THE RELATION BETWEEN THE SYNODIC AND SIDEREAL 
ROTATION PERIOD OF THE SUN 

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Abstract. The relation between the synodic and sidereal rotation period of the Sun for an arbitrary date of observation is derived taking into account details of the Earth’s motion. The transformation procedure between the synodic (apparent) and sidereal rotation period presented here can be performed without using the annual ephemerides. 

1. Introduction 

The synodic rotation period of the Sun can be determined from measurements of Doppler shifts close to the solar limb or from the apparent movement of ‘tracers’ such as sunspots, filaments, plages, etc., across the solar disc. Because the velocity of the Earth in its orbit varies, the relation between the synodic (apparent) period of the solar rotation and the sidereal period changes during the course of the year. If this is not taken into account, a systematic error can arise in studies of the solar rotation which are based on measurements performed in short time intervals. For example, if we assume that the Sun rotates once every 25.38 days (the Carrington sidereal period) the peak-to-peak change of the synodic period amounts to about 0.15 days (Graf, 1974). The procedure presented by Graf is based on a rather simple model of the Earth’s revolution, but it provides sufficient accuracy for actual purposes. However, the procedure presented requires information about the time of the mean perihelion for a given year. Let us note that in the paper mentioned there was a typing error in Equation (1): the second term on the right-hand side of that equation should have been multiplied by the numerical eccentricity factor \(e\), i.e., the equation should read as follows:  

\[
D = 1.0 - e \cos[T + e \sin(T)] ,
\]

where the quantities \(D\), \(T\), and \(e\) are defined as in the paper by Graf (1974), i.e., \(D\) = the calculated Earth–Sun distance in AU; \(e = 0.01675\), the eccentricity of the Earth’s orbit; \(T = (2\pi/365.25) \times (\text{DAY} - \text{PH})\), where \(\text{DAY}\) is the number of
days after January 0 and 0\textsuperscript{h} Ephemeris Time and PH is the time in days (after the same date) of the mean perihelion.

For any given time, the exact values of the parameters which are needed for the solar rotation rate determination can always be interpolated from the date given in the annual ephemerides. In this paper we present the simple formulae allowing one to compute directly the relation between the synodic and sidereal rotation period of the Sun for a given date without using ephemerides.

\section{Heliographic Coordinates}

The rotation of the Sun is very complicated and has many different aspects (Howard, 1984; Wöhl, 1990). The adopted system of heliographic coordinates is rather arbitrary, and corresponds to the average rotation period of the equatorial regions on the Sun (Green, 1988). The adopted values for the inclination, $i$, of the Sun’s equator to the ecliptic and the longitude $\Omega$ of the ascending node are:

\begin{align*}
    i &= 7^\circ 15' , \\
    \Omega &= 73^\circ 40' + 50.25''(t - 1850.0) ,
\end{align*}

where $t$ is the time expressed in years. Equations (1) and (2) specify the rotation axis of the Sun. The prime meridian is defined by the point on the solar equator which is assumed to rotate with a sidereal period of 25.38 days. This reference point was chosen to coincide with the ascending node on January 1, 1854 at 12:00 UT (expressed in Julian Day numbers as J.D. 2 398 220.0). The angular distance, $W$, of the reference point from the ascending node is defined in the direct sense (the Sun’s rotation is direct) and can be calculated from

\begin{equation}
    W = \frac{360^\circ}{25.36}(\text{J.D.} - 2 398 220.0) .
\end{equation}

For any other value of the sidereal $S$ period Equation (3a) may be written in the form

\begin{equation}
    W = \frac{360^\circ}{S}(\text{J.D.} - 2 398 220.0) .
\end{equation}

The heliographic coordinate system is specified by the heliographic coordinates ($L_0$, $B_0$) of the centre of the apparent solar disc and with the position angle, $P$, of the rotation axis. According to Waldmeier (1955) the following equations are valid:

\begin{align*}
    \sin B_0 &= \sin(\lambda_0 - \Omega) \sin i , \\
    \tan(L_0 + W) &= \cos i \tan(\lambda_0 - \Omega) ,
\end{align*}

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\[ P = - \arctan[\cos \lambda_0 \tan \epsilon] - \arctan[\tan i \cos(\Omega - \lambda_0)] , \]  

(6)

where \( \lambda_0 \) is the apparent longitude of the Sun referred to the true equinox of the date, and \( \epsilon \) is the obliquity of the ecliptic.

### 3. The Relation between the Synodic and Sidereal Rotation Period of the Sun

The synodic rotation period corresponds to a decrease in \( L_0 \) of 360°. Suppose that \( \Delta L_0 \) is the daily change of the heliographic longitude \( L_0 \) of the center of the solar disc. The synodic period, \( S_p \), is then

\[ S_p = \frac{360^\circ}{\Delta L_0} . \]  

(7)

Suppose that the observations take place on some arbitrary date. Let \( N \) be the moment of the beginning of the observation date, and \( N + 1 \) the moment of the end of the date (i.e., the moment of the beginning of the next day). Then from Equation (5) one finds

\[ L_0 + W = \arctan[\cos i \tan(\lambda_0 - \Omega)] \]  

(8)

for the moment \( N \), and

\[ L_0 - \Delta L_0 + W' = \cos i \tan(\lambda'_0 - \Omega') \]  

(9)

for the moment \( N + 1 \), where the values \( W', \lambda'_0, \text{ and } \Omega' \) are related to the moment \( N + 1 \) and \( \Delta L_0 \) is the daily change of the parameter \( L_0 \).

By subtracting Equations (8) and (9) one finds

\[ -\Delta L_0 + W' - W = \arctan[\cos i \tan(\lambda'_0 - \Omega')] - \arctan[\cos i \tan(\lambda_0 - \Omega)] . \]  

(10)

From Equation (3b) one obtains

\[ W' - W = \frac{360^\circ}{S} . \]  

(11)

Equations (10) and (11) combined with Equation (7) give

\[ \frac{360^\circ}{S} = \frac{360^\circ}{S_p} + \{ \arctan[\cos i \tan(\lambda'_0 - \Omega')] - \arctan[\cos i \tan(\lambda_0 - \Omega)] \} , \]  

(12)
where all the terms of Equation (12) are expressed in deg day$^{-1}$. The synodic period, $S_p$ (expressed in days), can be determined from the observations. Then the corresponding sidereal period, $S$ (expressed in days), can be found from Equation (12). Equation (2) gives the values of the longitude of the ascending node ($\Omega$ and $\Omega'$). The apparent longitude of the Sun can be calculated with sufficient accuracy by the following procedure (Meeus, 1988).

The geometric mean longitude, $L$, of the Sun (referred to the mean equinox of the date), the Sun’s mean anomaly, $M$, and eccentricity, $e$, of the Earth’s orbit are given by

$$L = 279.69668^\circ + 36000.76892^\circ T + 0.0003025^\circ T^2,$$  

(13a)

$$M = 358.47583^\circ + 35999.04975^\circ T - 0.000150^\circ T^2 - 0.0000033^\circ T^3,$$  

(13b)

$$e = 0.01675104 - 0.0000418T - 0.000000126T^2,$$  

(13c)

where $T$ is the time measured in Julian centuries of 36525 ephemeris days from the epoch 1900 January 0.5 ET (Ephemeris Time):

$$T = \frac{J.D. - 2415020.0}{36525}.$$  

(14)

If the given moment is expressed in Universal Time, we add to J.D. the value $\Delta T = ET - UT$ expressed in days. (From 1984 on, the expression $\Delta T = TDT - UT$ is used, where by TDT the terrestrial dynamical time is denoted.) The value of the difference, $\Delta T$, is tabulated in the *Astronomical Almanac*. For past years $\Delta T$ is about one minute.

With the values $M$ and $e$, the Kepler equation can be solved (all angles are expressed in degrees),

$$E = M + \frac{180^\circ}{\pi} e \sin E,$$  

(15)

using the iteration formula

$$E_{n+1} = E_n + \frac{M - E_n + \frac{180^\circ}{\pi} e \sin E_n}{1 - e \cos E_n},$$  

(16)

to find the eccentric anomaly, $E$. For the first approximation we use $E = M$. Then, the true anomaly can be obtained using

$$\tan \left( \frac{f}{2} \right) = \left( \sqrt{\frac{1+e}{1-e}} \right) \tan \left( \frac{E}{2} \right).$$  

(17)
The Sun’s true geometric longitude $\lambda$, referred to the mean equinox of the date, is then

$$\lambda = L + f - M .$$  \hspace{1cm} (18)

To find the apparent longitude of the Sun for the true equinox of the date, $\lambda_0$, it is necessary to correct $\lambda$ for nutation and aberration. With sufficient accuracy this can be performed by applying the formula

$$\lambda_0 = \lambda - 0.00569^\circ - 0.00479^\circ \sin(259.18^\circ - 1934.142^\circ T) .$$  \hspace{1cm} (19)

Using Equations (13)–(19) the values of the apparent longitude of the Sun for the beginning and the end of the observation day ($\lambda_0$ and $\lambda'_0$, respectively) can be calculated, and Equation (2) gives the values of the longitude of the ascending node for the beginning and the end of the observation day ($\Omega$ and $\Omega'$, respectively). When these values are known, Equation (12) provides a determination of the sidereal rotation period ($S$) from the observed value of the synodic rotation period ($S_p$):

$$S = S_p \left\{ 1 + \frac{S_p}{360^\circ} \left[ \arctan[\cos i \tan(\lambda'_0 - \Omega')] - \arctan[\cos i \tan(\lambda_0 - \Omega)] \right] \right\}^{-1} .$$  \hspace{1cm} (20)

4. Conclusion

When the solar sidereal rotation rate is studied, the measured synodic rotation rate is usually corrected by an average factor of 0.986 deg day$^{-1}$ to obtain the value of the sidereal rotation. However, if observations are performed in limited time intervals of the year, this can introduce a systematic error (up to 1% in the rotation rate) as some details and specificities of the Earth’s revolution are not taken into account. The procedure presented in this paper provides a transformation between the synodic and the sidereal rotation period with sufficient accuracy without using the ephemerides. This procedure should be taken into account when, e.g., possible temporal changes of the rotational rate are studied (see, e.g., Wöhl, 1990, and references therein).

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