DISCRETE HIGH-FREQUENCY $p$-MODES

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ABSTRACT

Observations report that the $p$-mode frequencies change with the solar activity cycle. Over the frequency range 0 to 8.5 mHz, shifts can be either positive or negative, appearing to exhibit a wave-like behavior with downturn occurring at $\sim 4.0$ mHz and upturn at $\sim 5.5$ mHz. A simple polytrope convection zone model overlaid with an isothermal chromospheric atmosphere threaded by a uniform horizontal magnetic field demonstrates frequency shifts that are qualitatively similar to those observed for both high and low frequencies, with shifts being a consequence of simultaneous increases in chromospheric field strength and temperature.

Subject headings: Sun: activity — Sun: magnetic fields — Sun: oscillations

1. INTRODUCTION

The discovery that $p$-modes change over the solar cycle offers the possibility of a new diagnostic signature of that cycle, complementary to the more familiar signatures such as sunspot numbers and the butterfly diagram. The first evidence that $p$-mode frequencies were cycle dependent arose from the Solar Maximum Mission satellite measurements of Woodard & Noyes (1985), which indicated a frequency shift of $-0.4 \mu$Hz in low-degree $p$-modes from 1980 (solar maximum) to 1984 (approaching solar minimum). The effect was without convincing theoretical explanation. Roberts & Campbell (1986) argued that if such a shift was brought about by changes in the Sun's magnetic field stored at the base of the convection zone, then that field must be of megagauss strength and so considerably stronger than had hitherto been suggested. Other, quite separate, theoretical considerations (e.g., Dziembowski & Goode 1989) suggested similarly high field strengths. If such strong fields do arise, then the operation of a dynamo, against the tendency for the field to be strongly buoyant, becomes problematic.

In the mid 1980s, the observational evidence for frequency shifts was not compelling, as it was based upon a limited data set. However, recent analyses of extensive data sets have dispelled that earlier uncertainty: $p$-modes vary over the solar cycle. This variation has been reported for both low-degree (Elsworth et al. 1990; Anguera Gubau et al. 1992; see the review by Pallé 1994) and high-degree modes (Liberth & Woodard 1990; Woodard & Libbrecht 1991; Woodard et al. 1991). Moreover, the variation at low degree is similar to that at high degree (Elsworth et al. 1994), which points toward a surface effect rather than an effect at depth as being primarily responsible for the frequency shifts (see also Libbrecht & Woodard 1990). Added to this, the shifts are correlated with surface magnetic activity, as indicated in magnetogram measurements and in activity latitudes (Woodard et al. 1991; Bachmann & Brown 1993).

New observational evidence suggests that shifts also occur in high-frequency modes (see, for example, Ronan et al. 1994a, b). Previous measurements of frequency shifts are confined to modes of frequency $\nu$ below $\sim 5$ mHz, whereas the observations by Ronan et al. are mainly in the range $4 < \nu < 8.5$ mHz, although they report $p$-mode ridges extending up as far as 10.5 mHz. High-frequency $p$-mode ridges, extending out beyond 8 mHz, have also been reported by Duvall et al. (1991, 1993). Moreover, the reported negative frequency shifts, of order $-10 \mu$Hz, are much stronger than those found at low $\nu$. These observations raise two basic questions: why are the $p$-mode ridges found out to such high frequencies, and what causes such a strong solar cycle frequency shift?

It has been argued theoretically (Campbell & Roberts 1989; Evans & Roberts 1990, 1991, 1992; Jain & Roberts 1993, 1994a, b, c) that the magnetism of the chromosphere has a strong effect on $p$-modes. In particular, an evolving model magnetic chromosphere may bring about theoretical frequency shifts, qualitatively similar to those reported in the observations. The frequency shifts are brought about by an increase in mean chromospheric magnetic field strength (Evans & Roberts 1992; Wright & Thompson 1992) combined with an increase in chromospheric temperature (Jain & Roberts 1993, 1994a, b; Hindman & Zweibel 1994). Magnetic and temperature effects have also been invoked in the explanation put forward by Goldreich et al. (1991) for zero-degree modes.

The calculations by Evans & Roberts (1990, 1992) and Jain & Roberts (1993, 1994a, b) were restricted for technical reasons, connected with the computation of hypergeometric functions, to modes with $\nu < 4.5$ mHz, and so did not consider what happens at high frequency. In the absence of a magnetic field, an isothermal atmosphere has an acoustic cutoff frequency $\nu_{c}$, such that modes with $\nu > \nu_{c}$ may propagate within the atmosphere, while modes with $\nu < \nu_{c}$ are evanescent (trapped). At the temperature minimum of the Sun, $\nu_{c} \approx 5.3$ mHz, and so modes above this frequency may propagate out of that part of the solar atmosphere. However, magnetism alters this simplistic view of propagation and trapping. In the presence of a magnetic atmosphere with a constant Alfvén speed, the modifications to this are only moderate, except for very high degree modes (Campbell & Roberts 1989; Jain & Roberts 1994c), but in the presence of a uniform magnetic field the effect is strong (Evans & Roberts 1990). Indeed, in the presence of a uniform horizontal magnetic field all modes are ultimately evanescent, simply because the Alfvén speed in such a medium becomes arbitrarily large high in the atmosphere (Wright & Thompson 1992; Johnston 1994). Thus, in a uniform magnetic field, trapped $p$-modes exist for frequencies above the acoustic

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cutoff of 5.3 mHz. Of course, the real Sun does not have a uniform magnetic chromosphere, and the actual Alfvén speed in its atmosphere is bounded; but the Alfvén speed nonetheless increases rapidly with height to reach a value of order 10^7 km s^{-1} in the corona, very much larger than either the Alfvén speed or the sound speed (\sim 7 km s^{-1}) in the low chromosphere. Accordingly, we must expect the magnetism of the solar atmosphere to have an appreciable effect on waves that penetrate into it.

To examine the frequency shifts to be expected from high-frequency (\nu > 5.3 mHz) modes, Johnston et al. (1994) carried out an approximate WKB treatment of the magnetic atmosphere (see also Wright & Thompson 1992; Johnston 1994) and found that high-frequency shifts are both larger in magnitude and opposite in sign to those at low-frequency (\nu < 5.3 mHz). A hint of this change in behavior was noticed earlier by Jain (1992; see also Jain & Roberts 1994e) for frequencies close to 5 mHz, but the effect becomes more pronounced at frequencies of order 6 mHz. This wave-like behavior in the frequency shift curve, exhibiting a slight rise phase at low frequencies, followed by a turnover at \sim 4 mHz with a plunge into negative shifts for higher frequencies, producing a trough at about \nu \approx 6.6 mHz, is remarkably consistent with the qualitative behavior found in the observations of Ronan et al. (1994a, b). In the theory, this impressive agreement is achieved at the expense of large changes in chromospheric temperature, changes that are unlikely to be as large in the Sun (although some temperature changes in the chromosphere are to be expected). Nonetheless, the agreement between theory and observations is sufficiently impressive so as to warrant further investigation.

Here we examine frequency shifts without making use of the WKB approximation. Instead, we carry out a numerical solution of the governing differential equation in the magnetic atmosphere and obtain frequency shifts in the range \nu = 0 to 10 mHz. Our frequency shifts confirm the trend found in the approximate treatment by Johnston et al. (1994). Our numerically determined frequency shifts, for conditions appropriate to the years covered by the observations, compare well with the results expected from a combination of the separate data sets of Libbrecht & Woodard (1990) for low \nu and Ronan et al. (1994a, b) for high \nu.

Finally, we note that our work has a bearing on the recent discussion of the nature of high-frequency modes. The standard discussion of high-frequency modes is centered on whether the modes are a consequence of reflections off the high-temperature gradient in the transition region (Balmforth & Gough 1990) or whether they are better understood as a consequence of the reinforcement of direct and indirect acoustic waves from a source below the photosphere (Kumar & Lu 1991; Kumar 1993). However, neither of these views has allowed for the effect of chromospheric magnetism. As we have noted above, magnetism may have a strong effect on acoustic cutoff. Changes over the solar cycle are a case in point. In the presence of a uniform field, the fact that the Alfvén speed increases with height in the stratified solar atmosphere dominates the behavior of acoustic-like disturbances (fast magneto-acoustic waves), causing all waves to be reflected whatever their frequency (see also von Uexküll & Kneer 1995).

2. NUMERICAL SOLUTIONS

We consider the model explored in Evans & Roberts (1990) and Jain & Roberts (1993), namely a polytropic field-free region overlain by an isothermal atmosphere within which is embedded a uniform horizontal magnetic field; see Jain (1992) for details.

For an isothermal chromosphere (z < 0), with a uniform magnetic field B in the x-direction, the vertical motions v_{z}(z) \exp i(\omega t - kx), with angular frequency \omega and horizontal wavenumber k, are governed by the equation (see Jain & Roberts 1994b for nonaligned effects)

\[ [\mathcal{A}_{1} + \mathcal{A}_{2} \exp (-\mathcal{A}_{3} z)] \frac{d^{2} v_{z}}{dz^{2}} + \mathcal{A}_{1} \frac{dv_{z}}{dz} + [\mathcal{A}_{4} - k^{2} \mathcal{A}_{2} \exp (-\mathcal{A}_{3} z)] v_{z} = 0 , \]  

(1)

where

\[ \mathcal{A}_{1} = \omega^{2} c_{e}^{2} , \quad \mathcal{A}_{2} = \nabla_{z}^{2} (\omega^{2} - k^{2} c_{e}^{2}) , \quad \mathcal{A}_{3} = \frac{1}{H_{e}} , \quad \mathcal{A}_{4} = (\gamma - 1) g k^{2} + \omega^{2} (\omega^{2} - k^{2} c_{e}^{2}) . \]

Here \( H_{e} \) is the scale height within the isothermal atmosphere, \( c_{e} \) is the sound speed, \( v_{z} \) is the Alfvén speed at the base of this region, \( \gamma \) is the adiabatic index, and \( g \) is the local gravitational constant.

In the nonmagnetic lower region (z > 0), representing the convection zone below and the governing equation (for \( \Delta \equiv \nabla \cdot \nu \)) is

\[ \frac{d^{2} \Delta}{dz^{2}} + \left( \frac{\gamma - 1}{c_{z}^{2}} \right) \frac{d \Delta}{dz} + \left[ \frac{(\omega^{2} - k^{2} c_{e}^{2})}{c_{z}^{2}} - \frac{g k^{2}}{\omega^{2}} \left( \frac{c_{z}^{2} - (\gamma - 1) g}{c_{e}^{2}} \right) \right] \Delta = 0 , \]

(3)

a prime denoting the derivative with respect to depth z; \( c_{z}^{2}(z) \) is the square of the sound speed in the field-free region. For a linear temperature profile,

\[ c_{z}^{2}(z) = c_{z}^{2}(1 + \frac{z}{z_{0}}) , \quad z > 0 , \]

(4)

the solution of equation (3) was first given by Lamb (1932). In terms of \( v_{z} \), we have

\[ (\omega^{2} - g k^{2})v_{z} \exp \left( i \omega t + i z \right) = [k c_{z}(\omega^{2} + g k) - \gamma \omega v_{z}^{2}] \times U(-a, m + 2, 2kz + 2kz_{0}) \]

- \[ 2a c_{z} k U(-a + 1, m + 3, 2kz + 2kz_{0}) \].

Here \( \varphi \), is an arbitrary constant, \( U \) is a confluent hypergeometric function, and \( m \) is the polytropic index. The parameter a is given by

\[ 2a = \frac{\omega^{2} m + 1}{g k} \frac{m + 1}{\gamma} + \left( \frac{m - m + 1}{\gamma} \right) \frac{g k}{\omega^{2}} - (m + 2) . \]

(6)

The solution (5) is obtained by demanding a finite kinetic energy density at large depths, i.e., by requiring that \( \frac{1}{2} \rho(z) v_{z}^{2} \to 0 \) as \( z \to \infty \), for equilibrium density \( \rho(z) \).

Equation (1) can be transformed into a standard hypergeometric equation (see Nye & Thomas 1976; Evans & Roberts 1990) and the solution for \( v_{z} \) matched with equation (5) across the interface \( z = 0 \) to give a dispersion relation. This approach was considered in Evans & Roberts (1990) and Jain & Roberts (1993). Unfortunately, frequency shifts were not calculated beyond 5 mHz because only real parameters in the calculation

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of the hypergeometric function was considered. Here, we solve equation (1) numerically, for frequencies beyond 5 mHz. For physically real solutions, we require that \( \nu \) and \( \partial \nu / \partial z \) tend to zero as \( z \rightarrow -\infty \). The other boundary conditions are that \( \nu \) and the total pressure perturbation are both continuous across the interface \( z = 0 \).

The cyclic frequency \( \nu (\equiv \omega / 2 \pi) \) for each mode \((n, l)\) is calculated for a given chromospheric temperature \( T_c \) and magnetic field strength \( B_z \), producing a base frequency \( \nu (B_z, T_c) \). The calculation is repeated for a higher chromospheric temperature \( T_c' \) and magnetic field strength \( B_z' \), for the same pair \((n, l)\), thus producing cyclic frequencies \( \nu (B_z', T_c') \). The calculated frequency change \( \Delta \nu \) is the difference between these two sets: \( \Delta \nu \equiv \nu (B_z', T_c') - \nu (B_z, T_c) \). The parameters \( c_{\text{opt}} \), \( m \), \( \gamma \), and \( z_0 \), describing the convection zone \((z > 0)\), are fixed (see Jain & Roberts 1993), and the degree \( l \) of the mode is determined by \( k^2 = l(l + 1)/R^2 \), where \( R \) is the solar radius.

Figures 1a–1b show the calculated frequency shift \( \Delta \nu \) (in \( \mu \text{Hz} \)) as a function of base frequency \( \nu \) (in \( \mu \text{Hz} \)) for modes of degree \( l = 100 \). The chromospheric magnetic field strength and temperature for the base frequency are taken as \( B_z = 10 \) G and \( T_c = 4170 \) K. The calculations are repeated for a higher magnetic field strength \( B_z' = 15 \) G, and the chromospheric temperature \( T_c' \) is varied between 4170 and 6500 K, as indicated in the figure caption.

It is clear from Figure 1a that frequency shifts at a given frequency decrease with increasing temperature \( T_c' \) (see also Hindman & Zweibel 1994). This is apparent for frequencies greater than 4 mHz in Figure 1a but can also be clearly seen for lower frequencies if we magnify the low-frequency regime of the curves. This is shown in Figure 1b.

Observe that in Figure 1b frequency shifts increase as a function of frequency \( \nu \), reach a maximum and then fall to a minimum at about \( \nu \approx 6.7 \) mHz. Thus, combining Figures 1a and 1b produces a wavelike behavior (with downturns and upturns) in the frequency shifts. Other choices of chromospheric field strength and temperature produce qualitatively similar results. In Figure 2 we illustrate the case of a base magnetic field of \( B_z = 20 \) G evolving to \( B_z' = 30 \) G, for the same temperature variations as adopted in Figure 1a. The two figures are very similar.

Finally, we consider the effect of changing the degree \( l \) of the mode, examining the case \( B_z = 10 \) G, \( B_z' = 15 \) G, \( T_c = 4170 \) K, and \( T_c' = 5000 \) K, corresponding to curve (c) of Figure 1a. Figure 3 shows the result for values of \( l \) between 30 and 150. The calculated frequency shifts have an upturn at around \( \nu \approx 6.7 \) mHz, for all values of \( l \), and the magnitude of the frequency shift \( \Delta \nu \) increases with increasing \( l \).

3. DISCUSSION

The observations of Ronan et al. (1994a, b) show a decrease in the frequency shifts up to \( \sim 5.5 \) mHz, falling to about \( -10 \) \( \mu \text{Hz} \), and an increase after that, reaching zero around 6.5 mHz. The observed frequency shifts also increase with degree \( l \) although the exact relationship is not clear.

The magnetic field strength changes in the chromosphere from 1988 to 1991 are not expected to be large, since both these data are near solar maximum; in our calculations, we have considered field changes from \( B_z = 10 \) G to \( B_z' = 15 \) G, and from \( B_z = 20 \) G to \( B_z' = 30 \) G. For the larger magnetic field changes, correspondingly slightly larger temperature changes are required to produce a wavelike behavior in the frequency shifts. For example, field changes from \( B_z = 20 \) G to \( B_z' = 30 \) G produce frequency shifts of the order of \( -10 \) \( \mu \text{Hz} \) near the upturn point, for temperature changes from \( T_c = 4170 \) K to \( T_c' = 5500 \) K (compared with 5000 K in the weaker field illustration). Thus, the qualitative behavior of the frequency

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shifts remains the same with the change in the values of magnetic field strengths and temperatures.

It is interesting to note in Figure 3 that all the $\Delta V$ curves pass through the same zero shift point, at $v \approx 8.2$ mHz. Changing the values of $B_p$, $B'_p$, $T_p$, and $T'_p$ changes the value of the zero shift point. The results of Figure 3 suggest that it may be helpful in the observations to present frequency shifts for specific, rather than averaged, values of $l$, since it is clear from the figure that the dependence of $\Delta V$ upon $l$ varies considerably with frequency $v$.

Radiative damping due to thermal radiation becomes important in the chromosphere. The high-frequency $p$-modes have large line widths compared to their frequency spacing (Kumar 1993), which suggests that these modes might be significantly damped. No such damping mechanism that assumes adiabatic perturbations is considered in the calculations.

We have here concentrated on a model of frequency shifts as produced by changes in chromospheric magnetism and temperature, modeled simply by a uniform horizontal magnetic field and an isothermal atmosphere. It would be of considerable interest to investigate the combined roles of temperature changes with radiative damping, to see whether this diminishes the requirement of large temperature changes. But such an investigation represents a major extension to our model and so will be left for a future work.

In conclusion, it is evident from our results that magnetism is important in the discussion of solar $p$-modes. It modifies the acoustic cut-off effect and offers a possible explanation of the solar cycle changes in $p$-mode frequencies.

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