The sensitivity of Doppler imaging to line profile models

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Abstract

In recent years there have been a number of successful reconstructions of the surfaces of fast rotating stars using Doppler imaging. As a large proportion of the imaged stars show large high-latitude spots, some doubt remains as to whether these high-latitude spots might be artefacts. In this paper we investigate how sensitive Doppler imaging is to the exact shape of the mapping line profile and its variation as a function of limb angle. We find that Doppler imaging is surprisingly robust against errors in the limb dependence of the profile, but that errors in the shape of the profile, and in particular the neglect of blends at considerable distances from the mapping line, will lead to spurious banding in the reconstructed images.

Key words: techniques: image processing - stars: activity - stars: imaging.

1 Introduction

A large proportion of stars that have been Doppler imaged in recent years seem to show large spots straddling or covering the poles (Vogt & Penrod 1983; Vogt 1988). This sparked some controversy on how believable these pictures were and whether the 'polar spots' were real features or just artefacts of the image reconstruction (Piskunov, Tuominen & Vilhu 1990; Strassmeier et al. 1991, 1993). One of the reasons for this controversy was the initial lack of an explanation why magnetic flux should surface at the stars' poles and the belief that most stars will show similar behaviour and characteristics to the Sun. There are now, however, theories to explain high-latitude activity. Schüssler & Solanki (1992) have suggested a model where the preferred spot latitudes depend on the rotation velocity and the depth of the convection zone of the star. In a rapidly rotating star the Coriolis force can be large enough to dominate over the magnetic buoyancy force, so that magnetic flux tubes will follow paths nearly parallel to the axis of rotation and surface at high latitudes, provided that the convection zone is deep enough. Recent MHD model calculations seem to support this conjecture for some pre-main-sequence stars (Caligari et al. 1994).

Another objection raised against the polar spots seen on Doppler maps touches the technique of Doppler imaging more specifically. The main problem posed by polar features is that they are symmetrical in time, i.e. that they do not move through the line profile, and only 'fill in' the line core. For polar spots to be detected reliably one needs to know the exact shape of the specific intensity profile at all points on the stellar disc for each particular mapping line and star, particularly around the line core. There is a host of possible mechanisms and errors that can affect the line shape, such as e.g. anisotropic turbulent motions and depth-dependent phenomena or just poor fits to the continuum of the target star. It has also been pointed out that fast rotators and stars on which one expects to find large spots tend to be chromospherically very active, and that hence some of the mapping lines might be filled in with chromospheric emission which would look like a polar spot on image reconstructions. The purpose of the work described in this paper was to examine some of these mechanisms and to study their effects on the reconstructed images.

In the following section we will give a brief description of Doppler imaging and of our particular reconstruction code. In Section 3 we compare different approximations for the shape of the mapping line and its behaviour as a function of limb angle. Section 4 describes how these different approximations affect the shape of the rotationally broadened line profile and illustrates the effects of the line profile changes on the image reconstructions. Our findings are discussed and summarized in the last two sections.

2 Doppler Imaging

Doppler imaging utilizes the fact that inhomogeneities in the surface brightness distribution will lead to deformations in a rotationally broadened absorption-line spectrum. The velocity shift of the profile deformation with respect to the line centre gives an instantaneous one-dimensional projection of the location of the inhomogeneity on the stellar surface. One can hence determine the position of a stellar spot by inverting time series of high-resolution spectra.

Due to the noise present in the data and the limited resolution there is no unique surface distribution that would fit the spectra and one is faced with an ill-conditioned inversion problem.
A very successful method has been the iterative forward techniques as first applied by Khokhlova (1975) and Khokhlova & Ryabchikova (1975). In an iterative procedure an initial surface image is translated into a flux profile which is then compared to the dataset and optimized. For this translation we divide the stellar surface into a grid of pixels $i$ each with area $w_i$. The flux profiles are obtained by integrating the specific intensity profiles from all visible surface pixels, taking into account the appropriate rotational shift at each phase.

Our code uses a two-temperature model. All spots have the same temperature and we neglect any contributions from the penumbrae. This allows us to characterize the stellar surface in terms of ‘spot filling factor’ rather than bolometric power. The spot filling factor, $f_s$, is the fraction of the pixel area occupied by spots. The advantage of this has been illustrated by Collier Cameron (1992). Because of rotational blurring and finite signal-to-noise ratios, the assumption that a unique temperature can be assigned to each pixel on the basis of its radiated bolometric power does not hold. A change in the surface area due to blurring will force the bolometric flux to adjust. As the combined spectrum of two neighbouring pixels with different temperatures will be different from the spectrum of the same two pixels at the effective temperature corresponding to the averaged bolometric flux, the model spectrum will contain specific intensity contributions from temperature components that are actually not present on the stellar surface. This can lead to miscalculation of the bump amplitude and can alter the reconstructed image in an unpredictable way. The reason for this is that the functional dependence of the line equivalent width on temperature is non-linear and is indeed double-valued for many of the common Doppler imaging lines in the temperature range we consider here.

In our approach all information about the line physics is stored in a three-dimensional lookup table. It contains pre-calculated monochromatic specific intensities as functions of wavelength, foreshortening and temperature on a linear interpolation grid. This method circumvents the time-consuming task of calculating line profiles in the innermost loop of the program.

An advantage of the forward iterative method is that the mismatch between the data computed from the current image, $D(f)$, and the observed data, $F$, can be easily quantified in terms of the observational error, $\sigma$, for example in the form of the $\chi^2$ statistic:

$$
\chi^2(f) = \sum_{k=1}^{M} \left( \frac{F_k - D_k(f)}{\sigma_k} \right)^2
$$

where $M$ is the total number of observations. Due to the ill-conditioned nature of the inversion one needs to introduce a further image constraint in order to select a particular image from the multitude of possible images with different grades of complexity. The commonest choices are either to select the smoothest image with the aid of Tikhonov's functional (Tikhonov 1963) or, alternatively, to use maximum entropy to select the image where there is least correlation between different parts of the image, unless the data demand it (Piskunov et al. 1990). The general form of Tikhonov's regularizing function is

$$
r_t(f) = \int \int_{\text{surface}} |\nabla f(M)|^2 \, dS(M)
$$

whereas the entropy is given by

$$
r_E(f) = -\int \int_{\text{surface}} f(M) \log(f(M)) \, dS(M).
$$

We use the maximum entropy criterion as described by Skilling & Bryan (1984). It can be defined in terms of the spot filling factor as

$$
S(f) = -\sum_{i=1}^{n} w_i \left[ f_i \log \frac{f_i}{m} + (1 - f_i) \log \frac{1 - f_i}{1 - m} \right]
$$

where $m$ is the default fraction of spot coverage and is set to a very small, positive value. Maximization of the entropy subject to the $\chi^2$-constraint is then equivalent to solution of the equation

$$
w_i \left[ \log \frac{m}{1 - m} - \log \frac{f_i}{1 - f_i} \right] = \lambda \frac{\partial \chi^2(f)}{\partial f_i}.
$$

The Lagrange multiplier, $\lambda$, is fixed by demanding that $\chi^2 \approx M$. This formulation ensures that a uniform photometric brightness is maintained, so that bright spots cannot develop.

In the past, there have been extensive investigations into the kind of artefacts that can be introduced in the image reconstruction by misjudging some of the stellar parameters, such as the inclination angle or the projected rotation velocity (see for example Vogt, Penrod & Hatzes 1987; Rice, Wehlt & Khokhlova 1989). Most of the errors either produce rotationally invariant artificial structure in the form of bands or affect the latitudinal resolution by shifting existing structure towards the equator or the polar regions. In this paper we investigate what effects errors in the physics of spectral line formation have on the image reconstruction. We were particularly interested in the behaviour of the line profile as a function of limb angle and in the artefacts that might be introduced when neglecting its variability.

3 THE LOOKUP TABLES

In our approach, all information about the physics of the star is contained in the lookup tables where the profile of the mapping line is stored as a function of temperature and limb angle. Differences between the lookup tables should hence affect the image reconstructions. The main points of divergence between lookup tables can be either the line profile shape as such or the way the profile shape and the continuum vary as a function of limb angle. We decided to compare three different approaches of generating lookup tables that are commonly used by Doppler imaging groups. By comparing them with each other we hope to be able to pinpoint the most important variables that will affect the image reconstructions. In the following section we present the different techniques we used to generate lookup tables.

3.1 LTE spectral synthesis

The spectral intensities are calculated using a modified version of SPECTRUM, an LTE spectral synthesis code originally written by P. Dufort. We obtained SPECTRUM through the UK SERC Collaborative Computational Project No. 7 for the Analysis of Astronomical Spectra, CCP7 (Jeffery 1989). Having started life as a spectral synthesis code for early-type stars, SPECTRUM only
considered radiative and Stark broadening when computing the overall damping profile.

Our modified version includes van der Waals damping as well as modified opacity routines and has additional routines that calculate specific intensities on a grid of limb angles instead of disc-integrated fluxes. This approach therefore takes into account the variation of the profile as a function of limb angle. It also allows more complicated phenomena such as depth-dependent or radial-tangential microturbulence to be included. The atmospheric models used have been taken from the ATLAS9 grid of dwarf model atmospheres (Kurucz 1991), which were also obtained through CCP7.

Clearly it is desirable to obtain lookup tables that are as accurate as possible. This means that we want to know the continuum intensity and the shape of the profile as a function of position on the star, taking into account the various physical effects that might characterize each specific star. In practice there are, however, some limitations on spectral synthesis. Many of the atomic parameters such as gf-values and damping coefficients are still poorly determined. There are also uncertainties concerning the abundances of most stars. Although much progress has been made in recent years, the accuracy of model atmospheres for cool stars is still debatable. Another problem is linked to the presence of molecules in cool stellar atmospheres and their effect on the opacity. We have so far not included any molecular lines in our spectral synthesis program, but it can in principle be done.

3.2 Template star profiles

In an alternative approach we used observed flux profiles of bright template stars to approximate the specific intensity profiles. The limb angle-dependent intensity profile was obtained by multiplying the normalized spectrum of a template star with the continuum intensity at each limb angle. The continuum intensities (as functions of limb angle, wavelength and temperature) were calculated with the spectral synthesis code described above. Alternatively, we calculated the linear and quadratic limb darkening coefficients using either the tabulations of Wade & Rucinski (1985) or our continuum intensities, and used these to scale the flux profiles.

An important difference between full intensity calculations and scaled flux calculations is the variability of the profile shape. The scaled flux approach assumes that the line profile is constant over the whole disc. Performance of a full spectral synthesis, however, shows that the depth and equivalent width of the profile change considerably as a function of limb angle. Instead of remaining constant, the residual intensity at the line centre and equivalent width decrease for decreasing limb angles. This implies that assumption of a constant line profile shape overestimates the contribution of the limb and underestimates the contribution of the disc centre to the line absorption. Fig. 1 shows that for \( T = 5000 \) K and \( \lambda = 643.9 \) nm the effect is most pronounced below approximately \( \mu = 0.4 \), where \( \mu \) is the cosine of the limb angle, also called the foreshortening. The equivalent width and the residual intensity have been normalized by division through their respective values at the disc centre. Fig. 1 also shows the deviation between the LTE continuum intensity and the linear limb darkening fit to it.

For the final fitting it is useful to have lookup tables with a sequence of different equivalent widths centred around the measured equivalent width of the profile of the target star. We altered the equivalent width of the original flux spectra in a rather crude way by subtracting or adding some continuum to the normalized profile. These profiles were subsequently renormalized, resulting in an increased or decreased equivalent width. The profiles so obtained were then scaled with the continuum intensity as described above.

3.3 Gaussian profile

In a third approach, we assumed that the mapping line could be approximated by a Gaussian. We neglected any blends and measured the line depth and equivalent width from the observed template star spectra. The continuum was scaled in the same way as for the template star lookup tables. The template star and the Gaussian profile approaches both assume implicitly that the shape of the line profile is independent of limb angle.

4 COMPARISONS BETWEEN DIFFERENT LOOKUP TABLES

In this section we present comparisons between different approximations for the mapping line profile and its variability as a function of limb angle. The comparisons were generally done as follows. We produced lookup tables using each of the three methods described in Section 3 above in turn, and used them to compute the rotationally broadened line profile for an un-spotted model image of the stellar surface. The resulting profiles were intercompared. Next, we created a model stellar image with a known spot configuration. We followed the same procedure as before. Fig. 1 shows the residual intensity (filled polygons) and equivalent width (stars) as a function of limb angle for Ca i 643.9 nm at 5000 K. The open polygons show the deviation of the real limb-darkening law from its linear fit.

Figure 1. The residual line intensity (filled polygons) and equivalent width (stars) as a function of limb angle for Ca i 643.9 nm at 5000 K. The open polygons show the deviation of the real limb-darkening law from its linear fit.
procedure used by Collier Cameron & Unruh (1994) to determine the best-fitting values of the line equivalent width and the projected equatorial rotation speed $v \sin i$. This involves minimizing the total spot area in the reconstructed image relative to these parameters. The procedure is justified on the grounds that, in our two-temperature model, any artefact that results from attempting to fit the model to the data in the presence of systematic errors must generally introduce more spots into the reconstructed image. In most of the tests presented here we used projected rotation velocities between 50 and 90 km s$^{-1}$ and an inclination, $i$, of 60°. The results, however, hold for a large range of inclination angles; for higher inclination angles, the ambiguity between the northern and southern hemispheres will increase, and for lower inclination angles one obtains less and less information on lower regions and the area around the equator. For the following tests we selected three mapping lines with different line strengths and atomic parameters: two Ca i lines at 643.9 nm and 671.8 nm and the Fe i line at 666.3 nm. Their respective equivalent widths are of the order of 0.032, 0.019 and 0.011 nm. The Fe i line has a strong blend in its blue wing which increases the equivalent width to about 0.015 nm.

4.1 The variation of the line profile as a function of limb angle

We first investigated how variations in the shape and equivalent width of the line profile with limb angle affect the rotationally broadened line profile. For this purpose we generated LTE spectral synthesis lookup tables for mapping lines with different strengths. Using the same atomic parameters we then also created disc-integrated flux profiles and scaled these according to the continuum intensities of the LTE spectral synthesis lookup tables. The lookup tables so obtained are hence identical, except for the behaviour of the line profile and its equivalent width as a function of limb angle. As pointed out in Section 3.2, neglect of the variability of the profile shape causes underestimation of the contribution of the disc centre to the line flux and results in a shallower rotationally broadened line profile. For the three mapping lines we tested, we found this effect to be strongest for the Ca i line at 643.9 nm. Fig. 7, where the line equivalent width is plotted versus limb angle, confirms this, as it shows that the equivalent width of the Ca i line at 643.9 nm declines most rapidly towards the limb. Fig. 2 compares the rotationally broadened profiles of the constant-profile approximation (dotted lines) with the spectral synthesis approach for $v \sin i = 50$ km s$^{-1}$ and $v \sin i = 90$ km s$^{-1}$. The lower equivalent width of the constant profile compared to the full spectral synthesis profile might in part be due to small numerical errors in the disc integration for the calculation of the flux profile. It turns out that the shape of the synthetic synthesis profile can be matched well using constant line profiles. In the case of the profile with $v \sin i = 50$ km s$^{-1}$, this requires a decrease in the rotational velocity of about 0.5 km s$^{-1}$ and an increase of the equivalent width by less than 2 per cent.

Figure 2. Rotationally broadened profiles, $v \sin i = 50$ km s$^{-1}$ (deep curves) and $v \sin i = 90$ km s$^{-1}$ (shallow curves), comparing the constant-profile approximation to a full spectral synthesis calculation for the Ca i line at 643.9 nm and $T = 5000$ K. The solid lines show the rotationally broadened profiles for the spectral synthesis calculation; the dotted lines represent the constant-profile approach.

To find out whether the mismatch between the rotationally broadened profiles is substantial enough to lead to artefacts in the image reconstruction, we followed the procedure outlined in Section 4 and generated a synthetic data set from the image shown in Fig. 3(a) for a projected rotation velocity of $v \sin i = 50$ km s$^{-1}$ and the spectral synthesis lookup table for Ca i. As the constant-profile lookup table is too shallow, one would expect that structure at high latitudes would be suppressed, whereas lower lying features should be emphasized. We therefore selected an input image with a polar spot, expecting the polar feature to weaken in the reconstruction. If we keep all the stellar parameters fixed, the reconstructed image shown in Fig. 3(c) shows some faint artefacts, most of them clustered close to the equator. The polar spot, however, remains and the four lower latitude spots are well recovered. Most of the artificial low-latitude structure disappears when the rotation velocity is decreased by 0.7 km s$^{-1}$ or, alternatively, by 0.4 km s$^{-1}$ if the equivalent width of the constant-profile lookup table is increased by 1 per cent (Fig. 3(d)). The resulting image is then virtually identical to the ideal reconstruction as shown in Fig. 3(b). These findings may seem surprising in the light of an earlier investigation by Piskunov & Rice (1993) who find that, for Ca i at 643.9 nm and other highly temperature-sensitive lines, a weak artificial polar spot appears in their reconstructions when they neglect the change in the line profile shape. The different artefacts observed by Piskunov & Rice (1993) and us are due to the different choices of the image parameter, i.e. temperature and spot filling factor, in our respective Doppler imaging codes. As mentioned above, neglect of the variation of the shape of the line profile as a function of limb angle results in a flatter rotationally broadened profile. If the temperature is used as an image parameter, the profile of a temperature-sensitive line, such as Ca i at 643.9 nm, can be deepened by lowering the temperature. A flatter profile can hence be ‘offset’ by introducing a gentle temperature gradient whereby the temperature decreases from the equator to the pole. (For some lines, e.g. ionized iron lines at solar temperatures, where the equivalent width will increase with increasing temperature, one would expect to observe a temperature gradient in the opposite direction.)

4.2 Different limb darkening laws for the continuum intensity

In this section we investigate how well linear and quadratic limb darkening laws approximate the true behaviour of the
Figure 3. Artefacts due to the neglect of the variability of the profile shape as a function of limb angle. The dataset was produced with Ca i at $\lambda = 643.9$ nm, $T = 5000$ K, $v\sin i = 50$ km s$^{-1}$ and a signal-to-noise ratio of 340. The values in brackets give the fraction of the stellar surface covered by spots. (a) Input image A (0.067), (b) 'ideal' reconstruction of A (0.063) taking account of the profile variation, (c) reconstructed image when the profile variation is neglected and the stellar parameters are fixed (0.086), (d) similar to (c), but now the rotation velocity has been decreased to 49.6 km s$^{-1}$ and the equivalent width of the constant-profile lookup table has been increased by about 1 per cent (0.067).
continuum intensity. The value of the continuum intensity at 640 nm and 5000 K is plotted in Fig. 4 for 10 limb angles. The solid line is the linear limb darkening and dashed line the quadratic limb darkening approximation. The agreement between linear and quadratic limb darkening laws is very good for limb angles, $\theta$, with $\cos \theta > 0.4$. Closer to the limb the differences become more noticeable. We also looked at limb darkening laws with grossly over- and underestimated limb darkening coefficients. The two dotted lines in Fig. 4 show the slope of the limb darkening law when the limb darkening coefficient is increased or decreased by 30 per cent. We scaled disc-integrated flux profiles according to different
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limb darkening laws to obtain lookup tables and used these to produce rotationally broadened profiles.

Fig. 5 shows how the different limb darkening laws from Fig. 4 affect the broadened profiles at projected rotation velocities, $v \sin i$, of 50 and 90 km s$^{-1}$. The quadratic and linear limb darkening laws produce almost identical rotationally broadened profiles for all the lines and rotation velocities we tested. This is not surprising as the linear and quadratic limb darkening laws only differ significantly when approaching the limb. The projected surface area of all pixels with $\cos \theta \geq 0.4$ corresponds to about 15 per cent of the total projected surface area. Due to this projection effect as well as the lower intensity of the pixels close to the limb, the flux difference between the outer pixels is not large enough to be noticeable in the rotationally broadened profile. On the scale shown here, the profiles with linear and quadratic limb darkening laws are indistinguishable; only the profiles obtained with linear limb darkening laws are plotted. The solid line is the rotationally broadened profile obtained with the correct limb darkening coefficient. The dotted and dashed lines are with the over- and underestimated limb darkening coefficients as shown in Fig. 4. Underestimation of the limb darkening coefficient, i.e. assumption of a flatter limb darkening curve, will overestimate the contributions at the limb and result in shallower rotationally broadened profiles. Similarly, overestimation of the limb darkening coefficient will result in deeper and narrower profiles. Fig. 5 also shows that the deviations in the profiles are much more prominent for lower rotation velocities.

As already observed in Section 4.1, these profile changes are very similar to the changes one observes when altering the rotational velocity or the equivalent width of the mapping line by small amounts. As the exact value of the rotational velocity is not known for real stars, the best-fitting value of the rotational velocity will depend on other profile parameters such as the equivalent width and the limb darkening law used. This raises the question whether a combination of systematic errors in the above quantities might accidentally lead to the creation of artificial structure or the suppression of real features.

We generated synthetic data using the linear limb darkening law that we obtained from the spectral synthesis program. We first reconstructed images from the resulting data set assuming a quadratic limb darkening law. As expected, the reconstructed images were identical to the ones obtained with the input lookup table. We then tried reconstructing images using increasingly flatter and steeper limb darkening laws. To fit the data with steeper limb darkening laws we had to increase the $v \sin i$ value of the star, and to fit the flatter limb darkening laws we had to decrease it. Differences between the individual reconstructed images were very small and hardly noticeable by eye. It turned out, however, that the more the lookup table used for the reconstruction deviated from the input lookup table, the more the spot area increased with respect to the ‘ideal’ reconstruction. More noticeable differences only appeared when the slope of the limb darkening was misjudged by as much as about 30 per cent. The exact value depends on several parameters such as the line equivalent width and $v \sin i$. For lower rotational velocities and smaller equivalent widths the misfits become more severe.

Fig. 6 shows some reconstructed images using different linear limb darkening coefficients, $\mu$. It illustrates that errors in the limb darkening coefficients tend to suppress or emphasize some of the existing features, rather than produce many spurious spots. Depending on the noise level, noise signatures at particular latitudes might also be fitted. This can lead to
Cal 643.9 nm linear limb darkening (u = 0.438, vsini = 50 km/s)

Figure 6. Effects of incorrect limb darkening coefficients for Ca i at λ = 643.9 nm, T = 5000 K, v sin i = 50 km s⁻¹ and a signal-to-noise ratio of 340. The values in brackets give the fraction of the stellar surface covered by spots. (a) Image reconstructed from a dataset (linear limb darkening coefficient u = 0.62, input image A) assuming that u = 0.44 and v sin i = 50 km s⁻¹ (0.088), (b) input image B (0.062), (c) 'ideal' reconstruction of B (0.057), (d) reconstruction of B using u = 0.81 and v sin i = 51.5 km s⁻¹ (0.058).
Figure 6 – continued

small additional features, usually clustered in a particular latitude band. For very high signal-to-noise ratios one observes banding that can be slightly structured in accordance with the phase coverage. The more common signatures of errors in the limb darkening laws, however, are large empty areas where no structure is allowed to grow. For flatter limb darkening laws, structure at high latitudes is suppressed and features at the low latitudes which are the sole contributors to the wings of the rotationally broadened profile become more prominent. We created a synthetic dataset with input image A and linear limb darkening law. The input image and the 'ideal' reconstruction are shown in Figs 3(a) and 3(b). The image reconstructed
from this dataset – using a flatter limb darkening law without adjusting the rotational velocity – is shown in Fig. 6(a). The kind of artefact that appears is similar to what is seen on a weaker scale when the variability of the line profile is neglected. This prompted us to test the fits for lower rotational velocities. It turned out that, when the rotational velocity is decreased to 48.8 km s$^{-1}$, the artificial banding disappears and we obtain a picture that is virtually indistinguishable from the ideal reconstruction shown in Fig. 3(b). To show the effects of steeper limb darkening laws we produced synthetic data from the spot configuration shown in Fig. 6(b), image B. The ideal reconstruction is shown in Fig. 6(c). The reconstructed image that we obtained when using lookup tables with a steeper limb darkening law and without altering the rotational velocity showed a very strong extended polar spot as well as some fainter artificial structure at the equator. Existing structure at latitudes between 20° and 60° was suppressed. Apart from the now much weaker polar feature, the spurious structure disappeared when the rotational velocity was increased to 51.5 km s$^{-1}$.

4.3 Anisotropic microturbulence

Originally, microturbulence was introduced to explain the enhanced equivalent width in the flat parts of the curve of growth. As modelling techniques improved and three-dimensional hydrodynamics and radiative transfer were included in the simulation code, small-scale velocity fields were observed that could explain the effects previously attributed to microturbulence (Dravins & Nordlund 1990). Nevertheless, microturbulence remains a useful parametrization of the velocity fields, provided that they are on length scales shorter than the photon mean free path. Microturbulent motions enhance the equivalent width of intermediate to strong lines by increasing the frequency range over which absorption can take place.

Solar microturbulence measurements seem to suggest that the tangential component of the microturbulence exceeds the radial component at all depths by roughly 1 km s$^{-1}$ (Gray 1988). Following Gray, the total microturbulence, $\zeta$, was parametrized in terms of the radial and tangential microturbulences, $\zeta_r$ and $\zeta_t$:

$$\zeta^2 = \zeta_r^2 \cos^2 \theta + \zeta_t^2 \sin^2 \theta.$$  \hfill (6)

This parametrization reflects the shift of observability from radial motions to tangential motions when moving towards the limb. Only radial motions will produce an observable Doppler shift at the disc centre. At the limb, on the other hand, the Doppler shifts will be due to tangential velocities. The ‘physicality’ of this parametrization is certainly doubtful, but it can be seen as a first attempt to approach the problem of anisotropic surface motions.

Fig. 7 shows how radial-tangential microturbulence alters the equivalent width of the mapping lines depending on their strength. We have normalized the equivalent widths by dividing them by the disc-integrated equivalent width of the line. The plot shows the normalized equivalent widths as a function of the limb angle for three lines: Ca i at $\lambda = 643.9$ nm and an equivalent width of 0.033 nm; Fe i at $\lambda = 666.3$ nm and an equivalent width of 0.011 nm; and Ca i at $\lambda = 671.8$ nm and an equivalent width of 0.016 nm. An increase of the tangential microturbulence shifts the maximum equivalent width towards the limb, but it can be seen that the different lines are affected in a different way according to their strength. The two weaker lines reach maximum equivalent width at a foreshortening coefficient, $\mu$, of about 0.5, the stronger 643.9-nm line at about 0.6. The main difference, though, is in the shape of the curve. Whereas the strong line shows an almost constant equivalent width when approaching the disc centre, the two weak lines show a very marked decrease. At the disc centre, the values of their equivalent widths are comparable to the equivalent widths at a limb angle of 0.2.

The effect of the radial-tangential microturbulence on the rotationally broadened profile can be seen in Fig. 8. It shows the profiles of the Ca i line at 643.9 nm and the Fe i line at 666.3 nm for a rotational velocity of 50 km s$^{-1}$. The solid lines are for isotropic and the dotted lines for radial-tangential microturbulence. The shift of the maximum equivalent width towards the limb effectively weakens the contribution of the disc centre to the line flux and results in shallower rotationally broadened profiles. If, conversely, the radial component is stronger than the tangential, the disc centre will contribute to a larger extent to the line flux and the profiles will show steeper wings. As before, these effects can be corrected to some extent by varying the $v \sin i$ of the star and the equivalent width of the line.

Figs 7 and 8 also show that the effect of anisotropic microturbulence becomes stronger compared to the actual line depression with decreasing equivalent width. If a star had anisotropic surface motions substantial enough to be picked up as profile deformations, one would expect small systematic differences between reconstructed images obtained with lines of different strengths.

We generated synthetic data with a tangential microturbulence value of 3.0 km s$^{-1}$ and a radial microturbulence value of 1.0 km s$^{-1}$. The projected rotational velocity, $v \sin i$, was set to 50 km s$^{-1}$ and the input image is shown in Fig. 6(b). From these data we reconstructed images using a lookup table with isotropic microturbulence. The results are shown in Fig. 9. As a higher tangential microturbulence strengthens the contributions of the limb with respect to the contributions of the centre, the profile will be shallower. If one tries to fit the data using the same rotational velocity and equivalent width, one obtains a large artificial polar structure. One can, however, improve the fit by increasing the rotational velocity or, if the
Figure 8. Comparison of rotationally broadened profiles with isotropic microturbulence (solid lines) and radial-tangential microturbulence (dotted lines). The top graph is for Fe i at 666.3 nm, the bottom graph for Ca i at 643.9 nm. Here, the radial and tangential microturbulence values are 1.5 and 2.5 km s\(^{-1}\) respectively. The isotropic microturbulence value is 2 km s\(^{-1}\).

equivalent width is normalized, the equivalent width of the line. As one does not in practice know the rotational velocity or the equivalent width of the line with absolute certainty, departures of, say, ±1 km s\(^{-1}\) in the rotational velocity will usually not arouse suspicion.

Apart from the small-scale microturbulent motions, the motion of large gas cells, 'macroturbulence', will also contribute to the broadening of the final disc-integrated profile. In contrast to microturbulence, macroturbulent motions do not affect the shape of the local line profile. Instead, the profile for each moving gas cell is Doppler shifted by an amount corresponding to the speed of the cell. Similar to rotation, macroturbulent motions therefore alter the shape of the disc-integrated profile, but they cannot alter the equivalent width of the profile or lead to any 'saturation' effects.

For most cool dwarfs, macroturbulence will not be an important contributor to the line broadening, but its effect increases with decreasing stellar gravity and increasing temperature. Whether it can be detected on a star depends strongly on the rotational velocity of the star. For fast rotators, Doppler shifts induced by macroturbulent motions will be largely swamped by the rotational Doppler shifts. Only in slower rotators, in particular in G- and earlier-type subgiants, does macroturbulence need to be accounted for (Gray 1988). The radial-tangential model for macroturbulence as suggested by Gray (1976) has been successfully applied to model the line profiles of slowly rotating stars (Toner & Gray 1988). It has also been included in a number of Doppler imaging studies (Saar, Piskunov & Tuominen 1992; Strassmeier et al. 1991 and references therein). We are, however, not aware of any systematic studies of its effect on the image reconstruction.

4.4 Blends

So far, we have looked at approximations that regard the limb dependence of the profiles. In this section we investigate the importance of the shape of the disc-integrated flux profile. We wanted to know how strong a close blend had to be in relation to the mapping line before it had to be taken into account either by using a template star profile or by spectral modelling.

Whether a blend can seriously affect the profile reconstruction depends on several parameters such as the rotational velocity of the star, the strength of the blend and its distance from the line centre. As Doppler shifts are proportional to the velocity, one would expect that close blends and asymmetric lines have a stronger effect at slow rotation speeds. At higher rotation rates, on the other hand, blending of neighbouring lines at considerable distances becomes a more important problem. If it is possible to account for blending sufficiently, the number of prospective Doppler imaging lines increases considerably.

We produced a series of lookup tables with two superimposed Gaussian profiles, varying the distance of the line centres and the relative strengths of the two lines. These lookup tables were used to generate synthetic data from which images were reconstructed using single Gaussian profiles.

We found that for close blends the rotation profile dominates so that the asymmetries are washed out. For \( v \sin i = 90 \text{ km s}^{-1} \) we had to move the second Gaussian to a distance of about 40 km s\(^{-1}\) before significant effects became noticeable. At \( v \sin i = 50 \text{ km s}^{-1} \) artefacts appeared when the blend was at a distance of approximately 15 km s\(^{-1}\). Fig. 10 shows three profiles with Gaussian blends at −30 km s\(^{-1}\), −60 km s\(^{-1}\) and −90 km s\(^{-1}\). These profiles were used to generate synthetic data for a star with \( v \sin i = 90 \text{ km s}^{-1} \) and an inclination of 60°. The rotationally broadened profiles for an unsptotted star are shown in Fig. 11. As the distance of the blend from the mapping line centre increases, the line centre of the rotationally broadened profile is shifted towards the blend and the continuum level at the blended side is lowered. Note that the equivalent width of the rotationally broadened line is artificially decreased for more distant blends as they will not fully contribute to the equivalent width and also lower the apparent continuum.

The apparent shift of the geocentric velocity of the star does not necessarily pose a problem in itself for the image reconstructions. If the shift is large, however, the signatures due to the spot will appear at different places.

When reconstructing images from the 'blended' data we observed that the reconstruction procedure had to emphasize particular latitudes on the star in order to fit the data. In very extreme cases this looked like a dark band. As the distance between the mapping line and the blend increased, the banding moved from low latitudes towards the pole. For a
projected rotational velocity of 90 km s$^{-1}$, $\chi^2$ was comparable to the goodness of fit obtained with the input lookup table only when the blends were closer than about $\pm 40$ km s$^{-1}$. For blends further out, we could not get complete convergence and had to allow higher values of $\chi^2$. The reconstructed images are shown in Fig. 12; note that the blend at $-90$ km s$^{-1}$ that intuitively might seem least important gives a very marked effect, mainly because the signatures in the blueshifted single Gaussian would seem to appear too early and need to be stretched redwards.

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Figure 10. Examples of blended Gaussian profiles. The blends are at $-30 \, \text{km s}^{-1}$ (solid line), $-60 \, \text{km s}^{-1}$ (dotted line) and $-90 \, \text{km s}^{-1}$ (dashed line). The dot-dashed line marks a single Gaussian.

Figure 11. The rotationally broadened Gaussian profiles from above. The rotational velocity was assumed to be $90 \, \text{km s}^{-1}$. As above, the dot-dashed line marks a single Gaussian. The solid line marks the blend at $-30 \, \text{km s}^{-1}$, the dotted line the blend at $-60 \, \text{km s}^{-1}$ and the dashed line the blend at $-90 \, \text{km s}^{-1}$.

5 DISCUSSION

We found that the reconstruction procedure is very stable against considerable errors in the line profile calculations and in the limb darkening coefficients. When reconstructing images from data using approximated lookup tables, we found that our images are closest to the images obtained with the input parameters and lookup tables when we adjust $v \sin i$ and the equivalent width of the line so that the spot area is minimized. One would expect that such an approach will introduce further artefacts, such as favouring high-latitude spots as they have smaller surface areas. In our experience this has not been the case, as we are using the minimum spot coverage not as a constraint, but merely in order to decide which picture to use finally. The main problem with this approach is that most of the rotationally invariant features will be removed or ignored. On the other hand, unless one knows the exact profile of the star to be mapped, which at the present is very difficult, these need to be treated with extreme care.

It is useful to distinguish between two different kinds of approximations or errors. Most of the errors, such as for example errors in the limb darkening coefficients, will affect the profile 'symmetrically'. The profile will become broader and flatter or narrower and deeper, but it remains symmetric. For high rotational velocities, in fact all velocities high enough for Doppler imaging to be used, the signatures due to these errors are very similar to the ones obtained when using slightly smaller or larger rotational velocities and can thus be suppressed. Some of the limb-dependent effects can in very extreme cases lead to artefacts such as dark rings or polar spots and crowns. It is also important to realize that different errors in the intensity profiles can lead to similar artefacts and hence reinforce each other.

More serious, however, are asymmetric errors such as the neglect of blends. These easily lead to artefacts as one is trying to reconstruct an intrinsically asymmetric profile with a symmetric one. We found that blends only need to be considered once they are at a certain distance from the mapping line. Very close blends are sufficiently smeared out in the rotationally broadened profile so that the line centre will be shifted slightly towards the blend. This will result in a difference in the overall velocity shift (the geocentric velocity) between unblended and blended lines, but the rotationally broadened line is still sufficiently symmetric so that we can use a single symmetric line for the reconstructions without forcing artefacts on to the stellar surface. In most of our tests where the equivalent width of the blend varied between 10 per cent and 50 per cent of the equivalent width of the mapping line, we found that the ratio of the distance between mapping line and blend (expressed in units of velocity) to the projected rotational velocity had to be of the order of 1/3 for artefacts to appear.

Our findings, that close blends will be dominated by the rotation profile and will in fact be undetectable in the reconstructed images, may be able to shed some light on the discussion of the chromospheric 'filling-in' of the line core. One could compare the effect of the core emission to the effect that would be produced if one had two equally strong close blends. As these cannot be detected in a sufficiently fast rotator, the chromospheric emission might in fact not be responsible for polar spots. In the case where one fears chromospheric pollution of the mapping line, it is advisable to compare the equivalent widths of the mapping line and other photospheric lines of the same species with the equivalent widths of a suitable comparison star.

We also noticed that a considerable number of artefacts will disappear earlier than the features that are actually present on the surface of the star when the $\chi^2$-criterion is relaxed.

6 CONCLUSIONS

For unblended lines and weak blends we found that Gaussian profiles yielded very good image reconstructions and were often indistinguishable from either the template star profiles or the spectral synthesis reconstructions. Nevertheless, the use of single Gaussian profiles restricts the applicability of Doppler imaging unnecessarily, especially at high rotational velocities where the number of unblended lines is
Figure 12. Artefacts due to the neglect of blends. (a) Reconstructed images from data produced with a blended Gaussian profile (blend at $-30$ km s$^{-1}$). The geocentric velocity had to be shifted to $-3.5$ km s$^{-1}$. (b) A blended Gaussian where the blend is now at $-60$ km s$^{-1}$. The geocentric velocity was shifted to $-5$ km s$^{-1}$. For pictures (c) and (d) the smaller Gaussian was at $-90$ km s$^{-1}$. (c) shows the comparison between the synthetic data produced with the blend (dots) and synthetic data that would be obtained with the same input image but using a single Gaussian with a geocentric velocity shift of $-5.5$ km s$^{-1}$ (histogram). It can be seen that, due to the high shift in the geocentric velocity, the 'humps' associated with the spots lag behind when a single Gaussian is used. (d) The final image when the data from (c) are fitted.
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Figure 12 - continued

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(d)

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Figure 12 - continued
rather limited. We find that spectral synthesis and template star lookup tables are far more flexible. Although the spectral synthesis approach is very elegant, there are still many uncertainties involved, such as the presence of molecules in the atmospheres of cool stars, and particularly in the spots. Most spectral synthesis approaches tend to neglect molecular absorption lines and also have to leave out large numbers of unidentified lines. We have shown that the neglect of blends – even relatively weak blends at considerable distances – introduces a large penalty in the reconstructions. For the particular lines we tested, the errors introduced by neglecting the variation of the line profile with limb angle were rather small, and the assumption of linear limb darkening laws did not lead to any noticeable artefacts. For stars where the surface inhomogeneities are due not to particular over- or under-abundances of chemical elements, but mainly to a temperature difference, we therefore think that template star lookup tables will yield the most consistent results.

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