = Af(x, y) \cdot \cos \omega t,
$$

(70)

where $A$ is the amplitude of the perturbation and $\omega$ is its frequency.

We assume free boundaries for $x = 0$, $x = X_{\text{max}}$, $y = 0$, and $y = Y_{\text{max}}$. Equations (58)–(65) are nondimensionalized by using a characteristic length scale $H$, the sound speed $c_s$, and the density $\rho_0$ outside the flux tube. Equations (58)–(65) are solved numerically by using a modified Lax-Wendroff scheme (Rubin & Burstein 1967) with an artificial viscosity according to Richtmyer & Morton (1967, chap. 13). The tests and accuracy of such an MHD code have been described by Shibata (1983, Shibata & Uchida 1985, Matsumoto et al. 1988). The mesh sizes are $\Delta x = X_{\text{max}}/(N_x - 1)$ and $\Delta y = Y_{\text{max}}/(N_y - 1)$, where $N_x$ and $N_y$ are the numbers of mesh points in the $x$- and $y$-directions. The total number of mesh points is $(N_x \times N_y) = (203 \times 203)$, the total area is $(X_{\text{max}} \times Y_{\text{max}}) = (20 \times 20)$ in a typical model in units of the characteristic length scale.

4.2. Numerical Results

As mentioned above, in our numerical simulation we restrict ourselves to the regime of the filamentation process in the interaction of magnetic flux tube with acoustic waves. The parameters of simulation models presented in this paper are summarized in Table 1. Here $\delta_x$ and $\delta_y$ are the interaction terms, $r_0$ is the initial radius of the flux tube, $\omega$ is the temporal frequency of the interaction terms, and $q$ is the artificial viscosity parameter. Models 1–3 represent the case where the interaction term has only an $x$-component. In models 2 and 3 we study the dependence of the evolution of the flux tube on the frequency of the interaction term, the initial radius $r_0$ of the flux tube, and the magnitude $q$ of the artificial viscosity, respectively. In model 4 we allow a $y$-component of the interaction term as well.

Figure 4 shows the initial ($t = 1$) and final ($t = 52$) states of the contours lines of the magnetic field $B$, the density distribution (log $\rho$), the velocity field $V = (V_x, V_y)$, and the current density $j = (j_x, j_y)$ for model 1. Initially, the vertical magnetic field with a smooth radial profile is concentrated in a flux tube with radius $r_0 = 2H$, the density inside the flux tube (also having a smooth radial dependence) is lower than outside for pressure equilibrium (isothermal plasma), and the velocity field shows the potential, incompressible flow imposed on magnetic flux tube according to equation (69). In the course of the evolution, the magnetic flux tube breaks down into four different magnetic structures ($t = 52$). The magnetic field of the final configuration is distributed into two major and two minor, nearly symmetrical, tubes. The density distribution and generated current density show the filamentation process as well. It is remarkable that magnetic field lines and plasma density lines in the "final" state are no longer collinear—a fact important for further evolution of a conglomerate of newborn flux tubes, as well as apparently for observed properties of magnetic structures.

In Figure 5 we show the final state of model 2. The frequency of acoustic perturbation is twice as high as in model 1. We again witness the splitting of the initial flux tube, but this time into two identical components, while in a previous case along two major newborn flux tubes there were two weaker elements. The appearance of two major flux tubes in both cases is provided by the fact that interaction force has only one component; the lower frequency (longer duration time) in a previous case provided the formation of two smaller satellites.

In model 3 we study the dependence on the artificial viscosity in our numerical code (Figs. 5 and 6). The simulations show that within the regime of $q = 3$ to $q = 6$ the evolution is not sensitive to the magnitude of the artificial viscosity, except maybe for the fact that higher viscosity, as predicted in theory, leads to a more pronounced effect of filamentation.

Finally, we investigate the case where the interaction terms have nonzero $x$- and $y$-components (model 4). This case is close to the theoretical analysis with the difference that theory assumes both components of the force equal to each other. The advantage of the numerical result compared with the analytical one is that here the evolution of the flux tube can be traced further into later stages: The filamentation process does not stop at the formation of the first four newborn flux tubes but goes further, showing the formation of a complex structure with many magnetic elements of different strength and size (Fig. 7).

5. SUMMARY

The "magnetosonic streaming" and its accompanying dynamic phenomena are general and should play an important role in various astrophysical objects where magnetic fields having a pronounced filamentary or domain structure are under the action of wave or convective motions. Filamentary structure of the star atmospheric plasma, for example, was one of the motivations of the theory of the filamentary construct of plasmas as developed by Kinney et al. (1994). Though it was found analytically with the analogy of the Faraday acoustic streaming effect (Ryutov 1986), the searching of this effect was stimulated by the observational evidence in solar atmosphere: photospheric magnetic flux tubes seen predominantly at intergranular boundaries experience persistent action of convective motions and acoustic wave trains. It is shown that at a nonlinear stage plasma motions result in the strong distortion of magnetic field topology, and thus, in changing the equilibrium conditions in medium. Depending on physical parameters of medium the interaction of flux tubes with plasma motions result in various effects, and, in particular, the generation of plasma flows and current drive, which, in turn, lead to the specific evolution of magnetic structures that manifests itself in clear morphological effect and evidently can be observed.

Prediction of theory is broken down into two major scenarios: either a magnetic flux tube experiences filamentation.
Fig. 4.—Results for model 1. Shown are (a) the contour lines of $B_0$ and (b) the density contours (log $\rho$) of undisturbed flux tube. (c)–(f) Results at $t = 52$: (c) the contour lines of magnetic field, (d) the density contours, (e) the current density $J = (J_x, J_y)$, and (f) the velocity field $V = (V_x, V_y)$. Total illustrated area is $(20 \times 20)$ in units of $H$. The contour level step width is 1.5 for (a) in the unit of linear scale and 0.2 for (b) in the unit of logarithmic scale. The maximal values of the current density and velocity vectors shown in panels (e) and (f) are in units of $(pc_\perp /H)^{1/2}$ and sound speed $c_s$, respectively. Numbers on top of each frame in (a) represent the time in units of $H/c_s$. 
tion process splitting into four newborn flux tubes or it vanishes diffusively in the ambient plasma.

Our computer simulation of the filamentation regime supports the analytical results and extends it to the stage when fragmentation process goes further: newborn flux tubes experience further splitting, thus forming a complex conglomerate of thin fluxes with quite uneven distribution of magnetic field and plasma density inside them.

Our next steps are visible in two major directions:

1. Further theoretical and computational studies that include the presence of background convective motions. We expect here several effects: rearrangement of newborn magnetic structures during their lifetime along the intergranular boundaries; coalescence of several flux tubes; explosive events in some particular flux tubes (Ryutova 1988).
2. Detailed comparison of theoretical results described in this paper with observed regularities in solar photosphere.

High-resolution observations of recent years (Title, Tarbell, & Topka 1987; Title et al. 1992; Muller 1990; Yi & Engvold 1993; Muller et al. 1994; Muller 1994, and references therein) gave us an outstanding possibility to learn about the structure and behavior of magnetic fluxes, complex velocity fields, and their evolution in time at the photospheric level. For the time being a rapid success in these observations is taking place. Multispectral images of magnetic flux tubes and time-sequence observations by Title et al. (1994) and Berger et al. (1994) revealed important features of the dynamics of solar atmosphere, which the authors summarize shortly as follows:

The images cover $60'' \times 80''$ with typical spatial resolution of $0.3''$ (218 km). Flux tubes appear as very high contrast subarcsecond structures located predominantly in intergranular lanes.

The flux tubes vary in shape from circular points with mean diameters of $0.35''$ (250 km) to elongated filaments...
with lengths of the order of arcseconds, which in some cases are resolvable as chains of circular flux tubes.

Thin magnetic flux tubes are associated with intergranular bright points whose intensity structure consists of a bright core with a dark surround. The diameter of the core is supposed to be as small as 0.15 (~100 km). Along the fine bright points, many examples of magnetic flux tubes over 300 km in diameter that are bright in continuum were found.

The dominant mode of flux tube evolution is fission of larger structures into small flux tubes and fusion of small flux tubes into conglomerate structures.

Lifetimes of free small flux tubes are on the order of 10 minutes. Large conglomerate structures can exhibit lifetimes on the order of hours.

As it is shown in § 2 qualitative agreement of the theoretical predictions with the observed regularities as well as order-of-magnitude quantitative estimates are quite promising. The further application of theoretical results for the specific observational data and possible infer of physical parameters will be presented elsewhere.

3. In § 3 we discussed the effect of "magnetosonic streaming" as a mechanism of generation of mass flows and currents along the magnetic structures. Quantitative estimates of the effect show that in the present state theory can be applied only for understanding the range of flow velocities in chromosphere and, in some extent, in low corona. These estimates are only preliminary and show that for the understanding of high-velocity and explosive events in transition zone and low corona it is necessary to consider nonstationary effects and dissipative instabilities.

4. Another important problem connected with the effect of magnetosonic streaming is the possibility of the local reconnections that may occur due to the strong distortion of the topology of magnetic field. This process can manifest itself as local brightening of small scale flux tubes with signature similar to bright points. According the observational data, strong local brightening associated with
structured magnetic fields (possibly “bright points”) is observed throughout the solar atmosphere from photospheric and chromospheric network bright points up to X-ray bright points in corona. They are observed in unipolar magnetic flux tubes as well as in small bipolar structures. It is especially interesting to study the impact of magnetosonic streaming in the presence of a magnetic field outside a given magnetic flux tube. Obviously, in this case the probability of local reconnections increases, and these effects may be considered as a possible explanation of bright points at various heights of solar atmosphere.

As mentioned above, the very idea of searching the effect of magnetosonic streaming came from the observational evidences in solar photosphere. However, the theory is quite general and can be applied to any astrophysical objects that can maintain oscillatory motions and contain structured magnetic fields or magnetic domains.

The computations were performed on the Cray Y-MP8/864 at the Computer Center of the University of Texas at Austin. This work was supported in part by the National Science Foundation grant ATM 88-11128, and US Department of Energy DE-FG05-80ET53088. One of us (M. R.) is grateful to Alan Title and Tom Berger for fruitful discussions.
APPENDIX A

RESONANT ABSORPTION OF KINK OSCILLATIONS

For long-wavelength kink oscillations, $kR \ll 1$, plasma can be considered as incompressible, $\nabla \cdot \mathbf{v} = 0$, and the motions of a flux tube will be a plane one with $\delta z = 0$. In this case we can introduce the stream function $\psi$ (see eq. [6]) and reduce the linearized set of MHD equations to a single equation for $\psi(r, \phi) = \chi(r) \exp (i\omega t - ikz - i\phi)$:

$$
\frac{\partial}{\partial r} \left[ \rho(r) - \frac{k^2 B^2(r)}{4\pi \omega^2} \right] r \frac{\partial \chi}{\partial r} - \left[ \rho(r) - \frac{k^2 B^2(r)}{4\pi \omega^2} \right] \chi(r) = 0 .
$$

(71)

This equation was obtained by Ryutova (1977) and solved for a simplified profile of flux tube parameters with a narrow transition layer shown in Figure 8. The dashed line is the magnetic pressure $[B^2(r)/8\pi]$ as a function of radius. The solid line is the plasma density. Both functions are assumed to be linear in the interval $[r, R + l]$ and $l \ll R$. The solution of equation (71) is the first-order Bessel function in the region $r < R$, and a first-order Hankel function outside the flux tube at $r > R + l$ (see also Ryutov & Ryutova 1976). So that in the long-wavelength approximation we have for $r < R$

$$
\chi = Ar ,
$$

(72)

and for $r > R + l$

$$
\chi = \frac{B}{r} .
$$

(73)

To find a solution in the transition region it is convenient to introduce a new variable $z = (r - R)/l$; $0 < z1$. Taking into account the smallness of the parameter $(l/R)$, we can rewrite equation (71) as follows:

$$
\frac{d}{dz} (z - z_0 - i\varepsilon) \frac{d\chi}{dz} - \frac{l^2}{R^2} (z - z_0 - i\varepsilon) \chi = 0 ,
$$

(74)

where

$$
z_0 = \left( \frac{k^2 B^2}{4\pi \omega^2} - \rho_i \right) \left( \frac{k^2 B^2}{4\pi \omega^2} + \rho_e - \rho_i \right)^{-1} .
$$

(75)

It is assumed, of course, that $0 < z_0 < 1$, otherwise the Alfvén resonance does not exist.

The equation (74) has a single-valued solution in the complex plane $z$ with a cut along the line $\text{Im} z = i\varepsilon$; $-\infty \leq \text{Re} z \leq z_0$ (see Fig. 8b). This solution can be expressed through Bessel functions. The smallness of the parameter $(l/R)$ allows us to represent it in the form

$$
\chi = C + D \ln (z - z_0 - i\varepsilon) .
$$

(76)

![Figure 8](image-url)

Fig. 8.—(a) Model of magnetic flux tube: magnetic field and plasma density are homogeneous almost across the whole radius and depend on radius linearly in a thin diffused boundary layer. (b) Complex plane $z = z(\varepsilon, k)$
Matching solutions (72) and (76) and their derivatives $aX/dr$ at $r = R$ and equations (73) and (76) and their derivatives at $r = R + l$ yields to the dispersion relation:

$$\ln \frac{1 - z_0}{z_0} + \frac{R}{l} \left( \frac{1}{1 - z_0} - \frac{1}{z_0} \right) + i\pi = 0.$$  \hspace{1cm} (77)

From this expression we find the frequency:

$$\omega = \frac{k B}{\sqrt{4\pi(\rho_i + \rho_s)}} \left( 1 + \frac{i\pi}{4} \frac{\rho_i}{\rho_i + \rho_s} \frac{l}{R} \right).$$  \hspace{1cm} (78)

We have chosen here the logarithm branch single-valued in the region with the cut along $\text{Im } z = i\epsilon$, mentioned above. This result shows that near the point where phase velocity of kink oscillations, $\omega/k = B/[4\pi(\rho_i + \rho_s)]^{1/2}$, becomes equal to local meaning of Alfvén velocity, strong absorption of kink oscillations occurs. For smooth enough flux tubes at the applicability limit when $l$ is of the order of $R$, we have even $\text{Im } \omega \sim \text{Re } \omega$. We remind the reader here that physically the nature of absorption is due to the pumping of the oscillation energy into resonance point where the dissipation occurs. The resonance absorption, by itself, does not transform the energy of oscillation to plasma heating, for example. It just results in the concentration of the initially smooth eigenfunction near the resonance point, or, in other words, in a conversion of oscillation energy to the Alfvén continuum. Then, the usual dissipation mechanisms turn on, and strongly oscillating perturbation damps out.

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