Is Our Knowledge of Solar Indices Enough to Explain Satellite Drag?

M. G. Deminov, E. V. Nepomnyashchaya, V. N. Obridko, D. B. Shelting, V. N. Shubin and V. A. Chizhenkov
IZMIRAN, 142092, Troitsk. Moscow Region, Russia

Abstract. The data on satellite drag have been analyzed for a near-circular orbit at an altitude of about 800 km. It is shown that the IACC–77 and MSIS–86 models afford high accuracy of calculations of the atmosphere density during the minimum, regular growth, and decay of solar activity (correlation coefficient \( r > 0.9 \)). During high solar activity, when the mean 10–cm flux hardly changes over a long time interval, the accuracy is much worse \( (0.2 < r < 0.7) \). We propose a method to improve the models by using a different relationship between the atmosphere density and solar activity in the epoch of maximum. The dependence is non-linear and contains \( F_{10.7} \) values averaged over the previous 81, 54, and 27 days. The method increases the accuracy of calculations with the MSIS–86 model in the epoch of solar maximum by 30–45% \( (0.75 < r < 0.83) \). In this period, the daily \( F_{10.7} \) values are insignificant to the atmosphere density. Taking into account the integral EUV flux \( (30–130 \text{ nm}) \) does not result in any noticeable improvement. It seems to be due to insufficient amount of EUV data for the period of high solar activity.

1. Introduction

The problem of satellite drag arose with the advent of the space era. On one hand, it was important to accurately estimate the lifetime of spacecraft. Remember the Skylab and Salyut space stations that were prematurely lost and fell down outside the estimated area. On the other, the analysis of satellite drag is a most reliable and direct way to verify our knowledge of the Earth atmosphere and its dependence on various solar and terrestrial phenomena. The existing model atmospheres are mainly semi–empirical. They are based on generalized experimental data on satellite drag, rocket, mass–spectrometer, and radar observations (e.g., see Jacchia 1977; Hedin 1987).

In spite of long pre–history, the accuracy of drag calculations is still low, and sometimes gross errors arise. This is due to the fact that the models described in (Jacchia 1977; Hedin 1987), use the \( F_{10.7} \) index — radio emission flux at 10.7 cm — to calculate the parameters of the upper atmosphere affected by solar activity. However the Earth atmosphere is mainly heated by the solar EUV radiation \( (10–175 \text{ nm}) \). As follows from the analysis of EUV and \( F_{10.7} \) data for solar cycles 21 and 22, the correlation between these values is the highest in the periods of relatively fast variations of solar activity (at the growth or decay of the cycle) and the lowest at the extreme points of the cycle (Nepomnyashchaya et al. 1995).
In this paper, the satellite drag data are used to consider density variations of the atmosphere at the maximum of solar activity.

2. Analysis of Observations and Comparison With the JACC–77 and MSIS–86 Models

Our study is based on the orbit evolution data from the Kosmos series satellites. These are light space vehicles with a mass of about 800 kg, the maximum revolution period of about 100 min, and the orbit inclination of about 74°. From the whole bulk of data, we have selected intervals when the satellite orbits were nearly circular (eccentricity not exceeding 0.004). The minimum altitudes of the orbits under consideration range within 760–800 km. The selected interval (from 1986 to 1992) covers all phases of solar cycle 22: the minimum (1986–1987), the growth phase (1987–1989), the maximum (1989–1992), and the decay phase (1992–1993). The total number of analyzed orbits is 224.

The rotation period of a satellite at the circular orbit varies with the atmosphere density at the orbital altitude (King–Hilly 1966):

\[
\frac{dT}{dt} = -3 \cdot \pi \cdot \delta \cdot (R_E + h) \cdot \rho,
\]

(0.1)

where \( T \) is the satellite rotation period measured at a time \( t \), \( \delta \) allows for aerodynamic parameters of the satellite, \( R_E \) is the radius of the Earth, \( h \) is the altitude of the orbit above the Earth, and \( \rho \) is the atmosphere density at the altitude \( h \) averaged over the period \( T \). At the altitude below 1000 km, the neutral components of the atmosphere (i.e. the thermosphere) have a density at least by an order of magnitude higher than the ionized components. Thus, for a circular orbit, \( \rho \) in equation (1) is the thermosphere density at the orbital altitude \( h < 1000 \) km averaged over one orbital period.

\[
\rho = \int \frac{\rho^*}{2\pi(R_E + h)} \cdot ds.
\]

(0.2)

Here \( \rho^* \) is the local thermosphere density, and integration is performed along the orbit. Substitution of (2) to (1) shows that the satellite drag is determined by the integral density along the orbit.

Assume that variation rates of the orbits in (1) do not depend, but on the satellite drag in the atmosphere. (In calculations, they are replaced by the difference ratios):

\[
\frac{dT}{dt} = \frac{\Delta T}{\Delta t},
\]

(0.3)

where \( \Delta T = T_{i+1} - T_i \) is the difference of the rotation periods in two successive records, and \( \Delta t = t_{i+1} - t_i \) is the time difference between two successive crossings of the equator. The least time interval between two orbit determinations is 10 days. To calculate the satellite drag, we have used discrete data on the orbital periods of Kosmos–1763 (July 1986–May 1989, 40 values), Kosmos–1937 (April 1986–May 1989, 43 values), Kosmos–1992 (January 1986–March 1993, 83 values), and Kosmos–2056 (January 1990–November 1992, 58 values).
Let us compare the calculated and the model densities. For this purpose, we choose two semi–empirical models — one mainly based on the satellite drag data (Jacobia 1977) and another, more complete, using mass–spectrometer, rocket, optical measurements, and incoherent scatter observations (MSIS–86, [7]). The principal attention is paid to the analysis of density variations in different phases of the solar cycle.

JACC–77 determines density variations of the atmosphere as a function of solar activity in terms of variation of $T_{exo}$ that has the form:

$$T_{1/2} = 5.48 \cdot F_{10.7}^{0.8} + 101.8 \cdot F_{10.7}^{0.4}. \quad (0.4)$$

Here, $T_{1/2}$ is the arithmetic mean of the diurnal extrema of the $T_{exo}$ distribution under quiet geomagnetic conditions; $F_{10.7}$ is the solar radio flux at 10.7 cm in the units of $10^{-22} \text{W} \cdot \text{m}^{-2} \cdot \text{Hz}^{-1}$; and $F$ is the weighted mean value of $F_{10.7}$ for
six solar rotations:

\[
F_{10.7} = \frac{\sum t \cdot F_{10.7}}{\sum t}, \quad w = \exp \left( -\frac{t - t_0}{\tau} \right),
\]

(0.5)

where \( t \) is the time, \( t_0 \) is the moment at which \( F_{10.7} \) is to be calculated, and \( \tau = 81 \) days (three solar rotations). Since the response of \( T_{exo} \) to variations of \( F_{10.7} \) is not instantaneous, the latter is taken with a lag of about one day.

MSIS–86 yields a more simple dependence of the neutral atmosphere on solar activity:

\[
N_i \sim a_1 \cdot \Delta F + a_2 \cdot (\Delta F)^2 + b_1 \cdot \Delta F + b_2 \cdot (\Delta F)^2,
\]

(0.6)

where \( \Delta F = F_{10.7} - F_{10.7} \) and \( \Delta F = F_{10.7} - 150 \). Here, \( F_{10.7} \) is the 10 cm radio flux measured on the previous day, and \( F_{10.7} \) is the \( F_{10.7} \) index averaged over 81 days and centered on the day for which the atmosphere parameters are calculated.

Fig. 1 compares the density variations in different phases of solar activity, obtained from the Kosmos–1763 and Kosmos–1992 satellite drag data, with the model values calculated from the JACC–77 and MSIS–86 models. As follows from equations (1) and (3), experimental data have the sense of the mean integral density for several revolutions of the satellite. Trajectory measurements were used to determine the satellite orbits, and the mean atmosphere densities for one revolution, \( \rho_i \), were calculated from the JACC–77 and MSIS–86 models. Dots in Fig. 1 denote the orbit variation rates (3) proportional with the mean atmosphere density between two successive crossings of the equator. The solid line illustrates the mean integral densities over the interval between two orbits, calculated from the models, \( \rho_n = \frac{\rho_{n+1} + \rho_1}{2} \). Two bottom panels illustrate variations of the solar activity index, \( F_{10.7} \). The left panel refers to the epochs of minimum and growth of solar activity (Kosmos–1763 data) and the right panel to the epoch of maximum (Kosmos–1992 data). In the minimum and the growth phases, one can readily see a good correlation between the densities obtained from the orbit variations and those calculated from the model atmospheres. The correlation coefficient for the JACC–77 model is \( r = 0.98 \) with the r.m.s. deviation \( \sigma = 4.0 \cdot 10^{-18} \) g-cm\(^{-3}\). For MSIS–86 these values are \( r = 0.97, \sigma = 4.0 \cdot 10^{-18} \) g-cm\(^{-3}\). The mean density calculated for this interval from the satellite drag data is \( 1.3 \cdot 10^{-17} \) g-cm\(^{-3}\). At the maximum of the cycle both models give large errors: for JACC–77, \( r = 0.54, \sigma = 2.0 \cdot 10^{-17} \) g-cm\(^{-3}\) and for MSIS–86, \( r = 0.41, \sigma = 2.2 \cdot 10^{-17} \) g-cm\(^{-3}\). The mean density is \( 6.4 \cdot 10^{-17} \) g-cm\(^{-3}\) (Kosmos 1992 data). In the epoch of decreasing activity, the correlation coefficients for JACC–77 and MSIS–86 calculated from the Kosmos–1992 and Kosmos–2056 data are \( r = 0.96, r = 0.92 \) and \( r = 0.95, r = 0.91 \), respectively.

Thus, the analysis shows that in the periods of low solar activity, as well as in the periods of its regular growth and decay, the JACC–77 and MSIS–86 models provide a fairly good agreement with the experimental densities calculated from the satellite drag data.
Figure 2. Comparison of $\rho$ derived from the Kosmos-2056 drag data (dots) and calculated from three models, JACC-77, MSIS-86, and (7), (solid lines) for the epoch of maximum of cycle 22. Variation of $F_{10.7}$ is shown at the bottom.
Figure 3. Comparison of $\rho_{\text{drag}}$ obtained from the Kosmos–1763, 1937, 1992, 2056 data and $\rho_{\text{mod}}$ calculated from JACC–77, MSIS–86, and (7) for the maximum of cycle 22 (1989–1992).

3. Atmosphere Density Modeling in the Epoch of Solar Maximum

Ultraviolet solar radiation (EUV) — the main source of heating of the Earth atmosphere — consists of two components. One is bound up with solar active regions and is generated in the solar corona. This radiation originates in high-temperature regions and correlates well with the solar 10 cm radio flux. Another component is due to radiation from the solar disk and proceeds from less heated regions. The first component displays fast day–to–day variations depending on the appearance and disappearance of active regions. The second component changes more slowly during the 11–year solar cycle. The 10 cm radio flux can be used as an easily accessible index of EUV radiation that involves both its components. As shown by Jacchia (1977), the EUV component is linearly related to $\overline{F}_{10.7}$ averaged over several solar rotations. The heating of the upper Earth atmosphere is rather an inertial process. Therefore at the maximum of solar activity, when $\overline{F}_{10.7}$ does not practically change over a long time interval, the EUV component from the solar disk can be expected to make predominant contribution to the atmosphere heating.

To verify this hypothesis, we must find a different dependence of the atmosphere density on solar activity for the epoch of solar maximum, than that used in MSIS–86:

$$\rho_0 = \rho_{m}^{MSIS} \cdot \left[ 1 + c_1 \left( \frac{\overline{F}_{81}}{F_0} \right)^{k_1} + c_2 \left( \frac{\overline{F}_{54}}{F_0} \right)^{k_2} + c_3 \left( \frac{\overline{F}_{27}}{F_0} \right)^{k_3} \right], \quad (0.7)$$

where $\rho_{m}^{MSIS}$ is the integral density along the orbit calculated from the MSIS–86 model, where the $F_{10.7}$ and $\overline{F}_{10.7}$ indices are constant and equal to 200. MSIS–86 has been chosen as a basic model, because it is most correct in taking into account the dependence on the geomagnetic activity index, $a_P(t')$. Here $t'$ is the characteristic response time of the atmosphere to geomagnetic activity. The
other input parameters of the model are strictly fixed by the orbit location in time and space. \( \overline{F}_{81}, \overline{F}_{54}, \) and \( \overline{F}_{27} \) are the \( F_{10.7} \) values for the previous 81 days (3 solar rotations), 54 days (2 rotations), and 27 days (1 rotation), respectively. So, in (5), \( \tau = 81, 54, \) and 27, respectively. The parameters of Equation (7), \( c_1, c_2, c_3, k_1, k_2, \) and \( k_3 \) were determined by the least square method. It was found that \( k_1 = 0.8, k_2 = 0.3, \) and \( k_3 = 0.5. \)

In the epochs of high solar activity, satisfactory correlation was obtained between the densities derived from the satellite drag data and the model calculations, for all four satellites. The results of the statistical analysis are shown in the Table.

<table>
<thead>
<tr>
<th>Satellite</th>
<th>JACC–77</th>
<th>MSIS–86</th>
<th>Model (7)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( r )</td>
<td>( \sigma,% )</td>
<td>( r )</td>
</tr>
<tr>
<td>Kosmos–1763</td>
<td>0.51</td>
<td>26.2</td>
<td>0.23</td>
</tr>
<tr>
<td>Kosmos–1937</td>
<td>0.68</td>
<td>27.5</td>
<td>0.57</td>
</tr>
<tr>
<td>Kosmos–1992</td>
<td>0.54</td>
<td>31.2</td>
<td>0.41</td>
</tr>
<tr>
<td>Kosmos–2056</td>
<td>0.61</td>
<td>36.6</td>
<td>0.54</td>
</tr>
<tr>
<td>1763, 1937, 1992, 2056</td>
<td>0.62</td>
<td>31.9</td>
<td>0.53</td>
</tr>
</tbody>
</table>

As seen from the Table, the accuracy of density calculations from MSIS–86 for the epoch of high solar activity can be increased by 30–45% by taking into account (7). It is clearly seen in Fig. 2, where the densities derived from the Kosmos–2056 drag data are compared with the model calculations (JACC–77, MSIS–86, and (7)) for the epoch of high solar activity. The diagrams shown in Fig. 3 represent the results of model approximation for all satellite drag data for the maximum of cycle 22 (1989–1992).

4. Discussion

Index \( k_1 = 0.8 \) in (7) coincides with the index of \( T_{1/2} \) in (4) (JACC–77 model). This nonlinear dependence of the exosphere temperature on \( \overline{F}_{10.7} \), obviously, increases the accuracy of the JACC–77 model compared to MSIS–86 (see the Table). If only the dependence on \( \overline{F}_{81} \) is taken into account, i.e. if in equation (7) \( c_2 = c_3 = 0 \), the correlation coefficient \( r \) does not exceed 0.7 for all cases cited in the Table. Therefore, additional terms for \( \overline{F}_{54} \) and \( \overline{F}_{27} \) increase the accuracy of the model. On the other hand, no noticeable improvement resulted from introducing the dependence on \( F_{10.7} \) for the previous day. It may be due to the fact that relative variations of \( F_{10.7} \) during high solar activity are insignificant.

The dependence of the thermosphere density on solar activity can be better allowed for by introducing additional solar activity indices. However, as shown in (Hoegy & Mahajan 1992), introducing the integral EUV flux (30–130 nm)
does not significantly improve the accuracy of model (7). Probably, it is due to insufficient volume of data for the period under consideration.

5. Conclusion

Thus, the available model atmospheres are suitable to describe the satellite drag at the growth or decay of the solar cycle. At the maximum of solar activity they do not provide good agreement with observations. An unexpected progress was achieved by more accurately taking into account the slowly changing radiation from the solar disk, rather than radiation from active regions. The procedure proposed in this paper increases the accuracy of density calculations for the epoch of solar maximum by 30–45%.

Acknowledgments. The work was supported by the Russian Foundation for Fundamental Research (grant N 94-02-03898).

References

Hedin, A. E. 1987, J. Geophys. Res. 92, N A5, 4649
King–Hilly, D. 1966, Theory of Orbits of Artificial Earth Satellites in the Atmosphere, Mir, Moscow
Nepomyashchaya, E. V., Chizhenkov, V. A., Shelting, B. D., & Shubin, V. N. 1995, Pisma v Astronomichesky Zhurnal 21, N4, 1

Group Discussion

Dryer: Are you satisfied that the satellite drag problem is now being satisfactorily handled with the 107 cm radio flux proxy?

Obridko: No. I suppose that the next step is to go to physical parameters, such as large-scale magnetic field of the Sun.