SOME COMMENTS ON THE PROBLEM OF SOLAR CYCLE PREDICTION

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Abstract. The paper provides a number of regression equations that can be used to calculate the height of the odd Wolf number cycle. The feasibility of the rule of Gnevyshev–Ohl is analyzed as applied to the geomagnetic aa-index. A modified rule of Gnevyshev–Ohl has been formulated to describe the behaviour of aa-indices. A new method is suggested for early prediction of the next solar cycle. In this method, the angular coefficient (straightline slope) of linear dependence of aa-indices on the Wolf number at the descending branch of the cycle has been used as a prediction index. It is shown to a high degree of certainty that the new prediction index is related to the height of the forthcoming cycle. While the methods based on the ratio of the even–odd cycles in a pair give very high values of cycle 23 maximum (203.2 ± 10.7), our new index, on the contrary, gives very low values (74.7 ± 6.9). There are some contradictory symptoms indicating that the forthcoming cycle 23 is likely to violate the regularities established for the past 125 years.

1. The Rule of Gnevyshev–Ohl

Gnevyshev and Ohl (1948) showed that the annual mean Wolf numbers summed up over 11-year cycles display an important regularity: if the Wolf number cycles are arranged in pairs in the sequence even–odd cycle, the second cycle in the pair is always higher than the first one. The correlation coefficient is 0.91. The only exception is the pair of cycles 4–5. The pairs of cycles arranged in the opposite way (odd–even cycle) do not display any sensible regularity. In such a pair the second cycle can be both higher and lower than the first one, with the correlation coefficient as small as 0.50. This means that the true physical cycle of solar activity lasts for 22 years (as it should do in accordance with the Hale law of alternating magnetic polarities), and begins with an even solar cycle.

Kopecký (1950) established that the rule of Gnevyshev–Ohl is valid also for the relation between corresponding Wolf numbers in the same phase of different cycles. With this addition, it can be called the rule of Gnevyshev–Ohl–Kopecký (GOK rule), and has already two exceptions – pairs 4–5 and 8–9 (for details see Vitinsky, Kopecký, and Kuklin, 1986).

The GOK-rule can be used directly for prediction purposes because the relationship between the heights of the solar cycles in a pair is rather close and satisfies the regression equation

\[ R_{M2k+1} = 34.7411 + 0.94054 \times R_{M2k} , \]

\[ n = 10, \quad r = 0.837, \quad se = 21.9, \quad cl > 99.5 . \]
Here $n$ is the number of points, $r$ is the correlation coefficient, $se$ is the standard observation error, and $cl$ is the confidence level. Equation (1) is valid for cycles from number 0 ($k = 0$) to number 21 ($k = 10$), except for the pair of cycles 4–5. The regression equation calculated for the last six pairs, beginning with cycle 10 ($k \geq 5$), has the form

$$RM_{2k+1} = 27.674 + 1.114 \, RM_{2k},$$

$$n = 6, \quad r = 0.974, \quad se = 9.60, \quad cl > 99.5. \tag{2}$$

We introduce an index, $a(RM)$, that is the ratio of the maximum annual mean Wolf numbers in the odd-following and the even cycles in a pair. Its average value is

$$a(RM) = 1.356 \pm 0.233 \tag{3}$$

for all cycles from 0 to 21 (except cycles 4–5) and

$$a(RM) = 1.437 \pm 0.148 \tag{4}$$

for cycles 10 to 21.

Figure 1 illustrates dependences for Equations (1) and (2), as well as the predicted values for cycle 23.
2. The Rule of Gnevyshev–Ohl for the Geomagnetic \( aa \)-Index

Ohl (1966) was the first to show that the recurrent geomagnetic activity at the decay of the solar cycle can be used as an efficient predictor of the intensity of the following cycle. Before proceeding to modification of this method, let us discuss the applicability of the rule of Gnevyshev–Ohl (G–O rule) to geomagnetic activity. In other words, let us check the validity of the statement: if the cycles of \( aa \)-indices are arranged in even–odd pairs, the second cycle in the pair is always higher than the first one. This question is not so simple, because the maxima and the minima of geomagnetic activity do not coincide with those of the Wolf numbers. Therefore, the beginnings and the ends of the geomagnetic cycles are not as unambiguous, as in the case of the Wolf number cycle. So, we have chosen the following variants of the sums and of the monthly means of \( aa \)-index values:

1. \( s(1) \) is the sum of monthly mean \( aa \)-indices over an interval between two consecutive Wolf number minima, and \( q(1) \) is the mean value over the same interval;
2. \( s(2) \) is the sum of the monthly mean \( aa \)-indices over an interval between two consecutive \( aa \)-index minima, and \( q(2) \) is the mean value over the same interval;
3. \( s(3) \) is the sum of the monthly mean \( aa \)-indices over an interval between two consecutive Wolf number maxima, and \( q(3) \) is the mean value over the same interval;
4. \( s(4) \) is the sum of the monthly mean \( aa \)-indices over an interval between two consecutive \( aa \) maxima, and \( q(4) \) is the mean value over the same interval.

The data of \( aa(M) \)-index are taken from Mayaud (1973) and then from subsequent annual IAGA reports. The epochs of the \( aa(M) \)-cycle minima and maxima are derived from the smoothed monthly data of the \( aa(M) \)-index, obtained in the same way as the smoothed monthly sunspot numbers.

Unfortunately, the data series we have got at our disposal begin at 1868, i.e., cover only 5 pairs of solar cycles. The results of calculations are presented in Table I, which gives for all four variants: \( i \) – the number of the cycle; \( t_0 \) – the first month in each summation interval; the sums (\( s \)) and the mean values (\( q \)); \( a_s \) and \( a_q \) – the second to the first cycle ratios of \( s \) and \( q \) in a pair. The Wolf number cycle 22 was assumed to begin in September 1986, and it reached a maximum in July 1989; the minimum of the \( aa \)-index occurred in January 1987, and the maximum in September 1991. It should be noted that the dates of extrema of the \( aa \)-index are rather tentative. However, in fact, our further conclusions are independent of a certain small inaccuracy that might arise in determining the calendar dates.

One comment should be made about the numbers of cycles in column 1. For variants 1 and 2, the conventional numbering is used. For variants 3 and 4, we use the number of the cycle in which the cycle defined by epochs of the maxima of Wolf number or \( aa \)-index starts. Thus, the descending branch of the current cycle is joined with the ascending branch of the following cycle.

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### Table I

Sums $(s)$ and means $(q)$ of monthly $aa$-indices in cycles 11–21

<table>
<thead>
<tr>
<th>$i$</th>
<th>$t_0$</th>
<th>$s(1)$</th>
<th>$q(1)$</th>
<th>$a_s$</th>
<th>$a_q$</th>
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<tr>
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<td>Dec. 1878</td>
<td>2022</td>
<td>15.09</td>
<td>1.06</td>
<td>1.01</td>
<td>Jan. 1879</td>
<td>2060</td>
<td>15.04</td>
<td>1.01</td>
<td>1.04</td>
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<td>13</td>
<td>Feb. 1890</td>
<td>2142</td>
<td>15.19</td>
<td>0.87</td>
<td>0.88</td>
<td>July 1890</td>
<td>2074</td>
<td>15.59</td>
<td>0.91</td>
<td>0.85</td>
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<tr>
<td>14</td>
<td>Jan. 1902</td>
<td>1853</td>
<td>13.43</td>
<td>1.10</td>
<td>1.28</td>
<td>Sep. 1901</td>
<td>1878</td>
<td>13.22</td>
<td>0.68</td>
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<td>2039</td>
<td>17.13</td>
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<td>Aug. 1913</td>
<td>1272</td>
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<td>2100</td>
<td>17.50</td>
<td>1.21</td>
<td>1.17</td>
<td>Oct. 1924</td>
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<td>2536</td>
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<td>June 1934</td>
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<td>July 1892</td>
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<td>2113</td>
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<td>23.96</td>
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In spite of limited data, the calculations have shown that:

(a) The G–O rule in its original form applied to the sums of $aa$-indices is valid only for variant 1, i.e., when the beginning and the end of the cycle are determined in the traditional way.

(b) The G–O rule in a modified form applied to the mean values of $aa$-indices is valid for variants 1 and 2.

(c) For variants 3 and 4, a new rule similar to that of G–O should be formulated as follows: the monthly mean geomagnetic activity is always lower in the interval from the Wolf number maximum or $aa$-index maximum in the odd cycle to that in the even cycle, than in the same interval from the even to the odd cycle.

(d) From Table I (as well as from Figure 2) one can see a strong long-term variation of $aa(M)$-index. Our $a$-index in Table I is greater than 1.0 not only for the even—odd cycle pairs, as required by the G–O rule, but also for some odd—
even pairs. This may be a consequence of a rapid secular increase of geomagnetic activity, in addition to the effect of the G–O rule.

As a result of secular variation combined with the effect of the G–O rule, the values of our $a$-index of the even–odd cycle pairs can be greater than those of the following odd–even cycle pairs (regardless of whether they are greater than 1.0 or not). In fact, this is true for both the sums and the means in variant 1, only for the means in variant 2, and is not true at all in variants 3 and 4.

(e) Moreover, a combined effect of the secular variation and the G–O rule results in a strong regression coupling between the cycles. We can calculate regression relations of odd-following cycle data versus even cycle ones, as well as even-following cycle data vs odd cycle ones for all four variants. Five of 8 regression relations for the sums (except for the even–odd pair in variant 4 and the odd–even pair in variants 2 and 4) and all 8 relations for the means reach confidence levels greater than 90%. However, if the selection criteria are more rigorous (the confidence level greater than 97.5% and the correlation coefficient greater than 0.925) then we deal only with the cases described in items (a), (b), and (c), i.e., the even–odd pair in variant 1 for the sums, the even–odd pair in variants 1 and 2 for the means, and the odd–even pair in variants 3 and 4 for the means.

(f) The G–O rule for the $aa$-index is most clearly pronounced in variant 1, i.e., when the beginning and the end of the cycle are determined in a traditional way. Only in this case are the additional rigorous criteria, stated in items (d) and (e), satisfied both for the sums and for the means. Figure 2 illustrates $s(1)$ and $q(1)$ as functions of the cycle number. The regression equations have the form

$$S(1)_{2k+1} = 456.832 + 0.892155 S(1)_{2k},$$

$$n = 5, \quad r = 0.955, \quad se = 160.1, \quad cl > 99.0,$$  \hspace{2cm} (5)
\[ q(1)_{2k+1} = 2.9847 + 0.9590 \, q(1)_{2k} , \]
\[ n = 5, \quad r = 0.931, \quad se = 1.776, \quad cl > 97.5 . \]  

(6)

The average ratio of \( s(1) \) in the following and the preceding cycles in an even–odd pair is \( 1.092 \pm 0.074 \), and the average ratio of \( q(1) \) is \( 1.1342 \pm 0.107 \) in an even–odd pair. This agrees with the results obtained by Hedeman and Dodson-Prince (1986).

Taking into account the high correlation coefficient, we give also the regression equation for the odd–even pair in variant 3:

\[ q(3)_{2k+2} = 2.822 + 0.9216 \, q(3)_{2k+1} , \]
\[ n = 5, \quad r = 0.997, \quad se = 0.357, \quad cl > 99.5 . \]  

(7)

3. New Method for Prediction of Future Solar Cycle from Geomagnetic Data

As mentioned above, Ohl (1966, 1968) was the first to suggest that characteristics of recurrent geomagnetic activity be used for predicting the height of the following solar cycle. Afterwards the method was repeatedly modified and improved by different authors (e.g., see Ohl, 1976; Ohl and Ohl, 1979; Thomson, 1990; Brown, 1990, and references therein). In fact, all modifications of the method are based on a positive correlation between the geophysical activity at the descending branch of the sunspot cycle and the height of the following cycle. It is not critical what geophysical index is chosen as a predictor. On the other hand, the level of geophysical activity, and especially its deviation from the statistical level corresponding to the current value of Wolf numbers, is of great importance. All modifications of the method use as a predictor the difference between the normalized geophysical parameter and the Wolf number.

Unfortunately, in spite of reliability and other advantages of the method, it is inapplicable for early prediction because it is based on the data series extending up to the first year after the minimum, i.e., it practically predicts the cycle that has already started.

In this paper, we suggest as a predictor the degree of dependence of the geophysical index on the Wolf numbers, i.e., \( p = \partial a a / \partial R \). To be more precise, we are going to use as a predictor the angular regression coefficient (straight line slope), \( p \), between the \( a a \)-index and the Wolf number derived for a relatively small interval at the descending branch of the cycle. Since the Wolf numbers at the descending branch decrease much faster than the \( a a \)-index (which even grows at the beginning of the branch), extremely high geophysical activity will obviously correspond to great negative values of the \( p \)-index, so that high correlation can be expected between \( p \) and the height of the following cycle.
Fig. 3. The maximum Wolf number in the forthcoming cycle, \( R_{i+1} \), as function of the \( p_i \) index.

For convenience of application of the method and for early prediction, we have used unsmoothed monthly mean values of \( aa \)-index and Wolf numbers, and tried to reduce to a minimum the selection interval over which the \( p \)-index was calculated. As the first month of the selection interval, we have used either the exact calendar date of maximum of the unsmoothed Wolf number (month 0), or the date a year and a half later (month 18). The latter was chosen proceeding from the consideration that the global magnetic field reversal in the Sun usually occurs 1–2 years after the Wolf number maximum (Makarov, Fatianov, and Sivaraman, 1983; Makarov and Sivaraman, 1983). The end of the selection interval was initially fixed at month 50 (i.e., the overall length of the selection interval was 51 or 33 months in the first and the second case, respectively). Then we tried shorter intervals (18–47 and 18–41) to enable earlier prediction.

The selection interval beginning 18 months and ending 50 months after the maximum of the Wolf number cycle, i.e., lasting for 33 months, proved to be most suitable, if we do not discriminate between the even and the odd cycles and do not eliminate the pair of cycles 4–5.

Figure 3 illustrates \( R_{i+1} \) as a function of \( p_i \). The regression equation has the form

\[
R_{i+1} = -743.5p_i + 152.4 ,
\]

\[
n = 11, \quad r = -0.924, \quad se = 18.1, \quad cl > 99.5 .
\]

Note that correlation with the height of the current maximum is practically absent, \( r = -0.170 \).

Taking into account all said above about the rule of Gnevyshev–Ohl, it is interesting to see how the proposed method works when separately applied to odd and even cycles. Table II presents the correlation coefficients of correlations.
TABLE II

<table>
<thead>
<tr>
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<th>0–50</th>
<th>18–50</th>
<th>18–47</th>
<th>18–41</th>
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<td>$p_{2k+1}$ and $RM_{2k+2}$</td>
<td>−0.962</td>
<td>−0.816</td>
<td>−0.855</td>
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<td>−0.0056</td>
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<td>−0.362</td>
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<td><strong>Even–odd pair</strong></td>
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<tr>
<td>$p_{2k}$ and $RM_{2k+1}$</td>
<td>−0.190</td>
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<td>$p_{2k}$ and $RM_{2k}$</td>
<td>−0.364</td>
<td>−0.941</td>
<td>−0.960</td>
<td>−0.977</td>
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</table>

between $p$-index and $RM$ of the following and the current cycle for different combinations of cycles and for different lengths of the selection interval for which the $p$-index was determined.

As seen from Table II, there is a difference in application of our method to even and odd cycles. In the odd cycle, the best correlation between the $p$-index and the $RM$ of the following cycle is achieved when a very long selection interval (51 months) is used. This dependence is illustrated in Figure 4(a). The regression equation has the form

$$R_{2k+2} = -661.76p_{2k+1} + 127.45 ,$$

$$n = 6, \quad r = -0.962, \quad se = 12.42, \quad cl > 99.5 .$$

Note that correlation with the maximum of the current cycle is very small (−0.400).

A somewhat different situation arises in the even cycles. Here, the correlation between the $p$-index and the maximum of the following cycle is the largest at a selection interval of 18–50 months, as in the case where the type of the cycle (even or odd) is not taken into account (see Equation (7)). This relation is illustrated in Figure 4(b). The regression equation has the form

$$R_{2k+1} = -735.12p_{2k} + 152.17 ,$$

$$n = 5, \quad r = -0.992, \quad se = 6.93, \quad cl > 99.5 .$$

Here, some remarks should be made:

(a) In the even cycles, the correlation of the $p$-index with the maximum Wolf number in the current cycle is also high, which naturally results from the rule of Gnevyshev–Ohl.
(b) Though the data series for the \textit{aa}-index are shorter (only cycles 11 to 22) than for the Wolf numbers, the statistics are at least not worse than in the case of the rule of Gnevyshev–Ohl for the Wolf numbers.

(c) The length of the descending branch in the odd cycles 11–21 ranges from 72 to 100 months with an average of 85.0 \(\pm\) 9.8 months, and in the even cycles 12–20 – from 65 to 91 months with an average of 80.8 \(\pm\) 9.8 months. A prediction based on our new method can be made well before the cycle minimum.

(d) There is a fundamental difference between the prediction methods based only on the sunspot number and those using geomagnetic activity data. As mentioned in Section 1, from the rule of Gnevyshev–Ohl it follows that the forecast can only be obtained for the odd-following cycles. In other words, the G–O rule says that the pairs of even and odd-following cycles are mutually dependent and the pairs of odd and even-following cycles are independent. Both Ohl's and our new method are based on geomagnetic activity. Therefore they can provide a relatively early forecast of the height not only for the odd, but also for the even cycles.

4. Prediction of Cycle 23

The unexpectedly high cycle 22 brought many scientists to believe that the next cycle 23 would be very high, or even higher than ever. This point of view was also supported by the author (see Obridko and Kuklin, 1994; Obridko \textit{et al.}, 1994; Obridko, Oraevsky, and Allen, 1994).

Some forecasts of cycle 23 are summarized in Table III, taken from Obridko \textit{et al.} (1994) with some minor changes. The same paper gives a detailed analy-
sis of the applied prediction methods. Without repeating here the whole analysis, we should note that the methods based on the relationship of the cycles in a pair (Wilson, 1988, 1992; Kopecký, 1991; Vitinsky, 1992; Rivin, 1992; Obridko et al., 1994; Obridko, Oraevsky, and Allen, 1994) yield high values of the forthcoming cycle. To this group belongs the prediction of Tritakis (1986). High values are also obtained when the phenomena observed in the preceding cycle are taken into account, in particular the behaviour of high-latitude filaments and faculae (Makarova, 1991; Makarov and Mikhailutsa, 1992). The predictions based on secular and super-secular variations of the 11-year cycle show rather low values (Shove, 1983; Chistyakov, 1983; Kontor et al., 1983). Kuklin (1993) regarded the relation between the neighbouring odd cycles as a one-dimensional logistic mapping of the process with intermittence, and obtained two versions of the forecast: abnormally high and abnormally low cycle 23 (see also Obridko et al., 1994).

In fact, it is easy to obtain from (1)–(4) the following values (consecutively):

\[ RM_{23} = 184.14 \pm 21.9 \], \hfill (1a)
\[ RM_{23} = 203.2 \pm 10.7 \], \hfill (2a)
\[ RM_{23} = 213.7 \pm 36.7 \], \hfill (3a)
\[ RM_{23} = 226.5 \pm 23.3 \]. \hfill (4a)

These values are so large that it seems reasonable to expect a very high cycle.
However, the situation becomes quite different if our new method is applied. To predict the height of cycle 23, we can use Equations (7) and (9). Over the interval of months 18 to 50 after the maximum of cycle 22, \( p = 0.1054 \). Substitute this into (7) and (9) to give, respectively,

\[
RM_{23} = 73.8 \pm 18.1, \quad (7a)
\]
\[
RM_{23} = 74.7 \pm 6.9, \quad (9a)
\]

which disagrees with the values inferred from the rule of Gnevyshev–Ohl. Somewhat larger values (87.9 \( \pm \) 20.0 and 92.8 \( \pm \) 14.2, respectively) are obtained if the selection intervals 18–47 are used.

Thus, the following conclusions can be made:

1. Two methods of similar statistical reliability yield basically different results. Today, we do not know which of them is preferable.

2. Both methods were correct in the past, but now we are facing an anomaly of the type of cycles 4–5 and 8–9. It may be either a break-down of regularities, or a mere fluctuation. In any case, the next cycle is expected to be absolutely abnormal.

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